UDK 624.21-21:624.042.8:528.482:004.92
Pregledni znanstveni članak

Modelling Behaviour of Bridge Pylon for Test Load Using Regression Analysis with Linear and Non-Linear Process

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ABSTRACT. This paper presents the procedure for dynamic system identification regarding behaviour of bridge pylon for test load, and the numeric example had been illustrated by examination of "Sloboda" bridge in Novi Sad. Since pylon shifts, occurred during test load, were long-period in nature, static GPS method had been applied for measurements. To identify dynamic process of the construction, auto-regression model with external input (ARX) had been selected. The process had been approximated as linear and non-linear. Establishing model degree had been performed by autocorrelation function and parameter significance test. It had shown that the shifts of pylon along the longitudinal axis of the bridge, occurring due to the load action, must be described as a result of non-linear process; while shifts, occurring orthogonal to longitudinal axis of the bridge, occurring due to the temperature change, are the result of linear process. Model fitting was also analyzed, observing the pylon as both rigid and deformable body. Higher percentage of fitting (alignment) had been achieved when the construction had been viewed as a deformable body.

Keywords: dynamic systems identification, regression analysis, linear and non-linear process, pylon.

1. Introduction

There is a common practice of describing dynamic processes of construction objects using natural laws. The construction is usually being observed as a linear, mechanical system with a certain degree of freedom, which is capable of linear movement only. Describing the process using natural laws requires detailed knowledge of the entire process, meaning knowledge of relations between input and output signals and reaction of object (transfer function).

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The most reliable models are the models obtained by measuring the particular construction or prototype. Within the systems theory, in the second half of 20th century, a system identification theory had been developing, covering procedures of establishing and verification of mathematical model of dynamic system, according to measurements. The goal of system identification is not establishing the "exact" model, instead, it is description of dynamic system or some characteristics therein, with the satisfactory accuracy. Differential or difference equations, establishing functional link between input and output signals; presented in systems theory using models of transfer functions or state-space.

Deformation analysis, performed by geodetic professionals, used to entail geometric interpretation only (determination of construction shape and dimensions, including shift vectors). Pioneer of physical interpretation of deformation processes of natural and man-made constructions, based on geodetic measurements, is Pelcer (1978). Intensive work of geodesists on these issues continues from mid-1990's. To present dynamic system, state-space model is commonly utilized, belonging to the group of linear models (Mastelić-Ivić and Kahmen 2001, Kuhlmann 2003, Eichhorn 2006). Some processes, however, cannot be described in precise enough manner, using linear approximation; instead, the process is to be described as non-linear (Kovačič et al. 2009).

GPS technology utilization for deformation analysis had started end-1990's (Ogaja 2002). This technology provides for observation of long-period changes, caused by pressure and temperature changes or slow tectonic processes. It also provides for observation of short-period changes, occurring due to the gusts, earthquakes or traffic. Static method is to be applied for long-period changes, having the accuracy similar to the terrestrial methods. The first experiments in observation of long-period deformation on dams using statistic method had been published by Hollmann and Velsch (1992) and DeLoach (1989).

In this paper the transfer function is determined for "Sloboda" bridge pylon in Novi Sad, based on geodetic measurements. Bridge pylon had been observed as a deformable dynamic system with multiple input and multiple output (MIMO – Multiple Input Multiple Output system). Input signals were force components in pylon cables and exterior temperature. Force components had been altered, due to bridge load, by driving trucks with known load. Output signals were pylon points coordinates (Y and X coordinates), measured during the experiment, which are function of load change and time. "Black box" modelling procedure had been applied, since model structure and parameter values were unknown. System was modelled using regression models, for the first case where the process is considered to be linear, and for the second case of non-linear process. Model degree was initially established by autocorrelation and cross-correlation of time series, and by parameter significance in second iteration. Parameter estimation had been performed using the least squares method.

2. Bridge Structure

"Sloboda" bridge had been constructed from 1976 to 1981 over the Danube River. It connects Novi Sad and Sremski Karlovci. Bridge structure was partially damaged in 1999, with the bridge being reconstructed during the period 2003–2005, followed by test load, whose data were used to model the behaviour of the pylon.

