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SIGMA-NOTATION AND THE EQUIVALENCE OF P AND NP CLASSES

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Abstract: *The aim of this paper is to show that σ–notation, based on the positionality principle, doesn't just answer the question of equivalence of P and NP problem classes, but also represents the key to the solution of (some) more general problems from the domain of multivalued logics. However, the achievement of such an aim in its full scope is hardly possible within the limits of this paper. Therefore, the following plan shall be realized:*

Firstly, the logic algebra fragment necessary for the solution of the P and NP classes equivalence problem shall be expounded.

Secondly, the necessary symbols and definitions shall be introduced to show that the calculations in the domain of the multivalued logic can be directly executed within the framework of σ–notation.

Keywords: *σ-notation, P=NP problem, SAT problem, multivalued logics.*

1. INTRODUCTION

As it is known from [1], to prove the equivalence of the classes of problems mentioned above, it would suffice that some (any) of the NP–complete problems can be solved in time bounded by the polynomial of the dimension of the problem. Here we shall consider the satisfiability problem in its classical formulation [1], which requires the answer to the following question: is there a sequence of variable values, which makes a given statement in CNF true? If the answer is positive, the statement in CNF is called satisfiable, if it is negative — unsatisfiable (or contradictory); if, apart from the recognition of satisfiability, the showing of the associated sequence of the propositional variable values is also required, the problem is called a SAT problem. This particular problem is going to be considered in the course of further explanation.

In 1971 S. Cook proved in [2] the SAT problem being NP–complete. In that way, proving that some (any) of the SAT class problems can be solved in time bounded by a polynomial of the dimension of the problem became sufficient for the proof of the equivalence of P and NP problem classes.

The SAT problem shall be solved by continous realization of conjuctions over disjuncts with the help of some special notation ([3]), whose necessary elements are described below.

2. SOME SYMBOLS AND DEFINITIONS

Beside the binary number system Zh we shall use quaternary Vh, octal Ah and hexadecimal Sh number system. In order to avoid repeated stating of the number system the digits are written in, the symbol θ shall be used to display a binary zero, a $\bar{\theta}$ to display a binary one. We are also introducing the following symbols

$$\begin{aligned}
Zh &\rightleftharpoons \{\theta,\, \bar{\theta}\},\\
Vh &\rightleftharpoons \{\nu,\, \tau,\, \bar{\tau},\, \bar{\nu}\} \rightleftharpoons \{\theta\,\theta,\, \bar{\theta}\,\theta,\, \theta\,\bar{\theta},\, \bar{\theta}\,\bar{\theta}\},\\
Ah &\rightleftharpoons \{\omega,\, \downarrow,\, \oplus,\, /,\, \bar{/},\, \bar{\oplus},\, \bar{\downarrow},\, \bar{\omega}\} \rightleftharpoons \{\theta\,\nu,\, \bar{\theta}\,\nu,\, \theta\,\tau,\, \bar{\theta}\,\tau,\, \theta\,\bar{\tau},\, \bar{\theta}\,\bar{\tau},\, \theta\,\bar{\nu},\, \bar{\theta}\,\bar{\nu}\},\\
Sh &\rightleftharpoons \{0,\, 1,\, 2,\, 3,\, 4,\, 5,\, 6,\, 7,\, \bar{7},\, \bar{6},\, \bar{5},\, \bar{4},\, \bar{3},\, \bar{2},\, \bar{1},\, \bar{0}\} \rightleftharpoons\\
&\rightleftharpoons \{\theta\omega, \bar{\theta}\omega, \theta\downarrow, \bar{\theta}\downarrow, \theta\oplus, \bar{\theta}\oplus, \theta/, \bar{\theta}/, \theta\bar{/}, \bar{\theta}\bar{/}, \theta\bar{\oplus}, \bar{\theta}\bar{\oplus}, \theta\bar{\downarrow}, \bar{\theta}\bar{\downarrow}, \theta\bar{\omega}, \bar{\theta}\bar{\omega}\},
\end{aligned}$$

respecting the convention that a pair of literals–digits in the second pair of curly brackets defines the corresponding literal-digit in the first pair of curly brackets.

Literals-digits from Zh, Vh and Sh shall be used for writing binary vectors, which shall be written in double angle brackets, a notation implicitly including the concatenation of literals-digits. For example, according to these principles, the following holds: $\langle\langle\bar{\theta}\,\theta\,\theta\,\theta\,\theta\,\bar{\theta}\,\bar{\theta}\,\theta\rangle\rangle = \langle\langle\tau\,\nu\,\bar{\tau}\,\tau\rangle\rangle = \langle\langle 16\rangle\rangle$.

Double angle brackets are used for writing the vectors of the length 2^n (that is, of rank n), which are called finite σ–operators. At this the following convention shall be respected: if $\langle\langle\alpha\rangle\rangle$— is some vector of rank k (length 2^k), then the notation $\langle\langle\alpha_j\rangle\rangle$ denotes that each coordinate of the vector

$\langle\langle\alpha\rangle\rangle$ appears j times, and therefore vector $\langle\langle\alpha_j\rangle\rangle$ has the rank $j+k$. The notation $\langle\langle\alpha^i\rangle\rangle$ denotes that the vector $\langle\langle\alpha\rangle\rangle$ appears 2^i times in concatenation. In that way, the notation $\langle\langle\alpha^i_j\rangle\rangle$ — denotes a vector of the rank $k+j+i$, under the condition that $\langle\langle\alpha\rangle\rangle$ — is a vector of rank k.

We would like to draw attention to the fact that the double angle brackets point to the coordinates appearing in the exponential ammount, and also to the fact that the upper, as well as the lower indices of a vector (or/and its coordinates) point to the exponential character of the appearance, with the indices implied (if not given, they equal zero).

Literals-digits from Ah are used for denoting the s–operator (see [3], pp. 25 and 26), but shall here be used only as $\bar{/}$ and $\bar{\downarrow}$ for denoting the bitwise conjuntion and disjunction respectively.

In order to give the SAT problem a table form, we shall write disjuncts in the rows of a table, so that the number of columns would equal the number of variables, and the number of rows the number of disjuncts. The table shall be filled in the following way: The literal with the smallest index i in the given disjunct we shall associate with $\bar{\tau}_{i-1}$ if it equals x_i, and τ_{i-1} if it equals $\bar{x}_i$. This we shall enter in the $(i-1)$–th column. In the other columns, which appear on the right side of the $(i-1)$–th column we shall enter T in the j–th column, if x_{j+1} (T^* for the $\bar{x}_{j+1}$) appears in the disjunct.

The explanation of this form of the SAT problem shall be given later, at the begin of paragraph 3.

In connection with the tables we shall use the following terms:*the length of the row* and *rank of the row*. The length of the row in the table with n columns 0, 1, 2, ... , $n-1$ and the beginning τ_{i-1} or $\bar{\tau}_{i-1}$ equals $n-i+1$, while the rank equals $n-i-k+1$, where k — is the sum of M productions (which are implied) in the row, i.e. the rank of the row equals the number of the literals appearing in the disjunct.

