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PROPAGATION OF NONLINEAR PHENOMENA IN A MEASUREMENT SEQUENCE

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ABSTRACT

Measurements provide one with results, in the form of both quantitative estimates of measured quantity along with attributed quantitative probabilistic analysis. Measurement is prescribed precisely in order to enable researchers, experts or other measurers to obtain maximum confidence in its results. In that way, the probability of obtaining unpredicted or unwanted consequences is minimised. Yet, owing to a rather large number of degrees of freedom in a typical measurement sequence, its nonlinear character and nonlinear couplings, in general it is not known in what amount a variation in measurement conditions brings about significantly larger variations in measured quantities or its derivatives.

In this article we treat in some details the aforementioned influence of variations and argue about possible results. In order to illustrate the treated influences we present results of a rather simple and common measurement of surface roughness of solid state objects. It is argued that there is no significant augmentation of variations in results of initial measurements throughout measurement sequence.

KEY WORDS

nonlinearity, measurement, complexity, roughness

CLASSIFICATION

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INTRODUCTION

Measurement, one of the cornerstones of modern science and technology, is a process conducted in a prescribed way. Prescriptions have been developed organically, during centuries of development. The purpose of prescriptions is to reduce the complexity and complicatedness of environment in which a measurement system is formed, i.e. measurement takes place. The reduction is of twofold character: (i) number of degrees of freedom of the environment, which influences final result, is minimised and (ii) quantitative estimate of residual penetrated environment complexity is relatively small.

In that way a measurement sequence[1] forms a part of the complement to complex systems. Complex systems are rather broad set of systems showing extremely large consequences of induced minute changes in its structure and/or dynamics [1]. That is in most cases traced back to existing nonlinear couplings among elements, i.e. the subsets of a complex system.

While on the one hand measurement sequence and complex systems share some similarities, their substantial difference is in the range of variation of the end result caused by minute changes in the environment, or intra-system variables.

In this article, we analyse in detail that topic. In particular, we start from the generic model of a measurement sequence and relate its element and their relations with elements and relation which one would encounter in a complex system. We apply that analysis onto a particular experiment with accompanied measurements having direct and significant practical importance.

Corresponding, generic model of a measurement sequence is developed in the second section. Reduced version of that generic model is formed in order to measure surface roughness, and its results are presented and analysed in the third section. Fourth section contains summary, conclusions and projections of further work.

GENERIC MODEL OF A MEASUREMENT SEQUENCE

Result of measurement is a set of values of a quantity attributed to a measured quantity, together with any other available relevant information [2]. A measurement result is generally expressed as a single measured quantity with a measurement uncertainty. If the measurement uncertainty is considered to be negligible for some purpose, the measurement result may be expressed as a single measured quantity value. In many fields, this is the common way of expressing a measurement result. However simple the definition may seem, it implies a thoroughly developed and structured context, which is nowadays covered by legislated industry standards or bodies having jurisdiction.

In particular, any referent quantity is established on the basis of consensus of a scientific community, following a large number of conducted experiments with unanimous interpretations. Along with thereby gathered experience, a referent quantity implies the existence of measuring equipment, Figure 1. Last but not least, referent quantity implies the development of scientific and technical thought which enabled all underlying activities and which requires the establishment of a referent quantity. In that way, a simple definition of a measurement implies interrelatedness of scientific, technical and social moments.

Emphasised relations among elements of a measurement sequence as shown in Figure 1 are in general of diverse amount, duration and sensitivity to variations. Furthermore, relations shown are only direct relations. Other, let us call them, indirect relations would in fact include all possible combinations of relations among listed elements. As an example, analysis & interpretation can bring conditions that influence, or change the very procedures, or measured quantity or equipment. Naturally, in order to understand whole set of relations, one needs to understand their implications and, in more general way, their meaning from the point of view of the environment to which measuring sequence belongs

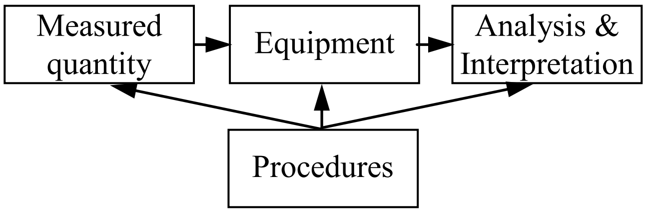

Figure 1. Measurement sequence starts from a procedure which prescribes choosing and preparation of measuring equipment, as well as of a body or process carrying measured quantity and subsequent analysis and reporting of results.