The bridge structure consists of:

- eight concrete piers with different thickness, two being on river banks and other in river bed;
- main bridge structure over the Danube River, with the total length being 591 m;
- two central metal pylons on each side of the main span;
- the approach composite structure, consisting of four spans towards Novi Sad and three spans towards Sremski Karlovci. Length of all segments is identical, being 60 m.

The main bridge structure is a cable-stayed structure with central metal pylons. Pylons are 60 m high and are embedded in the deck structures. Three cables link each pylon and structure. Three rows of cables are anchored at the heights: 36 m, 46 m and 56 m from the base of the pylon on the deck. The cables are anchored along the main span at: 54 m, 102 m and 150 m, measured from pylon axis; and along the side span at: 60 m, 90 m and 120 m from the pylon axis. Panoramic images of wider environment of “Sloboda” bridge are shown in Fig. 1, and graphical presentation of longitudinal section of the bridge is shown in Fig. 2.

Fig. 1. *Panoramic images of “Sloboda” bridge.*

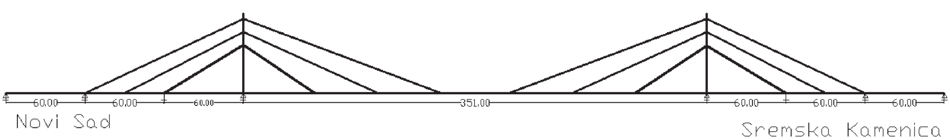

Fig. 2. *Longitudinal section of bridge with stabilization disposition.*

3. Program and Procedure of Testing

Bridge structure had been tested by trial static and dynamic loadings, according to the legislation in force in the Republic of Serbia in September 2005. Testing program had covered measurement of global and local deformations, including dynamic characteristics of the construction. This paper presents determination of Novi Sad bridge pylon shifts, categorized as global deformation, determined using geodetic methods.

3.1. Testing of Pylon Shifts

Testing of pylon top shifts with test load was performed using static GPS method. Designed geodetic control network consist of four points: one point on each bridge pylon and one point on both river banks of Danube River, as shown in network sketch (Fig. 3). Two Trimble 4600LS receivers had been placed on pylon tops, and one HIPER and LEGACY receiver on network points (geodetic pillars) on river banks of Danube River, towards Novi Sad and towards Sremski Karlovci. Measurement time and values of input signals for the pylon closer to Novi Sad are shown in Table 1.

Table 1. *Observations schedule with input parameter values.*

Epoch number	Time	Force in cables on:			Temperature °C	Load phase
		56 m	46 m	36 m		
		F56 [MN]	F46 [MN]	F36 [MN]		
1	6^h00^{min} – 6^h25^{min}	0	0	0	17	Unload
2	6^h25^{min} – 6^h50^{min}	0	0	0	17	Unload
3	6^h50^{min} – 7^h15^{min}	0	0	0	17	Unload
4	7^h15^{min} – 7^h50^{min}	–0.426	–0.414	1.137	18	1
5	7^h50^{min} – 8^h15^{min}	–0.426	–0.414	1.137	18	1
6	8^h15^{min} – 8^h40^{min}	–0.426	–0.414	1.137	19	1
7	8^h40^{min} – 9^h05^{min}	–0.426	–0.414	1.137	20	1
8	9^h15^{min} – 9^h40^{min}	0	0	0	20	Unload
9	9^h40^{min} – 10^h05^{min}	0	0	0	21	Unload
10	10^h20^{min} – 10^h45^{min}	–0.130	–0.647	0.951	22	2
11	10^h45^{min} – 11^h10^{min}	–0.130	–0.647	0.951	23	2
12	11^h15^{min} – 11^h40^{min}	0	0	0	24	Unload
13	11^h40^{min} – 12^h05^{min}	0	0	0	25	Unload
14	12^h05^{min} – 12^h30^{min}	1.318	–1.413	–0.413	27	3
15	12^h40^{min} – 13^h05^{min}	1.263	–0.759	–1.050	28	4
16	13^h15^{min} – 13^h40^{min}	0	0	0	28	5
17	13^h50^{min} – 14^h15^{min}	0	0	0	28	6
18	14^h15^{min} – 14^h40^{min}	0	0	0	28	7
19	14^h40^{min} – 15^h05^{min}	0	0	0	29	Unload
20	15^h05^{min} – 15^h30^{min}	0	0	0	29	Unload