By permuting rows and columns of the table, a standard shape of the table is acheived, meaning that the columns are aligned from the left to the right, in order of nondecreasing values of the expression $E = E_T * E_{T^*}$, where E_T i E_{T^*} — represents the corresponding number of appearances of T (including τ) in each column. The rows in the table are aligned according to nondecreasing values of their length, the same condition applying to the blocks (and their subblocks) of the subtable with the rows of the same length, which means that the rows in the block (subblock) with the beginning in the k–th row are aligned according to nondecreasing of their length, with the exception of the k–th column.

3. NOTATIONS FOR THE SAT PROBLEM

The left part of the table 3 can serve as an example of stating the SAT problem by using generating rules from the set

$$Q_3 = \{T, T^*, M\},$$

because the effect of every generating rule from the set Q_3 on the given vector $\langle\alpha_j\rangle$, by definition transforms the same vector in the concatenation of two vectors, the result being the double value of the dimension of the initial vector.

$$\langle\alpha_j\, T\rangle = \langle\alpha_j\, \bar{\theta}_j\rangle, \quad \langle\alpha_j\, T^*\rangle = \langle\bar{\theta}_j\, \alpha_j\rangle, \quad \langle\alpha_j\, M\rangle = \langle\alpha_j\, \alpha_j\rangle. \tag{1}$$

Definitions (1) correspond to known definitions (see [3], p. 87).

If the values of logic algebra functions $f(X_j)$ on the variable row $X_j = (x_1, x_2, \ldots, x_j)$, can be represented by the vector $\langle\alpha_j\rangle$, that shall be represented in the following way: $f(X_j) \simeq \langle\alpha_j\rangle$.

Theorem 1 If the logic algebra function $f(X_j)$ is represented by the vector $\langle\alpha_j\rangle$, then the following functions, with the domain X_{j+1}, can be represented as following:

$$f(X_{j+1}) = f(X_j) \simeq \langle\alpha_j M\rangle, \quad f(X_j)\bar{\downarrow}x_{j+1} \simeq \langle\alpha_j T\rangle, \quad f(X_j)\bar{\downarrow}\bar{x}_{j+1} \simeq \langle\alpha_j T^*\rangle.$$

This theorem is a fragment of the theorem 14 (see [3], p. 90).

The creation of the SAT problem table in the production system Q_3 can now be considered explained and justified.

The application of the theorem 1 and equivalence (1) on each row of the SAT problem in the system Q_3 makes it possible to notate every disjunction with an σ–operator in the system Vh, which is shown in the subtable of the same table 3.

The translation of the σ–operator table from the Vh system in the Sh system is realized in a relatively simple manner. It should suffice to keep in mind that for the integers $i \geq 0$ and $j \geq 0$ the following equivalences apply:

$$\begin{array}{lll} \langle\langle\tau^i_{j+1}\rangle\rangle = \langle\langle 3^i_j\rangle\rangle, & \langle\langle\tau^{i+1}\rangle\rangle = \langle\langle 5^i\rangle\rangle, & \langle\langle\nu_{j+1}\rangle\rangle = \langle\langle 0_j\rangle\rangle, \\ \langle\langle\bar{\tau}^i_{j+1}\rangle\rangle = \langle\langle \bar{3}^i_j\rangle\rangle, & \langle\langle\bar{\tau}^{i+1}\rangle\rangle = \langle\langle \bar{5}^i\rangle\rangle, & \langle\langle\bar{\nu}_{j+1}\rangle\rangle = \langle\langle \bar{0}_j\rangle\rangle. \end{array} \tag{2}$$

If next to the equivalence (2) the following equivalences are considered

$$\begin{array}{ll} \langle\bar{\tau}_j\, T\rangle = \langle\langle\bar{1}_j\rangle\rangle, & \langle\tau_j\, T\rangle = \langle\langle\bar{2}_j\rangle\rangle, \\ \langle\bar{\tau}_j\, T^*\rangle = \langle\langle\bar{4}_j\rangle\rangle, & \langle\tau_j\, T^*\rangle = \langle\langle 7_j\rangle\rangle, \end{array}$$

then it is possible to immediately move from the table in the Q_3 production system to the table in the Sh production system. In whichever way this is carried out, the SAT problem table in the Sh system shall be represented by the subtable of the table 3.

4. CALCULUS FOR THE SAT PROBLEM

In [3] there is a detailed consideration of the logic calculus in the system Vh. To avoid repeating ourselves, here we shall concentrate exclusively on the calculus in the Sh system for the SAT problem. That means that we have to concentrate on the σ–operators in the Sh system.

At first it should be noticed that the table 1, which realizes the $\overline{/}$ conjuction over the pair of vectors (literals-digits) α and β from Sh (in the table 1 the angle brackets around literals-digits are left out) holds.

The table 1 is used while applying the following obvious lemma

Lemma 1 If $\alpha,\, \beta \in Sh$ and $\langle\langle\alpha\rangle\rangle \,\overline{/}\, \langle\langle\beta\rangle\rangle = \langle\langle\gamma\rangle\rangle$, then
$\langle\langle\alpha^j\rangle\rangle\overline{/}\langle\langle\beta^j\rangle\rangle = \; = \langle\langle\gamma^j\rangle\rangle$, $\quad\langle\langle\alpha_j\rangle\rangle\overline{/}\langle\langle\beta_j\rangle\rangle = \langle\langle\gamma_j\rangle\rangle$ for $j = 1,\, 2,\, 3,\, \ldots$.

Table 1

$\overline{/}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{6}$	$\bar{7}$	7	6	5	4	3	2	1			
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1		$\bar{7}$	$\bar{7}$
2	2	0	0	2	2	0	0	2	2	0	0	2	2		$\bar{6}$	$\bar{7}$	$\bar{6}$
3	2	1	0	3	2	1	0	3	2	1	0	3		$\bar{5}$	$\bar{7}$	$\bar{7}$	$\bar{5}$
4	4	4	4	0	0	0	0	4	4	4	4		$\bar{4}$	$\bar{5}$	$\bar{6}$	$\bar{7}$	$\bar{4}$
5	4	5	4	1	0	1	0	5	4	5		$\bar{3}$	$\bar{7}$	$\bar{7}$	$\bar{7}$	$\bar{7}$	$\bar{3}$
6	6	4	4	2	2	0	0	6	6		$\bar{2}$	$\bar{3}$	$\bar{6}$	$\bar{7}$	$\bar{6}$	$\bar{7}$	$\bar{2}$
7	6	5	4	3	2	1	0	7		$\bar{1}$	$\bar{3}$	$\bar{3}$	$\bar{5}$	$\bar{5}$	$\bar{7}$	$\bar{7}$	$\bar{1}$
										$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{6}$	$\bar{7}$	$\overline{/}$

