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MODELLING CRISP AND FUZZY QUALITATIVE TEMPORAL RELATIONS

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Abstract: *Building a model for temporal knowledge representation and reasoning assumes choosing basic notions – primitives, the time instant or/and the time interval. This paper considers primitives for modelling crisp and fuzzy qualitative temporal relations. Based on fuzzified Allen's temporal relations between intervals, new relations between the fuzzy time point and the fuzzy time interval are proposed.*

Keywords: *fuzzy temporal knowledge, temporal relations, time point, time interval.*

1. INTRODUCTION

For many years, the problem of temporal knowledge representation and reasoning has been one of the central problems in the field of artificial intelligence and computer science [13], [11], [5], [7], [19], [10], [15], [17] etc. Different formal models have been proposed as solutions for effective temporal knowledge representation. The models have to include good epistemological properties, the ability to interact with other types of knowledge, and to efficiently manage temporal reasoning tasks, such as determining consistency, finding a consistent scenario and deducing new temporal relations. Let us give a brief review of some models in chronological order. J. McCarthy and P. Hayes introduced *situation calculus* [12] to represent events and their effects.

K. Kahn and G. A. Gorry proposed a *time specialist* - a program that can answer questions about temporal matters, but that otherwise it knows nothing of the problem domain in question [9].

J. F. Allen described a temporal representation that took the notation of a *temporal interval* as a primitive [1]. On the base of the thirteen possible relationships between intervals and the transitivity table, the inference technique about time was introduced.

The relationships between intervals were represented and maintained in a network where the nodes represented individual intervals. Each arc was labelled to indicate the possible relationship between the two intervals.

D. V. McDermott and T. L. Dean developed a scheme called *a time map* [3] for temporal reasoning. A time map was a graph where its vertices refereed to points or instants of time corresponding to the beginning and ending of events. To relate one point to another, a point-to-point constraint was used.

R. Pelavin and J. F. Allen, in 1986., extended a temporal logic in such a way that it could be used to support planning in temporally rich domains [15]. These are domains that include actions that take time, concurrent and simultaneous actions and actions performed by external agents or natural forces.

S. Ribarić introduced the t-timed Petri Nets as a model for temporal knowledge representation, planning and reasoning [14].

D. Dubois and H. Prade used Zadeh's possibility theory as a general framework for modeling *fuzzy temporal knowledge* [5].

A *temporal constraint network (TCN)* [4] as extension of network-based method which included continuous variables and the ability to deal with metric information, was proposed by R. Dechter, I. Meiri and J. Pearl in the year 1991.

A. Gerevini and L. Schubert (1995.) have addressed the problem of scalability in temporal reasoning by providing a collection of new algorithms for efficiently managing large sets of qualitative temporal relations [8].

Ribarić *et al.* [16] presented an object-oriented implementation of a model for fuzzy temporal reasoning etc.

Building a model for temporal knowledge representation and reasoning usually assumes choosing basic notions – primitives, the (fuzzy or non fuzzy) time instant or/and time interval. The reasoning process is then dependent on the chosen primitive. This paper considers primitives for modelling crisp and fuzzy qualitative temporal relations and introduces qualitative relations between the time point and the time interval.

The paper is organized as follows. In Section 2, we give crisp qualitative temporal relations that are valid between primitives, the time point and the time interval, and between two time intervals. Further, Section 3 provides the definition of the fuzzy time point and the fuzzy time interval, and describes the modelling of fuzzy qualitative relations. Section 4 gives an original approach to modelling fuzzy relations between time points and the fuzzy time interval.

2. MODELLING CRISP QUALITATIVE TEMPORAL RELATIONS

Let us introduce the basic elements for modelling qualitative temporal relations. The time scale, denoted by T, is a referential, universal set (this could be any linearly ordered set, e.g. R^+, Z^+). Actions or states and events are defined by assigning subsets of T to them.

A single element of T is interpreted as a crisp time point (or time instant) and it is used to describe an instantaneous event (without time duration).

According to Allen [1], if time points are allowed, intervals could be represented by their endpoints, where endpoints are single elements of T. The interval is an ordered pair of time points, with the first point less then the second. Table 1 gives five basic temporal relations between intervals defined by endpoints. Symbol x+ denotes the starting point and symbol x- denotes the ending point of the time interval X.