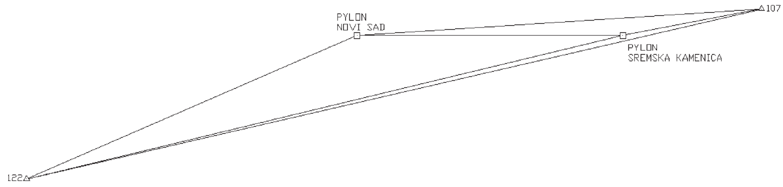

Fig. 3. *Geodetic control network.*

Trial load had been induced using 16 heavy trucks. Each loaded truck had had the approximate weight of 42 tons. Truck weights had been measured prior to testing. Intensity and truck position on the bridge (static load) had been calculate to cause 0.50–0.85% of maximum permitted structure strain. Straining using known load had induced the appropriate forces in the cables, with values being presented in Table 1. Static load induction had been performed in phases 1 through 7, with each phase having different load distribution and values (distribution of loaded trucks is shown in Fig. 4). Load phases 5, 6 and 7 did not cause forces in cables attached to Novi Sad pylon, as shown in Table 1. Said phases had been used for analysis of temperature impact on pylon shifts.

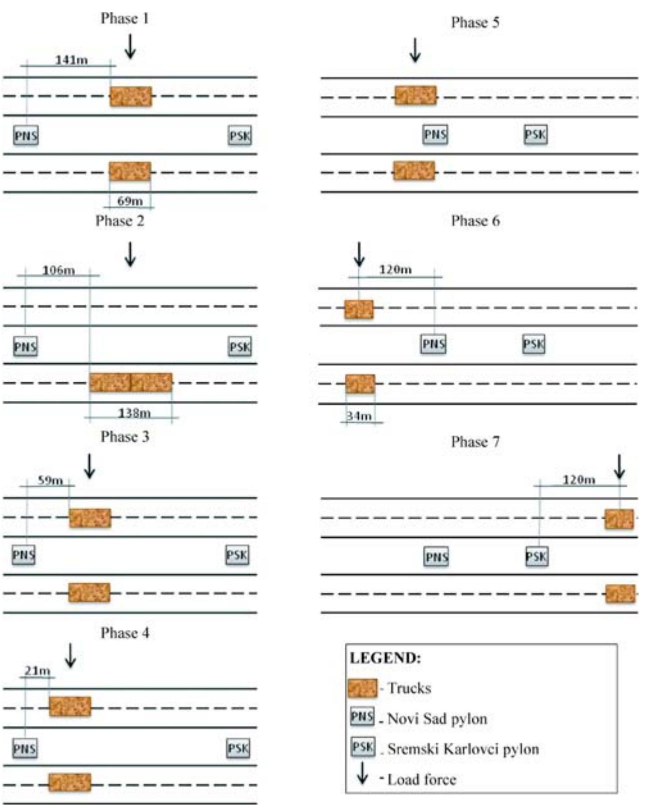

Fig. 4. *Load distribution per phases.*

Negative force value, as shown in Table 1, means that pylons are shifting towards each other, i.e. greater the stress of internal cables. The load carried by the following phases:

Phase 1: Bridge load had been distributed in both traffic lanes (8 heavy trucks in each, in bridge mid-section centre),

Phase 2: Asymmetrical load (all 16 heavy trucks in single traffic lane of bridge mid-section),

Phase 3: Load had consisted of 16 heavy trucks, distributed in both traffic lanes, 59 m away from Novi Sad pylon, towards bridge mid-section,

Phase 4: Load had consisted of 16 heavy trucks, distributed in both traffic lanes, 21 m away from Novi Sad pylon, towards bridge mid-section,

Phase 5: Load had consisted of 16 heavy trucks, distributed in both traffic lanes, next to Novi Sad pylon, towards the river bank,

Phase 6: Load had consisted of 16 heavy trucks, distributed in both traffic lanes, 120 m away from Novi Sad pylon, towards Novi Sad,

Phase 7: Load had consisted of 8 heavy trucks, distributed in both traffic lanes, 120 m away from Sremski Karlovci pylon, towards Sremski Karlovci.