In the lemmas which are going to be presented in further exposition, it is important to keep in mind (due to the use of the convention stated in paragraph 2) that the following equivalences (where $j \geq 0$ is an integer) always hold:

$$\begin{array}{llll} \langle\langle 0_{j+1}\rangle\rangle = \langle\langle 0_j 0_j\rangle\rangle, & \langle\langle 1_{j+1}\rangle\rangle = \langle\langle 3_j 0_j\rangle\rangle, & \langle\langle 2_{j+1}\rangle\rangle = \langle\langle \bar{3}_j 0_j\rangle\rangle, & \langle\langle 3_{j+1}\rangle\rangle = \langle\langle \bar{0}_j 0_j\rangle\rangle, \\ \langle\langle 4_{j+1}\rangle\rangle = \langle\langle 0_j 3_j\rangle\rangle, & \langle\langle 5_{j+1}\rangle\rangle = \langle\langle 3^1_j\rangle\rangle, & \langle\langle 6_{j+1}\rangle\rangle = \langle\langle \bar{3}_j 3_j\rangle\rangle, & \langle\langle 7_{j+1}\rangle\rangle = \langle\langle \bar{0}_j 3_j\rangle\rangle, \\ \langle\langle \bar{7}_{j+1}\rangle\rangle = \langle\langle 0_j \bar{3}_j\rangle\rangle, & \langle\langle \bar{6}_{j+1}\rangle\rangle = \langle\langle 3_j \bar{3}_j\rangle\rangle, & \langle\langle \bar{5}_{j+1}\rangle\rangle = \langle\langle \bar{3}^1_j\rangle\rangle, & \langle\langle \bar{4}_{j+1}\rangle\rangle = \langle\langle \bar{0}_j \bar{3}_j\rangle\rangle, \\ \langle\langle \bar{3}_{j+1}\rangle\rangle = \langle\langle 0_j \bar{0}_j\rangle\rangle, & \langle\langle \bar{2}_{j+1}\rangle\rangle = \langle\langle 3_j \bar{0}_j\rangle\rangle, & \langle\langle \bar{1}_{j+1}\rangle\rangle = \langle\langle \bar{3}_j \bar{0}_j\rangle\rangle, & \langle\langle \bar{0}_{j+1}\rangle\rangle = \langle\langle \bar{0}_j \bar{0}_j\rangle\rangle. \end{array}$$

Lemma 2 If $\varepsilon \in Sh$, then

$$\langle\langle 3_{j+1}\rangle\rangle \,\overline{/}\, \langle\langle\varepsilon^{j+1}\rangle\rangle = \langle\langle\varepsilon^j\, 0_j\rangle\rangle, \quad \langle\langle\bar{3}_{j+1}\rangle\rangle \,\overline{/}\, \langle\langle\varepsilon^{j+1}\rangle\rangle = \langle\langle 0_j\, \varepsilon^j\rangle\rangle.$$

Table 2

$\overline{/}$	1^1	2^1	3^1	4^1	5^1	6^1	7^1	$\bar{7}^1$	$\bar{6}^1$	$\bar{5}^1$	$\bar{4}^1$	$\bar{3}^1$	$\bar{2}^1$	$\bar{1}^1$
1_1	10	20	1_1	0_1	10	20	1_1	0_1	10	20	1_1	0_1	10	20
2_1	0_1	0_1	0_1	40	40	40	40	70	70	70	70	2_1	2_1	2_1
3_1	10	20	1_1	40	50	60	70	70	$\bar{6}0$	$\bar{5}0$	$\bar{4}0$	2_1	$\bar{2}0$	$\bar{1}0$
4_1	01	02	4_1	0_1	01	02	4_1	0_1	01	02	4_1	0_1	01	02
5_1	1^1	2^1	3^1	0_1	1^1	2^1	3^1	0_1	1^1	2^1	3^1	0_1	1^1	2^1
6_1	01	02	4_1	40	41	42	43	70	71	72	73	2_1	$\bar{3}1$	$\bar{3}2$
7_1	1^1	2^1	3^1	40	51	62	73	70	$\bar{6}1$	$\bar{5}2$	$\bar{4}3$	2_1	$\bar{2}1$	$\bar{1}2$
$\bar{7}_1$	0_1	0_1	0_1	04	04	04	04	$0\bar{7}$	$0\bar{7}$	$0\bar{7}$	$0\bar{7}$	$\bar{7}_1$	$\bar{7}_1$	$\bar{7}_1$
$\bar{6}_1$	10	20	1_1	04	14	24	34	$0\bar{7}$	$1\bar{7}$	$2\bar{7}$	$3\bar{7}$	$\bar{7}_1$	$1\bar{3}$	$2\bar{3}$
$\bar{5}_1$	0_1	0_1	0_1	4^1	4^1	4^1	4^1	$\bar{7}^1$	$\bar{7}^1$	$\bar{7}^1$	$\bar{7}^1$	$\bar{3}^1$	$\bar{3}^1$	$\bar{3}^1$
$\bar{4}_1$	10	20	1_1	4^1	54	64	74	$\bar{7}^1$	$\bar{6}\bar{7}$	$\bar{5}\bar{7}$	$\bar{4}\bar{7}$	$\bar{3}^1$	$\bar{2}\bar{3}$	$\bar{1}\bar{3}$
$\bar{3}_1$	01	02	4_1	04	05	06	07	$0\bar{7}$	$0\bar{6}$	$0\bar{5}$	$0\bar{4}$	$\bar{7}_1$	$0\bar{2}$	$0\bar{1}$
$\bar{2}_1$	1^1	2^1	3^1	04	15	26	37	$0\bar{7}$	$1\bar{6}$	$2\bar{5}$	$3\bar{4}$	$\bar{7}_1$	$1\bar{2}$	$2\bar{1}$
$\bar{1}_1$	01	02	4_1	4^1	45	46	47	$\bar{7}^1$	$\bar{7}\bar{6}$	$\bar{7}\bar{5}$	$\bar{7}\bar{4}$	$\bar{3}^1$	$\bar{3}\bar{2}$	$\bar{3}\bar{1}$

Lemma 3 If $\varepsilon \in Sh$ and $\langle\langle 3\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon\rangle\rangle = \langle\langle \eta\rangle\rangle$, $\langle\langle \bar{3}\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon\rangle\rangle = \langle\langle \mu\rangle\rangle$, then

$$\begin{array}{l}
\langle\langle 1_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \eta\, 0\rangle\rangle,\ \langle\langle \bar{1}_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \mu\, \varepsilon\rangle\rangle,\\
\langle\langle 2_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \mu\, 0\rangle\rangle,\ \langle\langle \bar{2}_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \eta\, \varepsilon\rangle\rangle,\\
\langle\langle 4_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle 0\, \eta\rangle\rangle,\ \langle\langle \bar{4}_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \varepsilon\, \mu\rangle\rangle,\\
\langle\langle 5_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \eta^1\rangle\rangle,\ \langle\langle \bar{5}_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \mu^1\rangle\rangle,\\
\langle\langle 6_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \mu\, \eta\rangle\rangle,\ \langle\langle \bar{6}_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \eta\, \mu\rangle\rangle,\\
\langle\langle 7_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle \varepsilon\, \eta\rangle\rangle,\ \langle\langle \bar{7}_1\rangle\rangle \,\overline{/}\, \langle\langle \varepsilon^1\rangle\rangle = \langle\langle 0\, \mu\rangle\rangle.
\end{array}$$