Table 1. Allen's temporal relations defined by endpoints

Interval relation	Equivalent relations on endpoints
X before Y	x+ < y-
X equal Y	(x- = y-) & (x+ = y+)
X meets Y	x+ = y-
X overlaps Y	(x- < y-) & (x+ > y-) & (x+ < y+)
X during Y	((x- > y-) & (x+ ≤ y+)) or (x- ≥ y-) & (x+ < y+))

Further subdivision of the *during* relation into *starts* and *finishes* provides a better computational model with seven temporal relations. Adding to each temporal relation its inverse (except to the relation *equal*), thirteen temporal relations [1] are obtained that can be used to express any relation that can hold between two intervals. Table 2 shows thirteen temporal relations defined by Allen [1].

Table 2. Allen's temporal relations

RELATION	SYMBOL	SYMBOL FOR INVERSE	PICTORIAL EXAMPLE
X before Y	<	>	xxxx yyyy
X equal Y	=	=	xxxx yyyy
X meets Y	m	mi	xxxx yyyy
X overlaps Y	o	oi	xxxx yyyy
X during Y	d	di	xxxx yyyyyyy
X starst Y	s	si	xxxx yyyyyyy
X finishes Y	f	fi	xxxx yyyyyyy

In many practical applications there is the need to introduce relations between a time point and a time interval. The models having a time point and a time interval as primitives have more modelling power. In some cases this could imply adding more complexity to the model.

In order to extend the modelling power of the temporal reasoning model we introduced five crisp relations [18] that can hold between a time point and a time interval. The five relations between a time point and a time interval can be derived based on thirteen Allen's temporal relations and on the assumption that the time interval degenerates into a time point, i.e. X+ $\rightarrow$ X -. Table 3 shows these new relations that hold between a time point and a time interval where • represents a time point and xxxxxx represents a time interval.

Table 3. Relations time points (tp) - time interval (x)

RELATION	SYMBOL	SYMBOL FOR INVERSE	PICTORIAL EXAMPLE
tp before Y	<	>	• xxxx
tp during Y	d	not exists	xx•xxx
tp starst Y	s	not exists	• xxxxxxx
tp finishes Y	f	not exists	• xxxxxxx

Thus, the modelling power is extended but in a way that does not imply extra complexity in the model. The proposed Allen's reasoning algorithm [1] based on the constraint propagation can be applied to the time point – time interval temporal logic. The implementation of this temporal reasoning model on a multi-agent system is given in [18].

3. MODELLING FUZZY QUALITATIVE TEMPORAL RELATIONS

3.1. Basic definitions

In this section some basic fuzzy set theory definitions [20], [21], [22], used in this paper, are given. The fuzzy set **A** on the universal set U is the function $\mu_A: U \rightarrow [0, 1]$ so that $\mu_A(x)$ represents the degree of membership of the element $x \in U$ to the fuzzy set **A**. The height of **A** is hgth(**A**) = $\sup_{x \in U} \mu_A(x)$. Fuzzy sets are denoted by bold letters. It is said that a fuzzy set is normal iff hght(**A**) = 1. The support of **A** is a crisp set, supp(**A**) = $\{x \in U \mid \mu_A(x) > 0\}$.

A fuzzy relation is a fuzzy set on the Cartesian product of universal sets $U_1 \times \ldots \times U_n$. If **R** is fuzzy relation on $X \times Y$ and **S** is fuzzy relation on $Y \times Z$, then sup-min composition of fuzzy relations **ROS** is given by

$$\mu_{R \circ S}(x, z) = \sup \min (\mu_R(x, y), \mu_S(y, z)).$$

In temporal reasoning it is necessary to model linguistic expression such as "much before", "slightly after", "approximately at the same time" etc. These expressions are modeled by fuzzy relations and here is example of modeling relation "approximately at the same time".

Example 1.