Designed duration of measurement epoch was 25 minutes each, to obtain fixed solutions of GPS basevectors. This minimal duration of measurement epoch also provides assess the effect of temperature, since the input step signal (load) is identical for the subject phase.

After vector processing, providing the fixed solutions, 2D adjustment had been performed in local coordinate system, where Y-axis had been defined along the longitudinal axis of the bridge, and X-axis along Danube flow.

All measurement epochs (the total of 20) had been adjusted together. Coordinates of pylon tops had been estimated for each epoch separately, and coordinates of geodetic pillars on river banks were considered unchanged for the entire observations period, since the induced load does not affect geodetic pillars. Adjustment procedure applied is identical with Carlsrue method for identification of stabile points, where non-stabile points get different coordinates per epochs, and stabile points have identical coordinates in any given epoch. This procedure of adjustment produces:

- number of measured variables of 101, with 5 vectors for each measurement epoch (1 between pylon tops and 4 between vectors of river bank geodetic pillars and pylon tops) and 1 vector between geodetic pillars for the entire observation period (20 x 5 + 1);
- total number of adjusted points are 42, 20 for each pylon top and 2 for geodetic pillar points (2 x 20 + 2).

Estimation of standard deviation from adjustment for all points subject to adjustment is:

- 1.1 mm to 2.2 mm along Y-axis;
- 1.2 mm to 3.9 mm along X-axis.

4. System Identification

Goal of the structure monitoring is to provide safety and protection against natural and artificial disasters for capital constructions. Successful protection and maintenance of the structure is provided using system identification procedure. Based on system identification, checking of the assumptions of static budget from the project is done, together with the correction of construction standards for safer future construction.

The system identification allows the prediction of the behaviour of the object as a function of time and different values of input signals, thus preventing potential risks and humanitarian and material damages.

System identification requires adhering to the certain procedures:

- Data analysis and detrending;
- Check of process linearity or non-linearity;
- Defining model structure, including:
  - Selection of model types (transfer function or states-space);
  - Defining model degree and checking for system delay;
- Selection of evaluation algorithm;
- Model validation.

Described experiment test pylon behaviour, time series of Novi Sad pylon top coordinates data were de-trended by subtracting mean of the entire time series for each coordinate separately. Autocorrelation of output signals (Y and X coordinates) had been calculated with the results presented in Table 2. Cross-correlations of input signals (force components along Y-axis in cables anchored on the appropriate heights: 56 m – mark *F56*, 46 m – mark *F46*, and 36 m – mark *F36*; and temperatures – mark *tem*) had been calculated against output signals, also shown in Table 2. Autocorrelation and cross-correlation lag being outside the confidence interval (confidence interval used is 2σ) indicates model degree, i.e. how many previous values of input or output signals influences the current value of output signal. The table also shows, in *Lag* column, how many previous values of input or output signal influence the output signal current value, i.e. value of autocorrelation or cross-correlation coefficient for subject lag outside the confidence interval. For example, value *1* means that the current value of output signal depends on the previous value; value *2* means that it depends on two previous values, and note *none* indicates that there is no dependence.

Autocorrelation and cross-correlation lags indicate the following conclusions on model degree (sum of all autocorrelation and cross-correlation lags for the particular output signal):

- Force component on 56 m height has the greatest influence on pylon top shifts along Y-axis, being the closest one to the top, up to the lag 4, followed by force on 46 m height up to the step 2, while cable force on 36 m height has *no* influence;

Table 2. *Autocorrelation and cross-correlation of signals.*

Correlation	**Lag**
Auto Y coordinate	*None*
Auto X coordinate	*1*
Cross Y and X	*None*
Cross Y and F56	*4*
Cross Y and F46	*2*
Cross Y and F36	*None*
Cross Y and tem	*None*
Cross X and F56	*1*
Cross X and F46	*None*
Cross X and F36	*1*
Cross X and tem	*3*

- There is no autocorrelation of Y coordinates time series, which was expected, since shifts of the structure due to change of load happen swiftly (step response), thus cannot be detected using static method, being a short-period changes;
- The greatest effect on shifts along X-axes brings temperature, up to the lag 3.

There is autocorrelation within X coordinates time series with lag 1, since temperature change is long-period fluctuation, which could be detected using the method applied. Influence of force components on mounting height of 36 m was correlated with shifts along X-axis with lag 1.