Table 3

$\overline{/}$	0	1	2	3	4	5	σ–oper. in Vh	σ–oper. in Sh	Resulting conjunction
1				$\bar{\tau}$	T^*	T	$\langle\langle\bar{\nu}_3\,\bar{\tau}_3\,\bar{\nu}_4\rangle\rangle$	$\langle\langle\bar{4}_3\,\bar{0}_3\rangle\rangle$	
2				$\bar{\tau}$	T	T^*	$\langle\langle\bar{\nu}_4\,\bar{\tau}_3\,\bar{\nu}_3\rangle\rangle$	$\langle\langle\bar{0}_3\,\bar{1}_3\rangle\rangle$	$\langle\langle\bar{4}_3\,\bar{1}_3\rangle\rangle$
3			$\bar{\tau}$		T	T	$\langle\langle\bar{\tau}_2^1\,\bar{\nu}_3\,\bar{\nu}_4\rangle\rangle$	$\langle\langle\bar{3}_1^1\,\bar{0}_2\,\bar{0}_3\rangle\rangle$	$\langle\langle\bar{3}_1^1\,\bar{3}_2\,\bar{1}_3\rangle\rangle$
4			τ	T^*		T	$\langle\langle(\bar{\nu}_2\,\tau_2)^1\,\bar{\nu}_4\rangle\rangle$	$\langle\langle 7_2^1\,\bar{0}_3\rangle\rangle$	$\langle\langle 2_2\,4_2\,\bar{1}_3\rangle\rangle$
5			$\bar{\tau}$	T		T^*	$\langle\langle\bar{\nu}_4\,(\bar{\tau}_2\,\bar{\nu}_2)^1\rangle\rangle$	$\langle\langle\bar{0}_3\,\bar{1}_2^1\rangle\rangle$	$\langle\langle 2_2\,4_2\,\bar{3}_2\,\bar{1}_2\rangle\rangle$
6			τ	T	T^*		$\langle\langle(\bar{\nu}_3\,\tau_2\,\bar{\nu}_2)^1\rangle\rangle$	$\langle\langle(\bar{0}_2\,\bar{2}_2)^1\rangle\rangle$	$\langle\langle 2_2\,4_2\,\bar{3}_2^1\rangle\rangle$
7			$\bar{\tau}$	T	T	T	$\langle\langle\bar{\tau}_2\,\bar{\nu}_2\,\bar{\nu}_3\,\bar{\nu}_4\rangle\rangle$	$\langle\langle\bar{1}_2\,\bar{0}_2\,\bar{0}_3\rangle\rangle$	$\langle\langle 2_2\,4_2\,\bar{3}_2^1\rangle\rangle$
8		$\bar{\tau}$			T^*	T^*	$\langle\langle\bar{\nu}_4\,\bar{\nu}_3\,\bar{\tau}_1^2\rangle\rangle$	$\langle\langle\bar{0}_3\,\bar{0}_2\,\bar{3}^2\rangle\rangle$	$\langle\langle 2_2\,4_2\,\bar{3}_2\,0_1\,\bar{3}^1\rangle\rangle$
9		$\bar{\tau}$			T^*	T	$\langle\langle\bar{\nu}_3\,\bar{\tau}_1^2\,\bar{\nu}_4\rangle\rangle$	$\langle\langle\bar{0}_2\,\bar{3}^2\,\bar{0}_3\rangle\rangle$	$\langle\langle 2_2\,0_1\,2_1\,\bar{3}_2\,0_1\,\bar{3}^1\rangle\rangle$
10		τ			T^*	T^*	$\langle\langle\bar{\nu}_4\,\bar{\nu}_3\,\bar{\tau}_1^2\rangle\rangle$	$\langle\langle\bar{0}_3\,\bar{0}_2\,3^2\rangle\rangle$	$\langle\langle 2_2\,0_1\,2_1\,\bar{3}_2\,0_2\rangle\rangle$
11		τ		T^*		T	$\langle\langle(\bar{\nu}_2\,\tau_1^1)^1\,\bar{\nu}_4\rangle\rangle$	$\langle\langle(\bar{0}_1\,3^1)^1\,\bar{0}_3\rangle\rangle$	$\langle\langle 2_2\,0_2\,\bar{3}_2\,0_2\rangle\rangle$
12		τ		T	T		$\langle\langle(\tau_1^1\,\bar{\nu}_2\,\bar{\nu}_3)^1\rangle\rangle$	$\langle\langle(3^1\,\bar{0}_1\,\bar{0}_2)^1\rangle\rangle$	$\langle\langle 4_1\,0_1\,0_2\,\bar{3}_2\,0_2\rangle\rangle$
13		τ	T^*			T^*	$\langle\langle\bar{\nu}_4\,(\bar{\nu}_1\,\bar{\tau}_1)^2\rangle\rangle$	$\langle\langle\bar{0}_3\,7_1^2\rangle\rangle$	$\langle\langle 4_1\,0_3\,7_1\,0_2\rangle\rangle$
14		$\bar{\tau}$	T		T		$\langle\langle((\bar{\tau}_1\,\bar{\nu}_1)^1\,\bar{\nu}_3)^1\rangle\rangle$	$\langle\langle(\bar{1}_1^1\,\bar{0}_2)^1\rangle\rangle$	$\langle\langle 4_1\,0_3\,6_1\,0_2\rangle\rangle$
15		$\bar{\tau}$	T^*	T^*			$\langle\langle(\bar{\nu}_2\,\bar{\nu}_1\,\bar{\tau}_1)^2\rangle\rangle$	$\langle\langle(\bar{0}_1\,\bar{4}_1)^2\rangle\rangle$	$\langle\langle 4_1\,0_3\,2_1\,0_2\rangle\rangle$
16	τ			T	T	T	$\langle\langle\tau^2\,\bar{\nu}_2\,\bar{\nu}_3\,\bar{\nu}_4\rangle\rangle$	$\langle\langle 5^1\,\bar{0}_1\,\bar{0}_2\,\bar{0}_3\rangle\rangle$	$\langle\langle 0\,1\,0_3\,2_1\,0_2\rangle\rangle$
17	τ		T^*		T^*	T^*	$\langle\langle\bar{\nu}_4\,\bar{\nu}_3\,(\bar{\nu}_1\,\tau^1)^1\rangle\rangle$	$\langle\langle\bar{0}_3\,\bar{0}_2\,(\bar{0}\,5)^1\rangle\rangle$	$\langle\langle\mathbf{0}\ \mathbf{1}\ \mathbf{0}_3\ \mathbf{2}_1\ \mathbf{0}_2\rangle\rangle$
18		τ	T			T^*	$\langle\langle\bar{\nu}_4\,(\tau_1\,\bar{\nu}_1)^2\rangle\rangle$	$\langle\langle\bar{0}_3\,\bar{2}_1^2\rangle\rangle$	$\langle\langle 0\,1\,0_3\,0_1\,0_2\rangle\rangle$
19	$\bar{\tau}$	T^*			T		$\langle\langle((\bar{\tau}\,\bar{\nu})^2\,\bar{\nu}_3)^1\rangle\rangle$	$\langle\langle(\bar{1}^2\,\bar{0}_2)^1\rangle\rangle$	$\langle\langle\ \mathbf{0}_4\rangle\rangle$