The linguistic expression "approximately at the same time" can be modelled by fuzzy relation **R** as follows.

$$m_R(s,t) = \max\left(0, \min\left(1, \left(\frac{d+r-|s-t|}{r}\right)\right)\right) = \begin{cases} 1 & |s-t| \leq d \\ 0 & |s-t| \geq d+r, \text{ where } d \geq 0 \text{ and } r > 0 \\ \dfrac{d+r-|s-t|}{r} & \text{else} \end{cases}$$

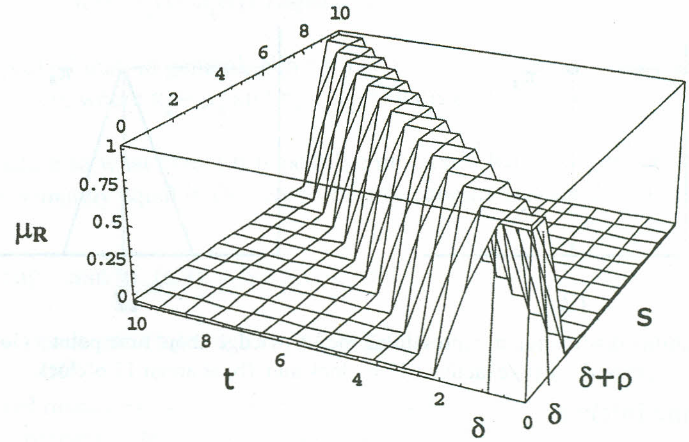

Figure 1. Fuzzy relation R = "approximately at the same time" for parameters $\delta = \rho = 1$.

Fig. 1 shows fuzzy temporal relation "approximately at the same time" as a function of temporal variables *t* and *s* ranging from 0 to10 time units. The parameters δ and ρ, denoted on Fig.1, give semantic meaning to the word "approximately". Linguistic expression "approximately equal" can be modelled by the same relation **R**.

3.2. Fuzzy time point

The time point (or time instant) is a time primitive and it describes an event, the beginning or end of some action/state. The fuzzy time point will be denoted by **a, b, c,** etc. As in [5], the knowledge about a time point **a** is presented in the form of a possibility distribution function π_a. The possibility distribution [21] is a function $\pi_a : T \rightarrow [0, 1]$, so that $\forall t \in T$, $\pi_a(t) \in [0, 1]$ is a numerical estimate of the possibility that the time point **a** is precisely *t*. Possibility distribution is associated with the fuzzy set **A** (more or less possible values of **a**) and $\pi_a(t)$ is by definition equal to $\mu_A(t)$ [21]. When a fuzzy point is linguistically described, **A** is the label of the point **a**.

Example 2.

Based on the above definition we can model temporal components of these two simple linguistic expressions.

> *"John arrived in front of the bank exactly at 12 o'clock"*, and
> *"John arrived in front of the bank at about 12 o'clock"*.

Let **a** denote the time point of John's arrival in front of the bank. The knowledge about time point **a** in the first sentence is presented by the possibility distribution π_a depicted on the Fig.2(a), and the knowledge about time point **a** in the second sentence is presented by the possibility distribution π_a depicted on the Fig.2(b).

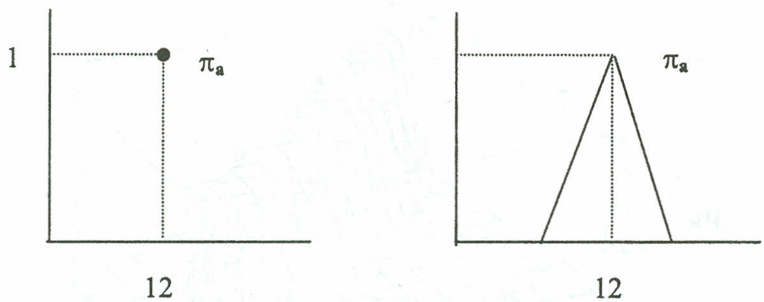

Figure 2. Possibility distribution π_a representing the knowledge about time point a (John's arrival) expressed by (a) exactly at 12 o'clock and (b) at about 12 o'clock.

3.3. Fuzzy time interval

The duration of an action or a state is described by fuzzy time interval. Dubois and Prade [5] have described fuzzy interval as a pair of fuzzy sets: **]A, B[** and **[A, B]**. The set **]A, B[** is a set of time points that are more or less certainly between **a** and **b**, and the set **[A, B]** is a set of time points that are possibly between **a** and **b**. These sets are given by:

$$\mu_{[A,B]}(t) = \sup_{s \le t \le s'} \min(\pi_a(s), \pi_b(s')) = [\mathbf{A}, +\infty) \cap (-\infty, \mathbf{B}], \quad (1)$$

and

$$\mu_{]A,B[}(t) = \min(\mu_{]A,+\infty)}(t), \mu_{(-\infty,B[}(t)) = (-\infty, \mathbf{A}]^C \cap [\mathbf{B}, +\infty)^C. \quad (2)$$

Figure 3. illustrates fuzzy time interval defined by Dubois and Prade in the case of non-overlapping and overlapping possibility distributions.