4.1. Regression Analysis

Regression analysis is widely used for system identification. Identification techniques, based on regression using the least square method are applied on both linear and non-linear processes with simple input and simple output (SISO – Simple Input Simple Output), as well as processes with multiple input and multiple output (MIMO – Multiple Input Multiple Output).

During the period in which measurements are performed for the purpose of identification, it is assumed that the parameters being estimated are either stationary or quasi-stationary. This stationary period must be greater than m x T; with m – number of parameters estimated, and T – sampling.

General form of parameterized linear time invariant model (LTI – Linear Time Invariant) is defined as follows (Ljung 1987):

$$y(t) = G(q,\theta)u(t) + H(q,\theta)e(t), \tag{1}$$

with:

$$G(q) = \sum_{k=1}^{\infty} g(k)q^{-k} \text{ – linear system transfer function,}$$

$$H(q) = 1 + \sum_{k=1}^{\infty} h(k) q^{-k} \text{ – disturbance transfer function,}$$

$g(k)$ – impulse response, and

q^{-k} – backward shift operator ($\mathfrak{Z}$ transformation operator)

θ – parameter or parameter vector.

One-step-ahead prediction shall be:

$$\hat{y}(t|\theta) = H^{-1}(q,\theta)G(q,\theta)u(t) + [1 - H^{-1}(q,\theta)]y(t). \qquad (2)$$

The easiest way to parameterize transfer functions *G(q)* and *H(q)* is fractional rational function, with parameters being polynomial coefficients of numerator and denominator. This example applies model structure in the form of error equation, being an autoregressive model with external input (ARX – Autoregressive with eXternal input).

4.1.1. Linear dynamic system identification using regression analysis

Common form of linear difference equation is:

$$y(t) + a_1 y(t-1) + \ldots + a_{n_a}(t - n_a) = b_1 u(t - n_k) + \ldots + b_{n_b} u(t - n_k - n_b + 1) \qquad (3)$$

establishing the link between current output $y(t)$ and finite number of previous output signals $y(t-k)$, $k = 1, \ldots, n_a$ and previous input signals $u(t-k)$, $k = n_k, \ldots, n_b$; possibly with current input signal, if system delay being $n_k = 0$, and model degree being $n = n_a + n_b - n_k$.

To estimate parameters, first 15 epochs of measurement had been used, being approximately 70% of total time series, as proposed in theory, with the remaining 30% of data being used for model validation. Validation had established that there is no system delay, $\mathbf{n}_k = \mathbf{0}$, meaning that system response is instant. The least squares method was used for parameters estimation.

As a criterion of adequacy of the identified model it is assumed that each parameter has been assessed that a significant, i.e. for its value to be greater than double value of estimated standard deviation of that parameter. To validate different degree model, fitting criteria and residual analysis had been applied. Fitting criteria using simulation is obtained using the following equation:

$$Fit\,[\%] = \left[1 \frac{|y - \hat{y}|}{|y - \bar{y}|}\right] \cdot 100, \text{ with:} \qquad (4)$$

y – measured output,

$\hat{y}$ – simulated model output and

$\bar{y}$ – measured output average.

Residual analysis had been performed by verification of:

- randomness – autocorrelation function must be within the confidence interval, residuals are not correlated, and
- independence – residuals are not correlated with input signals.

Initial structure of ARX model had been defined according to the autocorrelation and cross-correlation analysis (see Table 2); however, checking parameter significance and model validation had established that not all parameters are significant. Based on this analysis, the conclusion was made that the most appropriate error equations in this case are shown with (5):

$$\begin{aligned} Y(t) &= b_{11}F56(t) + b_{12}F46(t) + b_{13}F56(t-1) \\ X(t) + a_{21}X(t-1) &= b_{21}F36(t) + b_{22}\, tem(t). \end{aligned} \tag{5}$$

Parameters estimations for the most appropriate linear model, with parameters standard deviation are shown in Table 3.