Lemma 4 If $\varepsilon \in Sh$, then

$$\begin{array}{llll}
\langle\langle 1_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle \varepsilon^{j}\, 0_j\, 0_{j+1}\rangle\rangle, & \langle\langle \bar{1}_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle 0_j\, \varepsilon^{j}\, \varepsilon^{j+1}\rangle\rangle, \\
\langle\langle 2_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle 0_j\, \varepsilon^{j}\, 0_{j+1}\rangle\rangle, & \langle\langle \bar{2}_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle \varepsilon^{j}\, 0_j\, \varepsilon^{j+1}\rangle\rangle, \\
\langle\langle 4_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle 0_{j+1}\, \varepsilon^{j}\, 0_j\rangle\rangle, & \langle\langle \bar{4}_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle \varepsilon^{j+1}\, 0_j\, \varepsilon^{j}\rangle\rangle, \\
\langle\langle 5_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle (\varepsilon^{j}\, 0_j)^1\rangle\rangle, & \langle\langle \bar{5}_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle (0_j\, \varepsilon^{j})^1\rangle\rangle, \\
\langle\langle 6_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle 0_j\, \varepsilon^{j+1}\, 0_j\rangle\rangle, & \langle\langle \bar{6}_{j+2}\rangle\rangle \,\bar{\underline{/}}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle \varepsilon^{j} 0_{j+1}\, \varepsilon^{j}\rangle\rangle, \\
\langle\langle 7_{j+2}\rangle\rangle \,\bar{/}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle \varepsilon^{j+1}\, \varepsilon^{j}\, 0_j\rangle\rangle, & \langle\langle \bar{7}_{j+2}\rangle\rangle \,\bar{/}\, \langle\langle \varepsilon^{j+2}\rangle\rangle = \langle\langle 0_{j+1}\, 0_j\, \varepsilon^{j}\rangle\rangle.
\end{array}$$

Table 4

	σ–definitions in pseudosymbols		
1	$\lambda_4^1 \rightleftharpoons \langle\langle \bar{4}_3\, \bar{0}_3\rangle\rangle$		
2	$\lambda_4^2 \rightleftharpoons \langle\langle \bar{0}_3\, \bar{1}_3\rangle\rangle$		
3	$\lambda_3^3 \rightleftharpoons \langle\langle \bar{3}_1^1\, \bar{0}_2\rangle\rangle$,	$\lambda_4^3 \rightleftharpoons \langle\langle \lambda_3^3\, \bar{0}_3\rangle\rangle$	
4	$\lambda_4^4 \rightleftharpoons \langle\langle 7_2^1\, \bar{0}_3\rangle\rangle$		
5	$\lambda_4^5 \rightleftharpoons \langle\langle \bar{0}_3\, \bar{1}_2^1\rangle\rangle$		
6	$\lambda_3^6 \rightleftharpoons \langle\langle \bar{0}_2\, \bar{2}_2\rangle\rangle$,	$\lambda_4^6 \rightleftharpoons \langle\langle (\lambda_3^6)^1\rangle\rangle$	
7	$\lambda_3^7 \rightleftharpoons \langle\langle \bar{1}_2\, \bar{0}_2\rangle\rangle$,	$\lambda_4^7 \rightleftharpoons \langle\langle \lambda_3^7\, \bar{0}_3\rangle\rangle$	
8	$\lambda_3^8 \rightleftharpoons \langle\langle \bar{0}_2\, \bar{3}^2\rangle\rangle$,	$\lambda_4^8 \rightleftharpoons \langle\langle \bar{0}_3\, \lambda_3^8\rangle\rangle$	
9	$\lambda_3^9 \rightleftharpoons \langle\langle \bar{0}_2\, \bar{3}^2\rangle\rangle$,	$\lambda_4^9 \rightleftharpoons \langle\langle \lambda_3^9\, \bar{0}_3\rangle\rangle$	
10	$\lambda_3^{10} \rightleftharpoons \langle\langle \bar{0}_2\, 3^2\rangle\rangle$,	$\lambda_4^{10} \rightleftharpoons \langle\langle \bar{0}_3\, \lambda_3^{10}\rangle\rangle$	
11	$\lambda_3^{11} \rightleftharpoons \langle\langle (\bar{0}_1\, 3^1)^1\rangle\rangle$,	$\lambda_4^{11} \rightleftharpoons \langle\langle \lambda_3^{11}\, \bar{0}_3\rangle\rangle$	
12	$\lambda_2^{12} \rightleftharpoons \langle\langle 3^1\, \bar{0}_1\rangle\rangle$,	$\lambda_3^{12} \rightleftharpoons \langle\langle \lambda_2^{12}\, \bar{0}_2\rangle\rangle$,	$\lambda_4^{12} \rightleftharpoons \langle\langle (\lambda_3^{12})^1\rangle\rangle$
13	$\lambda_4^{13} \rightleftharpoons \langle\langle \bar{0}_3\, 7_1^2\rangle\rangle$		
14	$\lambda_3^{14} \rightleftharpoons \langle\langle \bar{1}_1^1\, \bar{0}_2\rangle\rangle$,	$\lambda_4^{14} \rightleftharpoons \langle\langle (\lambda_3^{14})^1\rangle\rangle$	
15	$\lambda_2^{15} \rightleftharpoons \langle\langle \bar{0}_1\, \bar{4}_1\rangle\rangle$,	$\lambda_4^{15} \rightleftharpoons \langle\langle (\lambda_2^{15})^2\rangle\rangle$	
16	$\lambda_2^{16} \rightleftharpoons \langle\langle 5^1\, \bar{0}_1\rangle\rangle$,	$\lambda_3^{16} \rightleftharpoons \langle\langle \lambda_2^{16} \bar{0}_2\rangle\rangle$,	$\lambda_4^{16} \rightleftharpoons \langle\langle \lambda_3^{16} \bar{0}_3\rangle\rangle$
17	$\lambda_2^{17} \rightleftharpoons \langle\langle (\bar{0}5)^1\rangle\rangle$,	$\lambda_3^{17} \rightleftharpoons \langle\langle \bar{0}_2\, \lambda_2^{17}\rangle\rangle$,	$\lambda_4^{17} \rightleftharpoons \langle\langle \bar{0}_3\, \lambda_3^{17}\rangle\rangle$

Strictly speaking, the equivalences stated above suffice for the realization of all conjuctions over disjunctions, notated by σ–operators in Sh system, in the SAT problem. However, for the sake of convenience, we shall write out lemma 2 (for $j = 0$) and lemma 3 in the table 2 (the brackets are again implied).

Now we shall further expound the illustrated example from the table 3.

The results of conjuction, showed in the last column of the table 3, are acquired relatively simply, and gain a rather *"mechanical"* character, under the condition of introducing recursive definitions with the use of pseudosymbols λ_j^i, μ_j^i, η_j^i, ... for σ–operators in double angle brackets. We should pay attention to the fact that the meaning of the symbols is somewhat different from the one stated at the beginning of the paper: the lower index in the pseudosymbols points to the rank of the vector, the upper— only to the index of the vector, i.e. λ^i, μ^i, η^i are in a fact i–th symbols, but are all of the row j. The next to the last column of the table 3 is shown in the table 4, the first computations stating the following:

$$
\begin{array}{ll}
1) & \mu_4^1 = \lambda_4^1 \,\overline{/}\, \lambda_4^2 = \langle\langle \bar{4}_3\, \bar{0}_3 \rangle\rangle \,\overline{/}\, \langle\langle \bar{0}_3\, \bar{1}_3 \rangle\rangle = \langle\langle \bar{4}_3\, \bar{1}_3 \rangle\rangle; \\
2) & \mu_4^2 = \mu_4^1 \,\overline{/}\, \lambda_4^3 = \langle\langle \bar{4}_3\, \bar{1}_3 \rangle\rangle \,\overline{/}\, \langle\langle \lambda_3^3\, \bar{0}_3 \rangle\rangle = \langle\langle \mu_3^2\, \bar{1}_3 \rangle\rangle, \\
 & \mu_3^2 = \langle\langle \bar{4}_3 \rangle\rangle \,\overline{/}\, \langle\langle \lambda_3^3 \rangle\rangle = \langle\langle \bar{0}_2\, \bar{3}_2 \rangle\rangle \,\overline{/}\, \langle\langle \bar{3}_1^1\, \bar{0}_2 \rangle\rangle = \langle\langle \bar{3}_1^1\, \bar{3}_2 \rangle\rangle; \\
3) & \mu_4^3 = \mu_4^2 \,\overline{/}\, \lambda_4^4 = \langle\langle \mu_3^2\, \bar{1}_3 \rangle\rangle \,\overline{/}\, \langle\langle 7_2^1\, \bar{0}_3 \rangle\rangle = \langle\langle \mu_3^3\, \bar{1}_3 \rangle\rangle, \\
 & \mu_3^3 = \langle\langle \mu_3^2 \rangle\rangle \,\overline{/}\, \langle\langle 7_2^1 \rangle\rangle = \langle\langle \bar{3}_1^1\, \bar{3}_2 \rangle\rangle \,\overline{/}\, \langle\langle 7_2\, 7_2 \rangle\rangle = \langle\langle 2_2\, 4_2 \rangle\rangle; \\
4) & \mu_4^4 = \mu_4^3 \,\overline{/}\, \lambda_4^5 = \langle\langle \mu_3^3\, \bar{1}_3 \rangle\rangle \,\overline{/}\, \langle\langle \bar{0}_3\, \bar{1}_2^1 \rangle\rangle = \langle\langle \mu_3^3\, \eta_3^4 \rangle\rangle, \\
 & \eta_3^4 = \langle\langle \bar{1}_3 \rangle\rangle \,\overline{/}\, \langle\langle \bar{1}_2^1 \rangle\rangle = \langle\langle \bar{3}_2\, \bar{0}_2 \rangle\rangle \,\overline{/}\, \langle\langle \bar{1}_2\, \bar{1}_2 \rangle\rangle = \langle\langle \bar{3}_2\, \bar{1}_2 \rangle\rangle; \\
5) & \mu_4^5 = \mu_4^4 \,\overline{/}\, \lambda_4^6 = \langle\langle \mu_3^3\, \eta_3^4 \rangle\rangle \,\overline{/}\, \langle\langle (\lambda_3^6)^1 \rangle\rangle = \langle\langle \mu_3^5\, \eta_3^5 \rangle\rangle, \\
 & \mu_3^5 = \mu_3^3 \,\overline{/}\, \lambda_3^6 = \langle\langle 2_2\, 4_2 \rangle\rangle \,\overline{/}\, \langle\langle \bar{0}_2\, \bar{2}_2 \rangle\rangle = \langle\langle 2_2\, 4_2 \rangle\rangle, \\
 & \eta_3^5 = \langle\langle \eta_3^4 \rangle\rangle \,\overline{/}\, \langle\langle \lambda_3^6 \rangle\rangle = \langle\langle \bar{3}_2\, \bar{1}_2 \rangle\rangle \,\overline{/}\, \langle\langle \bar{0}_2\, \bar{2}_2 \rangle\rangle = \langle\langle \bar{3}_2\, \bar{3}_2 \rangle\rangle = \langle\langle \bar{3}_2^1 \rangle\rangle; \text{ and so on.}
\end{array}
$$

In the subtable 3 (the last column of the table) the results of conjuctions are shown (as the result of the application of the calculus explained above): in the i–th row the result of conjuctions over σ–operators (from the next to the last row) starting with the first and ending with the i–th. The σ–operator from the last column of the 17–th row (separated from the rest)) therefore presents the result of conjuction of all 17 operators of the second-last subtable. The separated operator implies the subtable, whose result it is, satisfiable, with the satisfiability achieved on the value rows of the variables with the base ten indices 4; 42; 42, i.e. the rows:

$$(001000), \quad (010101), \quad (110101). \tag{3}$$

We should notice that the table 3 (with the 19 rows shown in it) is contradictory, because the resulting σ–operator equals $\langle\langle 0_4 \rangle\rangle$.

In order to illustrate another important idea on this simple example, let us assume that in the each row of the resulting table a maximum of R (in our example $R = 4$) literals–digits can be entered (with or without indices), and that the problems in 3 should be solved with this limitation borne in mind. In that case the last subtable from the table 3 would look as the first subtable of the table 5 (let us notice that the rows would stay unchanged up to the eigth row). By looking at the 17–th row, we reach the conclusion (based on the beginning of the separated operator) of satisfiability confirmed by the row of the corresponding variable values (the first row in 3).

If table 3 contains all 19 rows, then in the 19–th row of the table 5 the result is gained, with which we are returning to the 8–th row to acquire the σ–operators shown in the second subtable of the table 4. The result of the 18–th row tells us about the problem being contradictory.

Table 5

N oper.	1	N oper.	2
8	$\langle\langle 2_2\, 4_2\, \ldots\rangle\rangle$	8, 9	$\langle\langle 0_3\, \bar{3}_2\, 0_1\, \bar{3}^1\rangle\rangle$
9, 10	$\langle\langle 2_2\, 0_1\, 2_1\, \ldots\rangle\rangle$	10, 11, 12	$\langle\langle 0_3\, \bar{3}_2\, 0_2\rangle\rangle$
11	$\langle\langle 2_2\, 0_2\, \ldots\rangle\rangle$	13	$\langle\langle 0_3\, 0_1\, 7_1\, 0_2\rangle\rangle$
12, 13, 14, 15	$\langle\langle 4_1\, 0_1\, 0_2\, \ldots\rangle\rangle$	14	$\langle\langle 0_3\, 0_1\, 6_1\, 0_2\rangle\rangle$
16, 17	$\langle\langle \mathbf{0}\ \mathbf{1}\ \mathbf{0}_1\ \mathbf{0}_2\, \ldots\rangle\rangle$	15, 16, 17	$\langle\langle 0_3\, 0_1\, 2_1\, 0_2\rangle\rangle$
18	$\langle\langle 0\, 1\, 0_1\, 0_2\, \ldots\rangle\rangle$	18	$\langle\langle \mathbf{0}_4\rangle\rangle$
19	$\langle\langle \mathbf{0}_3\rangle\rangle$		

Examinations and calculations illustrated on individual examples have a universal character, which makes us conclude that in the general case the text of the resulting σ–operator for the standardized table of the SAT problem cannot have exponential growth. Let us name every σ–operator literal–digit with or without indices, different from $\langle\langle 0_s\rangle\rangle$, for $s \geq 0$, significant component.

Then we shold look at the lemmas 1, 2 and 4 and the tables 1 and 2 (which are sufficient to calculate conjuctions) and notice the following: a conjuction over two significant components cannot generate more then two significant components. The components $\langle\langle 0_s\rangle\rangle$ are absorbtive, because of which their appearance cannot facilitate a strong text growth. The volume of the text doesn't even have to be estimated: it should be noticed that for the solution of the SAT problem the acceptance of the important idea stated below would be sufficient, with putting $R = 2n$ in the general case. In our example it would be necessary to put $R = 12$, but in that case there would be no return (surplus), which was something we wanted to illustrate. All this being said, the statement of the following conclusion becomes possible.