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In a larger system, measurement sequence is part serving as a source of reliable information about (quantitatively expressed) conditions of some emphasised part. In that sense, results of measurement sequence should be stable, and if possible linearly dependent on variations in initial or boundary conditions. Overall, variations in measurement sequence should not induce augmentation of variations' consequence in a larger system which exploits measurement results, or occurrence of emergence as a limiting case of augmentation.

Encountered notions of emergence and augmentation of initial variation are regularly utilised in the context of complex systems. Complex systems are systems consisting of nonlinear coupled elements which are characterised by significant sensitivity to small variations. In other words, they show augmentation of variations' consequences and emergent phenomena as its limiting form. It is interesting to note, while substantial to utilise and non-trivial to analyse that measurement sequence as a part of a larger, complex system should have suppressed essential characteristics of a complex system. Preliminary analysis of diverse measurement sequences reveals that, in each and every case, the suppression was achieved in a different manner, based on a detailed understanding of all important elements.

CASE STUDY: MEASUREMENT OF SURFACE ROUGHNESS

Surface roughness is the property of surface of any solid state object, which is of importance for predicting and optimising exploitation of that object. Surface roughness includes several quantitative parameters, which are all representations of a surface roughness profile [3]. Surface roughness profile[2] is a height of a particular point on a surface, measured orthogonally from surface determined as the averaged tangential surface. It can be positive or negative number.

Two of the parameters expressing quantitatively surface roughness profile are the following[3]: arithmetic average of the absolute values of surface's heights *Ra* and maximal vertical distance between any two surface points *Rz*. They both belong to the peak & valley group of parameters. They are determined for a 2D profile of a surface and are in recent years broadened to 3D S-parameters, determined for a scan of part of a surface. However, since R-parameters are in use for a much longer time than S-parameters their use in the context of this article is more appropriate. Nevertheless, similar analysis can be performed with S-parameters. We skip details of sampling of surface in order to obtain representative values of R-parameters.

Let us consider as a particular example of measuring surface roughness parameters the case in which a portion of a metal object's surface should be covered with a protective layer of dye [4]. Quantity of the dye which is needed has twofold economic consequences. On the one hand, the thicker the layer of dye coverage, the higher the cost of surface's protection. On the other hand, the thinner the layer of dye coverage, the higher the probability of surface's corrosion.

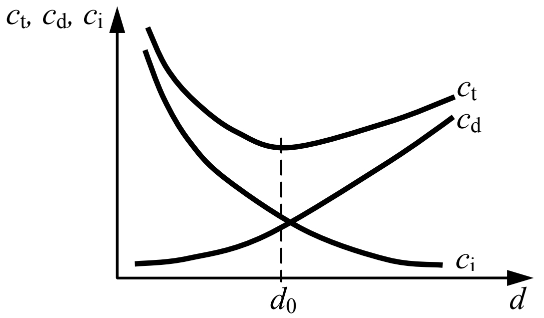

Figure 2. Qualitative representation of total (c_t), direct (c_d) and indirect (c_i) costs related to anti-corrosion protection of a surface, shown as a function of average thickness d of dye covering surface.

Since corrosion means degradation of the surface, its occurrence implies both insufficient thickness of dye coverage and also, relatively large costs for surface protection with added costs of degraded surface's repair. Let us call costs of dye and its covering as the *direct costs*. Then, let us call the *indirect costs* all costs occurring during corroded surface repair. Total costs of surface related processes are added direct and indirect costs. Qualitatively, situation is presented in Figure 2. Based on the previous considerations, one may introduce the function $c_t(d)$. In an optimization problem, its minima will bring about the thickness of dye coverage which minimises total costs related to surface coverage d_0.