Table 3. *Parameters estimation of linear model with standard deviations.*

Parameter	**Parameter estimation**	**Parameter standard deviation**
b_{11}	$-80.38\left[\frac{mm}{MN}\right]$	$9.88\left[\frac{mm}{MN}\right]$
b_{12}	$-117.41\left[\frac{mm}{MN}\right]$	$9.26\left[\frac{mm}{MN}\right]$
b_{13}	$33.07\left[\frac{mm}{MN}\right]$	$10.35\left[\frac{mm}{MN}\right]$
a_{21}	–0.35	0.12
b_{21}	$-11.90\left[\frac{mm}{MN}\right]$	$4.34\left[\frac{mm}{MN}\right]$
b_{22}	$6.76\left[\frac{mm}{^\circ C}\right]$	$1.42\left[\frac{mm}{^\circ C}\right]$

Under the assumption of linear system, the appropriate transfer function for Y coordinates is:

$$A_Y(q) = 1$$

$$B_Y(q) = (-80.38 + 33.56\,q^{-1})F56 - 117.41F46 \tag{6}$$

$$G_Y(q,\theta) = \frac{B_Y(q)}{A_Y(q)}, \quad H(q,\theta) = \frac{1}{A(q)} \text{ [Ljung (1987)]}$$

and for X coordinates:

$$A_X(q) = (1 - 0.35\,q^{-1})X$$

$$B_X(q) = -11.90F36 + 6.76\,tem \tag{7}$$

$$G_X(q,\theta) = \frac{B_X(q)}{A_X(q)}, \quad H(q,\theta) = \frac{1}{A(q)} \text{ [Ljung (1987)]}$$

Linear model fitting graph is shown in Fig. 5.

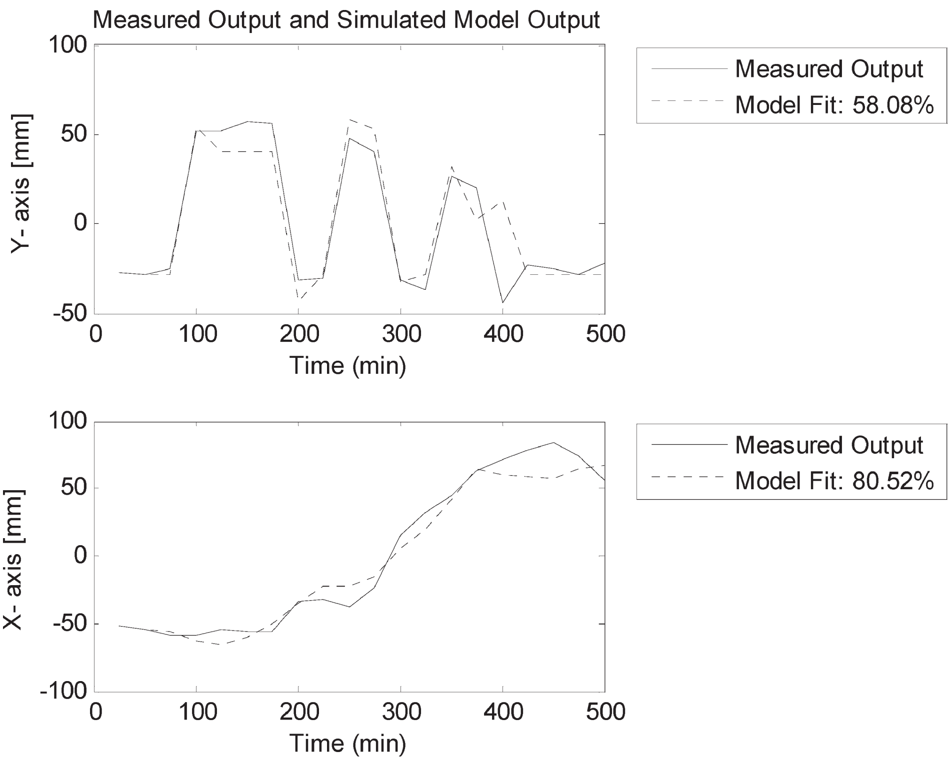

Fig. 5. *Linear regression model fitting graph.*

Model validation procedure had shown that the residuals were not correlated and are independent of input signals.

4.1.2. Non-linear dynamic system identification using regression analysis

Most of dynamic processes building structures have non-linear character. Approximation is being used when non-linear function is unknown, as in this case. The simplest form of approximation is polynomial.