Theorem 2 The class of NP–complete problems is equivalent to the class P.

With this theorem we are bringing our fragment of the notation and logic calculus, oriented on the SAT problem, to an end. Let us notice that, in case of necessity of solving more general classes of problems from the logic algebra domain (such as direct calculations demanded by logic problems, without reducing them to standard representatives, then the Zegalkin systems, criptography problems, etc.) it would, without major effort, be possible to widen the calculations to encompass the whole operator system.

$$Op = \{\downarrow,\, \oplus,\, /,\, \bar{/},\, \bar{\oplus},\, \bar{\downarrow}\},$$

That shall, however, not be demonstrated in this paper. Instead we shall move on to multivalued logic and show how the positionality principle can also be applied on such cases.

5. SYMBOLS AND DEFINITIONS FOR MULTIVALUED LOGICS

Let $PL_{k \cdot m}$ — be a symbol for positional multivalued logic, for integers k and $m \geq 2$. If $k = m$, we write PL_k.

In $PL_{k \cdot m}$ let us assume that the arguments of the function $f(x_1, x_2, \ldots, x_n)$ belong to the set $E_k = \{0, 1, 2, \ldots, k-1\}$, and values to the set $G_m = \{0, 1, 2, \ldots, m-1\}$.

The function $f(x_1, x_2, \ldots, x_n)$ is completely stated if its table is stated, in which the rows of arguments are notated as representations of numbers $0, 1, 2, \ldots, k^n - 1$ in the k–ary number system, with increase of the digit position from left to the right (attention has to be payed to the latter, since it is not an usual notation). The symbol f is interpreted as mapping determined by the table, and the symbols $x_1, x_2, \ldots, x_n$ — as the names of the columns. Over the columns, as over the one variable functions, unary operations are carried out, given by the permutations from the symetric group S_k.

All that has been said presents a complete repetition of classic conventions, and the mentioned positionality begins with the representations of f symbols with the positional s– operators and column notations via σ–operators.

For $PL_{k \cdot m}$ q–ary s–operators are vectors of the dimension $q \cdot k - 1$ with the coordinates from the set G_m. The numeration of the vector coordinates from left to the right begins with index 0, whereby in the case of $m \leq 10$ it is not necessary to separate the coordinates with a comma. In all other cases comma is obligatory.

Example I. In the case of PL_2 such binary, nontrivial s–operators are operators from Op, the first three of them having the following form:

$$\downarrow = \langle 100 \rangle, \quad \oplus = \langle 010 \rangle, \quad / = \langle 110 \rangle.$$

As far as the next three s–operators are concerned, they emerge as a result of the application of the operation of unary inversion:
— $\rightleftharpoons$ (01):

$$\bar{/} = \langle 001 \rangle, \quad \bar{\oplus} = \langle 101 \rangle, \quad \bar{\downarrow} = \langle 011 \rangle,$$

which represents the transposition from the group S_2.

Example II. In the case of PL_3, binary nontrivial s–operators are operators from the next two sets:

$$\begin{aligned} O\acute{p} = \; & \{\langle 00001 \rangle, \; \langle 00010 \rangle, \; \langle 00011 \rangle, \; \langle 00101 \rangle, \; \langle 00110 \rangle, \\ & \langle 01001 \rangle, \; \langle 00100 \rangle, \; \langle 01010 \rangle, \; \langle 01110 \rangle\}, \end{aligned}$$

$$\begin{aligned} O\acute{\acute{p}} = \; & \{\langle 00012 \rangle, \; \langle 00102 \rangle, \; \langle 00112 \rangle, \; \langle 00120 \rangle, \; \langle 00121 \rangle, \\ & \langle 00122 \rangle, \; \langle 01002 \rangle, \; \langle 01012 \rangle, \; \langle 01020 \rangle, \; \langle 01021 \rangle \\ & \langle 01102 \rangle, \; \langle 01112 \rangle, \; \langle 01120 \rangle, \; \langle 01201 \rangle, \; \langle 01210 \rangle\}. \end{aligned}$$

All the other nontrivial binary s–operators from PL_3 emerge from the ones counted in $O\acute{p}$ and $O\acute{\acute{p}}$ as the result of the application of the following unary operations (permutations from the group S_3):

$$- \rightleftharpoons (01),\ \neg \rightleftharpoons (02),\ \bullet \rightleftharpoons (12),\ \rightarrow \rightleftharpoons (012),\ \leftarrow \rightleftharpoons (021) \tag{4}$$

and the operation of conjugation (see [3] p. 29): (For the given vector $\langle R_n\rangle = \langle r_1, r_2, \dots, r_n\rangle$ conjugated with them is the vector $\langle R^*\rangle = \langle r_n, \dots, r_2, r_1\rangle$.

If α is the symbol for the first vector from $O\acute{\acute{p}}$, we get:

$\alpha = \langle 00012\rangle$, $\bar{\alpha} = \langle 11102\rangle$, $\overset{\neg}{\alpha} = \langle 22210\rangle$, $\overset{\bullet}{\alpha} = \langle 00021\rangle$, $\overrightarrow{\alpha} = \langle 11120\rangle$, $\overleftarrow{\alpha} = \langle 22201\rangle$,

$\alpha^* = \langle 21000\rangle$, $\bar{\alpha}^* = \langle 20111\rangle$, $\overset{\neg}{\alpha}^* = \langle 01222\rangle$, $\overset{\bullet}{\alpha}^* = \langle 12000\rangle$, $\overrightarrow{\alpha}^* = \langle 02111\rangle$, $\overleftarrow{\alpha}^* = \langle 10222\rangle$.

Example III. In $PL_{2\cdot 3}$ binary nontrivial s–operators are vectors:

$$\pi = (012),\ \bar{\pi} = (102),\ \overset{\neg}{\pi} = (210),\ \overset{\bullet}{\pi} = (021),\ \overrightarrow{\pi} = (120)\ \overleftarrow{\pi} = (201) \tag{5}$$

for which the following equivalences apply:

$$\pi = \overset{\neg}{\pi}^*,\ \bar{\pi} = \overleftarrow{\pi}^*,\ \overset{\bullet}{\pi} = \overrightarrow{\pi}^*.$$

Beside the vector (5), the application of unary operations (4) on Op also results in vectors:

$$\downarrow = \bar{/}^* = \langle 100\rangle,\ \bar{\downarrow} = /^* = \langle 011\rangle,\ \overset{\neg}{\downarrow} = \overrightarrow{/}^* = \langle 122\rangle,$$
$$\overset{\bullet}{\downarrow} = \overleftarrow{/}^* = \langle 200\rangle,\ \overrightarrow{\downarrow} = \overset{\neg}{/} = \langle 211\rangle,\ \overleftarrow{\downarrow} = \overset{\bullet}{/}^* = \langle 022\rangle$$

and selfconjugated vectors:

$$\overset{\neg}{\oplus} = \langle 212\rangle,\ \overset{\bullet}{\oplus} = \langle 020\rangle,\ \overrightarrow{\oplus} = \langle 121\rangle,\ \overleftarrow{\oplus} = \langle 202\rangle.$$

The application of simple, as well as the complex s–operator on the vector argument is well known (see [3] pp. 25 – 26) and shall therefore not be repeated here.