However, thickness d is an averaged quantity. Because of the nonzero surface roughness, on some surface position with coordinates (x, y) the position-dependent thickness $d(x, y)$ will vary. That is implicitly included in the Figure 2 and accompanied considerations. Owing to some realistic distribution surface roughness profile, for a given d, in general there will be parts on the surface with uncovered surface[4]. Indirect costs c_i are nonzero for larger d precisely because some parts of the surface are still left uncovered. Let us denote with A total area of all parts of the surface which are left uncovered after the surface is covered with the dye of average thickness d. Then $A = A(d)$, which can be inverted to $d = d(A)$ and consequently $c_t = c_t(A)$. Function $d = d(A/S)$ is the usual Abbott-Firestone function for quantity of material on a surface [5].

Let us denote with z the surface height of a surface profile measured from some referent point. If the probability distribution function of surface height z is $p(z)$, then one has

$$A = S\int_{-\infty}^{d} p(z)\mathrm{d}z\,, \tag{1}$$

where S is the total surface area. Since z is the surface height of a surface profile measured from some referent point, it will always be finite, so lower integration point in (1) can be equivalently stated as some finite, otherwise arbitrary, quantity $z_{\min}$.

In case of rather randomly roughened surface, probability density function is the normal distribution $\mathrm{N}(\mu, \sigma^2)$ of expected value μ and standard deviation σ:

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]. \tag{2}$$

In (2), precise values of μ and σ are to be found numerically from experimentally obtained surface roughness profiles, such as the one given in Fig. 3. Substitution of (2) into (1) brings about the following expression [6]:

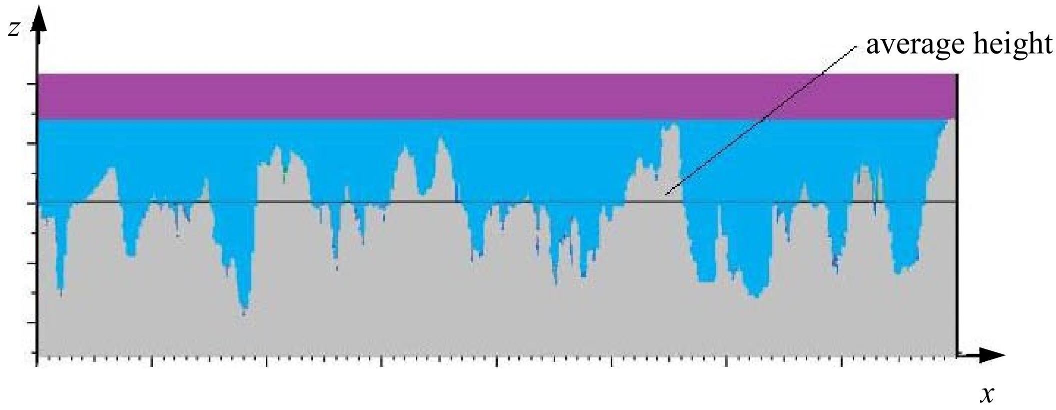

Figure 3. Typical example of surface roughness profile (grey) and dye coverage (blue and violet) [4]. Referent line form measuring surface heights is not shown for simplicity. Profile is obtained after scanning with stylus on some linear, otherwise arbitrary axis *x*.

$$A = \frac{S}{2}\left[1 + \mathrm{erf}\left(\frac{d + \mu}{\sigma\sqrt{2}}\right)\right], \tag{3}$$

so accompanied Abbott-Firestone function is:

$$d = \sigma\sqrt{2}\left[\mathrm{erf}^{-1}\left(2\frac{A}{S}\right) - 1\right] - \mu. \tag{4}$$