For non-linear polynomial initial model, apart from input signals used for linear model, squares sum of forces acting on the cables were used, anchored at height 56 m – mark *F56_2*, 46 m – mark *F46_2*, and 36 m – mark *F36_2*. There are a total of seven input signals. To estimate parameters, first 15 epochs of measurement had been used, being approximately 70% of total time series, as proposed in theory, with the remaining 30% of data being used for model validation. Validation had established that there is no system delay, $\mathbf{n}_k = \mathbf{0}$, meaning that system response is instant. The least squares method was used for parameters evaluation.

Checking parameter significance and model validation had established that not all parameters are significant. Based on this analysis, the conclusion was made that the most appropriate error equations in this case are shown with (8):

$$\begin{aligned} Y(t) &= b_{11}F56(t) + b_{12}F46(t) + b_{13}\,tem(t) + b_{14}F56\_2(t) + b_{15}F46\_2(t) \\ X(t) + a_{21}X(t-1) &= b_{21}F36(t) + b_{22}\,tem(t). \end{aligned} \tag{8}$$

Parameters estimations for the most appropriate non-linear model, with parameters standard deviation are shown in Table 4.

Table 4. *Parameters estimation of linear model with standard deviations.*

Parameter	Parameter estimation	Parameter standard deviation
b_{11}	$-58.45\left[\frac{mm}{MN}\right]$	$4.00\left[\frac{mm}{MN}\right]$
b_{12}	$-115.09\left[\frac{mm}{MN}\right]$	$4.09\left[\frac{mm}{MN}\right]$
b_{13}	$-0.81\left[\frac{mm}{^\circ C}\right]$	$0.37\left[\frac{mm}{^\circ C}\right]$
b_{14}	$37.21\left[\frac{mm}{MN^2}\right]$	$3.47\left[\frac{mm}{MN^2}\right]$
b_{15}	$-67.23\left[\frac{mm}{MN^2}\right]$	$4.43\left[\frac{mm}{MN^2}\right]$
a_{21}	-0.35	0.13
b_{21}	$-11.90\left[\frac{mm}{MN}\right]$	$4.53\left[\frac{mm}{MN}\right]$
b_{22}	$6.76\left[\frac{mm}{^\circ C}\right]$	$1.49\left[\frac{mm}{^\circ C}\right]$

Under the assumption of non-linear system, the appropriate transfer function for Y coordinates is:

$$A_Y(q) = 1$$

$$B_Y(q) = -58.45F56 - 115.09F46 - 0.81\,tem + 37.21F56\_2 - 67.23F46\_2 \quad (9)$$

$$G_Y(q,\theta) = \frac{B_Y(q)}{A_Y(q)}, \quad H(q,\theta) = \frac{1}{A(q)} \text{ [Ljung (1987)]}$$

and for X coordinates:

$$A_X(q) = (1 - 0.35\,q^{-1})X$$

$$B_X(q) = -11.90F36 + 6.76\,tem \quad (10)$$

$$G_X(q,\theta) = \frac{B_X(q)}{A_X(q)}, \quad H(q,\theta) = \frac{1}{A(q)} \text{ [Ljung (1987)]}$$

Non-linear model fitting graph is shown in Fig. 6.