Now we shall say a few words about the column notation via σ–operator. In a general case, when we are dealing with $PL_{k\cdot m}$, double angle brackets are used to notate a vector of the dimension k^n (i.e. of the rank n, where the base k is defined through PL), which shall be further referred to as σ–operators.

The following convention shall be upheld: if $\langle\langle\alpha\rangle\rangle$ — is a vector of the rank h (dimension k^h), then the notation $\langle\langle\alpha_j\rangle\rangle$ denotes that every coordinate of the vector $\langle\langle\alpha\rangle\rangle$ appears with the arity j (of the dimension k^j), because of which the vector α has the rank $h + j$. The notation

$\langle\langle\alpha^i\rangle\rangle$ denotes that the vector $\langle\langle\alpha\rangle\rangle$ itself appears in the concatenation h^i times. In that way, the notation $\langle\langle\alpha^i_j\rangle\rangle$ — is a vector of the rank $h+j+i$, under the condition of $\langle\langle\alpha\rangle\rangle$ — being a vector of the rank h.

Once again we shall emphasize that the double angle brackets point to the coordinates appearing in the exponential frequency, as well as the fact that the upper (as well as lower) vector indices (or/and its coordinates) point to the exponential character of the appeareance, and that the indices are always implied (if not stated, then equal to *zero*).

In already discussed $PL_{k\cdot m}$ the colums are:

$$x_j \;\underset{\rightharpoondown}{\simeq}\; \langle\langle\rho^{n-j}_{j-1}\rangle\rangle, \quad \overset{\triangle}{x}_j \underset{\rightharpoondown}{\simeq} \langle\langle\overset{\triangle}{\rho}\,{}^{n-j}_{j-1}\rangle\rangle, \tag{6}$$

where the vector is $\langle\langle\rho\rangle\rangle = \langle\langle 0,\,1,\,2,\,\ldots\,,\,k-1\rangle\rangle$, and the unary operation $\triangle$ — one of the permutations from the symmetric group S_k.

A special case (see. [3] p. 100) for PL_2 is:

$$x_j \;\underset{\rightharpoondown}{\simeq}\; \langle\langle\bar{\tau}^{n-j}_{j-1}\rangle\rangle, \quad \bar{x}_j \;\underset{\rightharpoondown}{\simeq}\; \langle\langle\tau^{n-j}_{j-1}\rangle\rangle,$$

where $\langle\langle\tau\rangle\rangle = \langle\langle 10\rangle\rangle$.

In the case of $PL_{3\cdot m}$, in correspondance with (5) and (6), the following holds:

$$x_j \;\underset{\rightharpoondown}{\simeq}\; \langle\langle\pi^{n-j}_{j-1}\rangle\rangle, \quad \bar{x}_j \;\underset{\rightharpoondown}{\simeq}\; \langle\langle\bar{\pi}^{n-j}_{j-1}\rangle\rangle, \quad \overset{\neg}{x}_j \underset{\rightharpoondown}{\simeq} \langle\langle\overset{\neg}{\pi}\,{}^{n-j}_{j-1}\rangle\rangle,$$
$$\overset{\bullet}{x}_j \underset{\rightharpoondown}{\simeq} \langle\langle\overset{\bullet}{\pi}{}^{n-j}_{j-1}\rangle\rangle, \quad \overrightarrow{x}_j \underset{\rightharpoondown}{\simeq} \langle\langle\overrightarrow{\pi}\,{}^{n-j}_{j-1}\rangle\rangle, \quad \overleftarrow{x}_j \underset{\rightharpoondown}{\simeq} \langle\langle\overleftarrow{\pi}\,{}^{n-j}_{j-1}\rangle\rangle.$$

Further, the calculations in $PL_{k\cdot m}$ along the scheme, analog to the scheme in paragraph 4 of this paper, should be explained in detail, but such a task would be difficult to realize within the limits of this paper, because of which this exposition shall be brought to an end by some simple illustrating examples, shown in the table 6, where:

$R = \langle 01120\rangle$, $Q = \langle 01201\rangle$, $W = \langle 01210\rangle$, whereby $F = W(R,\,Q) = \langle 02121\rangle$ holds.

It can be read from the table columns that

$$\begin{aligned}
W(R(x_1,x_2),\,Q(x_1,x_2)) &= F(x_1,x_2),\\
W(R(x_1,\overset{\neg}{x}_2),\,Q(x_1,\overset{\neg}{x}_2)) &= F(x_1,\overset{\neg}{x}_2),\\
W(R(\overset{\bullet}{x}_1,x_2),\,Q(\overset{\bullet}{x}_1,x_2)) &= F(\overset{\bullet}{x}_1,x_2),\\
W(R(\overset{\bullet}{x}_1,\overset{\neg}{x}_2),\,Q(\overset{\bullet}{x}_1,\overset{\neg}{x}_2)) &= F(\overset{\bullet}{x}_1,\overset{\neg}{x}_2).
\end{aligned}$$

These examples represent a confirmation of the results for PL_3, analog to the result of the theorem 3 (see [3] p. 77) for PL_2.

Table 6

x_1	x_2	R	Q	W	F		x_1	$\overset{\neg}{x_2}$	R	Q	W	F		$\overset{\bullet}{x_1}$	x_2	R	Q	W	F		$\overset{\bullet}{x_1}$	$\overset{\neg}{x_2}$	R	Q	W	F
0	0	0	0	0	0		0	2	1	2	1	1		0	0	0	0	0	0		0	2	1	2	1	1
1	0	1	1	2	2		1	2	2	0	2	2		2	0	1	2	1	1		2	2	0	1	1	1
2	0	1	2	1	1		2	2	0	1	1	1		1	0	1	1	2	2		1	2	2	0	2	2
0	1	1	1	2	2		0	1	1	1	2	2		0	1	1	1	2	2		0	1	1	1	2	2
1	1	1	2	1	1		1	1	1	2	1	1		2	1	2	0	2	2		2	1	2	0	2	2
2	1	2	0	2	2		2	1	2	0	2	2		1	1	1	2	1	1		1	1	1	2	1	1
0	2	1	2	1	1		0	0	0	0	0	0		0	2	1	2	1	1		0	0	0	0	0	0
1	2	2	0	2	2		1	0	1	1	2	2		2	2	0	1	1	1		2	0	1	2	1	1
2	2	0	1	1	1		2	0	1	2	1	1		1	2	2	0	2	2		1	0	1	1	2	2

6. CONCLUDING REMARKS

The proof of equivalence of P and NP class problems (the second one), expounded in this paper, is basically already present in [3] (see. 8.1), even if not in an explicit form. The main issue not explicated in [3], is the important idea of calculating with the bound R applied to the memory volume for literals-digits with or without indices. I have assumed that this result was going to be recognized without additional emphasis, which turned out to be unrealistic.

I should also mention, that although the first proof of the equivalence of P and NP class problem was carried out by the end of the last century, the second was carried out at the very beginnig of the present century. I should also state that it is possible to acquire a third proof, as a common result of the last two results. However, I believe that the more important issue is the application of the sigma–notation on superreduction, and even more important is the computer realization based on σ–notation, for its capability to make possible on existing computers what is expected of quantum computers.

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