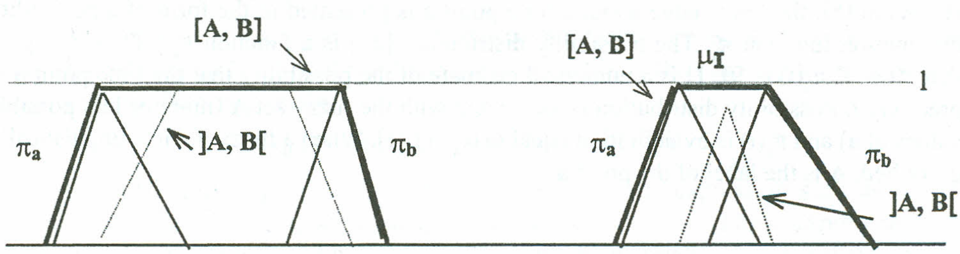

Figure 3. Fuzzy time interval defined by Dubois and Prade in the case of a) non-overlapping possibility distributions, b) overlapping possibility distributions

3.4. Ordering of fuzzy time points using necessity and possibility measures

Given two possibility distributions, restricting the possible values of two time instants, it is interesting to estimate to what extent it is possible or certain that the instant **a** takes place before the instant **b**. This estimation can be performed in terms of possibility and necessity measures [6].

The possibility and necessity that **a** is smaller than **b** is equal to

$$\Pi(\mathbf{a} \leq \mathbf{b}) = \sup_{s \leq t} \min(\pi_a(s), \pi_b(t)), \text{ and}$$

$$N(\mathbf{a} \leq \mathbf{b}) = 1 - \sup_{s \leq t} \min(\pi_a(s), \pi_b(t)), \text{ respectively.}$$

It can be proved that, in general, $N(\mathbf{a} \leq \mathbf{b})$ is equal to the height of $\mu_{]A, B[}(t)$ (defined in Section 3.3. by (2)), where $\pi_a = \mu_A$ and $\pi_b = \mu_B$ (Dubois *et* Prade, 1988).

The estimation to what extent it is possible or certain that two time points **a** and **b** are (at least) approximately equal in the sense of fuzzy equality relation **R** (such as in Example 1) is given by

$$\Pi(\mathbf{a} =_R \mathbf{b}) = \sup_{s \leq t} \min(\mu_R(s, t), \pi_a(s), \pi_b(t)), \text{ and}$$

$$N(\mathbf{a} =_R \mathbf{b}) = 1 - \sup_{s \leq t} \min(\mu_R(s, t), 1 - \pi_a(s), 1 - \pi_b(t)).$$

The detailed discussion about these indices and their properties is given in [5] and [6]. These indices are used to define the fuzzified version of Allen's temporal relations.

3.5. Fuzzified allen's temporal relations between intervals

The fuzzified version of these relations is discussed by Dubios and Prade in their paper [5]. They used the measure of necessity to estimate the degree of necessity or certainty that a given temporal relation holds between fuzzy time intervals **[a, b]** and **[c, d]**. Assuming that for every considered interval **[a, b]**, property $N(\mathbf{a} \leq \mathbf{b}) = 1$ holds, the definition of these relations is:

$N([\mathbf{a}, \mathbf{b}]\ \mathbf{before}\ [\mathbf{c}, \mathbf{d}]) = N(\mathbf{b} \leq \mathbf{c})$,
$N([\mathbf{a}, \mathbf{b}]\ \mathbf{overlaps}\ [\mathbf{c}, \mathbf{d}]) = \min(N(\mathbf{a} \leq \mathbf{c}), N(\mathbf{c} \leq \mathbf{b}), N(\mathbf{b} \leq \mathbf{d}))$,
$N([\mathbf{a}, \mathbf{b}]\ \mathbf{during}\ [\mathbf{c}, \mathbf{d}]) = \min(N(\mathbf{c} \leq \mathbf{a}), N(\mathbf{b} \leq \mathbf{d}))$.