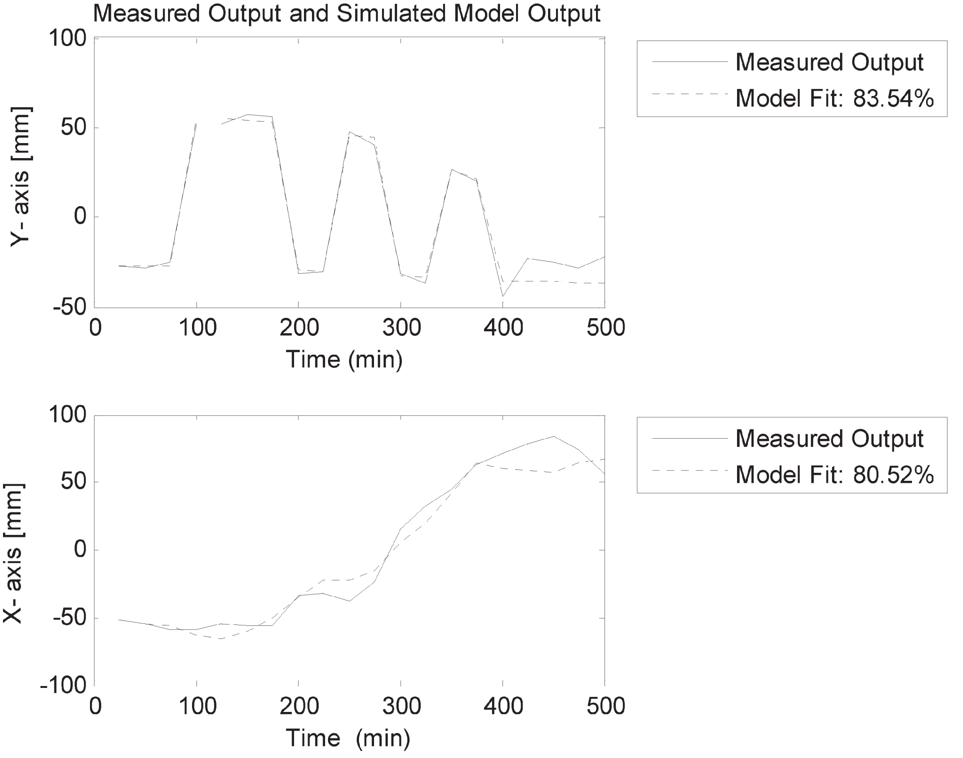

Fig. 6. *Non-linear regression model fitting graph.*

Model validation procedure had shown that the residuals were not correlated and are independent of input signals.

5. Conclusion

Monitoring of civil engineering construction is law-induced obligation, for the purpose of assuring safety and protection against natural and artificial disasters. The most reliable protection of constructions against unwanted consequences is timely modelling of construction behaviour, used to predict or simulate dynamic processed in time and against different values of input signals. The most appropriate model estimation is obtained through measurements on the very construction or prototype, subjected to the influence of the expected values of input signals. The entire procedure falls within the scope and theory of system identification.

This paper presents system identification methodology and procedures in detail, using transfer function. Using the example at hand, it is shown that the pylon behaviour along the longitudinal axis of the bridge is in line with non-linear laws, while shifts orthogonal to bridge axis are linear in nature. Fitting pylon behaviour along Y-axis when the process is observed as linear is 58%, which does not satisfy the model accuracy for practical application; while for the non-linear process, the fitting is 83.5%. Analysis of pylon behaviour along X-axis, by testing parameter significance, had established that the input signals, being square forces in cables, are negligible; rendering the process to be linear. The results shown indicate the conclusion that the pylon is to be considered deformable, instead of rigid body.

Further research in the future shall be based on comparison of various types of models: transfer function (shown in this paper), state-space (subspace method) and identification based on neuron networks. Field of state-space and transfer function both have physical foundations, providing for determination of parameter physical values. The approximation accuracy for dynamic processes using kinematic methods shall also be subject to further research.

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Modeliranje ponašanja pilona mosta na probno opterećenje primjenom regresijske analize kao linearan i nelinearan proces

SAŽETAK. U radu je opisan postupak identifikacije dinamičkog sustava ponašanja pilona mosta na probno opterećenje, a brojčani primjer ilustriran je ispitivanjem mosta "Slobode" u Novom Sadu. Kako su pomjeranja pilona pri probnom opterećenju dugoperiodična, za mjerenja je primijenjena statička GPS metoda. Za identifikaciju dinamičkog procesa konstrukcije izabran je autoregresijski model s vanjskim pobuđivanjem (AutoRegressive with eXternal input – ARX). Proces je aproksimiran kao linearan i nelinearan. Za utvrđivanje stupnja modela primijenjena je autokovarijacijska funkcija i test značajnosti parametara. Pokazano je da se pomjeranja pilona mosta, u smjeru duže osi, koja nastaju pod utjecajem opterećenja, moraju opisati kao rezultat nelinearnog procesa; dok su okomita pomjeranja, koja nastaju uslijed promjene temperature, rezultat linearnog procesa. Također je analizirano fitovanje modela (suglasnost) kada je pilon promatran kao kruto i kao deformatibilno tijelo. Ostvaren je veći postotak fitovanja kada je pilon promatran kao deformatibilno tijelo.

Ključne riječi: *identifikacija dinamičkih sustava, regresijska analiza, linearan i nelinearan proces, pilon.*

Prihvaćeno: 2011-08-22