In deriving (3), the assumption $d + \mu > 0$ were used. Otherwise, instead of (3) one obtains a form including Heaviside function, which we consider being not of a minor correction since practically important range of values has $d + \mu > 0$. For semi-qualitative considerations, and based on Figure 2, dependence of total costs on average thickness of dye coverage d can be modelled as

$$c_t = K \cdot \exp(-\alpha d) + \beta d. \tag{5}$$

Interpretation of constants in (5) is that K is the total cost of substitution of corroded surface with new one, while α and β are parameters denoting the dimensional equivalent of corrosion influence and rise of direct costs for a unit change of thickness, respectively. Implicit assumption in (5) is that area of the total surface S is relatively large so that some constant contribution to direct cost is negligible in the range of interesting thicknesses d.

In cases in which protective function of dye coverage is crucial, accompanied parameter α is relatively small, in the sense that minimum of total costs is shifted toward relatively larger values of d. However, in that range, shape of (4) shows relatively smooth growth for a decrease of A/S, which is faster than shift of d as described by (4). In that sense, one does not expect deviating dependence of any of the quantities c_t, d_0 or A/S onto one another, and similar considerations point to the fact that it is also valid for small variations in any of these parameters. Thus, analysed measurement, profiling of surface roughness using R-parameters, does not show some instabilities or significant augmentation of variation in initial parameters.

Similar analysis can be performed for S-parameters, and using numerical approach for any type of surface roughness profile, what would add to test of wide applicability of stated suppression of nonlinearity's propagation in that segment of measurements.

CONCLUSIONS

Measurement sequence has some elements of complex systems. In order to be useful, the measurement sequence should suppress nonlinear augmentation of variations in some of its elements onto final results and/or its derivatives. In the case of surface roughness profiling, on the semi-quantitative basis it is argued that such augmentation does not exist.

REMARKS

[1]In this article we utilise the notion measurement sequence for a subsystem belonging to conduction of a measurement of some quantity. We purposefully do not utilise the notion measurement system, as it is in general reserved for the totality of measured quantities, the examples of which are SI, CGS, MKS and other measurement systems.

[2]To be differentiated from surface primary profile and surface waviness profile.

[3]We denote R-parameters following the current valid standard. In older literature, based on previously valid standards, these parameters would be denoted with subscripts: R_a and R_z.

[4]Because of adhesion, initially the whole surface will be covered with dye, independently of the average thickness of coverage. However, afterwards coverage above and around the peaks in surface roughness profile will be degraded more rapidly, thus the probability that their coverage disappears is rather large. That eventual state is considered here.

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PROPAGACIJA NELINEARNIH POJAVA U MJERNOM NIZU

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SAŽETAK

Rezultat mjerenja je brojčana procjena mjerene veličine uz pridruženu kvantitativnu analizu vjerojatnosti. Mjerenja su propisane strukture upravo zato da omoguće istraživačima, stručnjacima i drugim mjeriteljima najveću moguću pouzdanost u rezultate. Na taj način, kao sljedeće svojstvo, vjerojatnost ostvarivanja nepredviđenih ili neželjenih posljedica je minimalna. Ipak, zbog relativno velikog broja stupnjeva slobode u tipičnom mjeriteljskom slijedu, njegovog nelinearnog karaktera i nelinearnih sprezanja, općenito nije poznato u kojem iznosu varijacija u uvjetima mjerenja dovodi do znatno veće varijacije u mjerenoj veličini ili njenim izvedenicama.

U ovom radu razmatramo potankosti navedenog utjecaja varijacija i diskutiramo o mogućim rezultatima. Kao ilustraciju razmatranih utjecaja prikazujemo rezultate koji se odnose na relativno jednostavno i uobičajeno mjerenje površinske hrapavosti objekata u čvrstom stanju. Pokazano je kako nema znatnih povećanja varijacija u početnim parametrima duž mjerniteljskog niza.

KLJUČNE RIJEČI

nelinearnost, mjerenje, kompleksnost, hrapavost