Using index $N(\mathbf{e} =_S \mathbf{f})$:

$N([\mathbf{a}, \mathbf{b}]\ \mathbf{equals}_{-R}\ [\mathbf{c}, \mathbf{d}]) = \min(N(\mathbf{a} =_R \mathbf{c}), N(\mathbf{b} =_R \mathbf{d}))$,
$N([\mathbf{a}, \mathbf{b}]\ \mathbf{meets}_{-R}\ [\mathbf{c}, \mathbf{d}]) = N(\mathbf{b} =_R \mathbf{c})$,
$N([\mathbf{a}, \mathbf{b}]\ \mathbf{starts}_{-R}\ [\mathbf{c}, \mathbf{d}]) = N(\mathbf{a} =_R \mathbf{c})$,
$N([\mathbf{a}, \mathbf{b}]\ \mathbf{finishes}_{-R}\ [\mathbf{c}, \mathbf{d}]) = N(\mathbf{b} =_R \mathbf{d})$.

Based on these definitions and the transitivity property of N(. ≤ .), some patterns of reasoning can be established [5].

4. QUALITATIVE RELATIONS BETWEEN FUZZY TIME POINT AND FUZZY TIME INTERVAL

Fuzzy time intervals are considered for modelling events that have duration and fuzzy time points are considered for modelling instantaneous events.

Temporal relations between fuzzy intervals that are explained in Section 3.5. can be extended for estimating qualitative relations between a fuzzy time point and a fuzzy time interval. There are five temporal relations that hold between the fuzzy time interval **[a, b]** and the fuzzy time point ***tp***: before/after, during, starts and finishes. They are defined using indices $N(\mathbf{a} \leq \mathbf{b})$ and $N(\mathbf{a} =_R \mathbf{b})$, defined in Section 3.4, where **R** is the fuzzy relation that defines the meaning of "approximately equal".

$N([\boldsymbol{tp} \textbf{ before } [\mathbf{a}, \mathbf{b}]) = N(\boldsymbol{tp} \leq \mathbf{a})$

$N(\boldsymbol{tp} \textbf{ during } [\mathbf{a}, \mathbf{b}]) = \min (N(\mathbf{a} \leq \boldsymbol{tp}), N(\boldsymbol{tp} \leq \mathbf{b}))$

$N(\boldsymbol{tp} \textbf{ starts}_R [\mathbf{a}, \mathbf{b}]) = N(\boldsymbol{tp} =_R \mathbf{a})$

$N(\boldsymbol{tp} \textbf{ finishes}_R [\mathbf{a}, \mathbf{b}]) = N(\mathbf{b} =_R \boldsymbol{tp})$.

The above definitions enable the introduction of a fuzzy time point in qualitative temporal reasoning. Thus, fuzzified interval temporal logic is extended by a fuzzy time point – fuzzy time interval logic.

The following example illustrates a qualitative temporal reasoning when an unprecise information about a time point and a time interval are given.

Example 3.

John's meeting starts at about 10 o'clock and finishes about 12 o'clock. He expects an important telephone call that is announced at about 10.15. What is the certainty that the call will occur during the meeting?

The knowledge about John's meeting is defined by two possibility distributions π_a and π_b representing the possible starting and ending times of John's meeting, respectively. Possibility distributions π_a and π_b are depicted on Fig.4. The knowledge about the time point representing the possible time of an important telephone call, about 10.15, is given by the possibility distribution π_c (Fig.4).

In this example we are not interested in constructing a fuzzy time interval of John's meeting from two possibility distributions π_a and π_b (such as in Fig. 3a), but we would like to evaluate the fuzzy temporal relation *during* using the necessity measure.

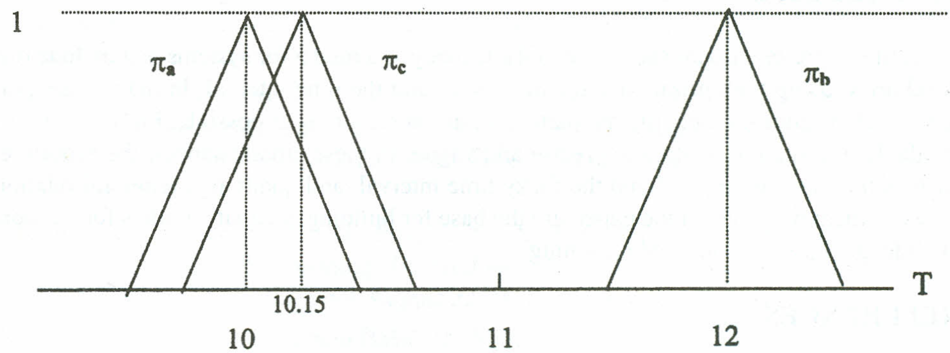

Figure 4. Possibility distributions π_a and π_b representing the start and end of Jon's meeting respectively, and π_c representing the possible time of the telephone call.

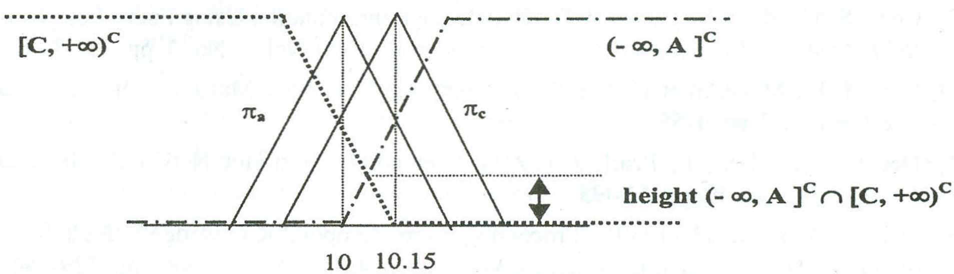

Figure 5. Evaluation process of the necessity measure N(a ≤ tp), where the knowledge about time point **a** is given by π_a and the knowledge about time point tp is given by possibility distribution π_c

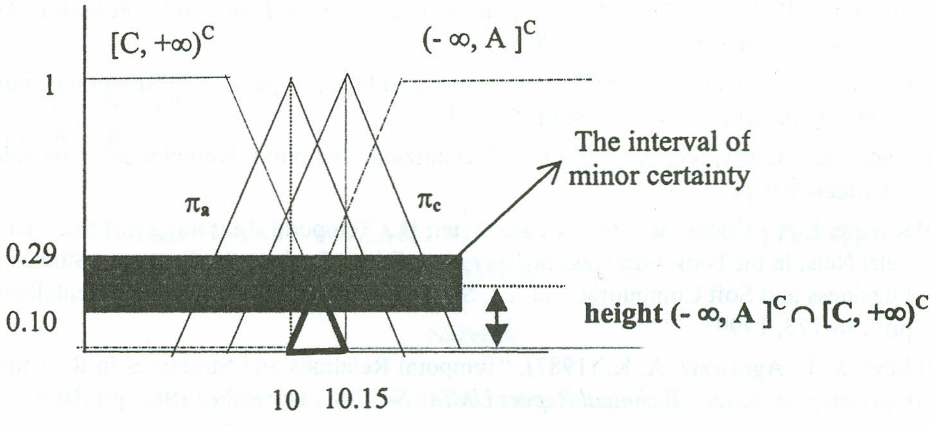

Figure 6. The interval of minor certainty

According to the scale given in [2], by which the interval of real numbers [0, 1] is divided into nine parts and a linguistic approximation is assigned to each part, the linguistic answer to the posed question in this example is – *There is a minor certainty that the telephone call will occur during John's meeting.* Fig. 6 shows the graphical deduction of the answer.

5. CONCLUSION

Better representational and inferential adequacy of a model for systems that include time is achieved using both primitives, the time point and the time interval. In many cases, due to the lack of information, only qualitative temporal reasoning is possible. Furthermore, the available information is often unprecise and vague. In these circumstances, the primitives, such as the fuzzy time point and the fuzzy time interval, and qualitative temporal relations between them proposed in the paper, are the base for building adequate models for temporal knowledge representation and reasoning.

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MODELIRANJE EGZAKTNIH I FUZZY KVALITATIVNIH TEMPORALNIH RELACIJA

Sažetak

Izgradnja modela za temporalnu reprezentaciju znanja i temporalno rezoniranje pretpostavlja izbor temeljnih pojmova – primitiva – vremenskog trenutka i vremenskog intervala. Ovaj članak razmatra primitive za modeliranje egzaktnih i fuzzy *kvalitativnih temporalnih relacija. Na temelju Allenovih temporalnih relacija između intervala' predložene su nove relacije između* fuzzy *vremenskih trenutaka i* fuzzy *vremenskih intervala*

Ključne riječi: fuzzy *temporalno znanje, temporalne relacije, vremenska točka, vremenski interval*