# Project Investment Decision-Making 

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#### Abstract

Project evaluation is the control of the planning and implementation of project activities with regard to the objectives to be achieved. In this paper we assume the objective to be efficient outcome and profit maximization. This means that project evaluation puts normative assessments into the context of planning and management and hence into the context of intentional action and cycles of action. The model for project evaluation we propose has two money holders who must decide how to invest their money in two investment funds (financial intermediaries) that, in turn, will use the money to bid to acquire ownership in two projects. The general case when the number of money holders, the number of funds, and the number of investments are arbitrary may be handled in a similar manner to the development below, but at a cost of greater complexity. As a result no mechanism to achieve the maximum outcome is present and different methods to find optimal structure under uncertainty and different cost structures are discussed.


Keywords: project evaluation, profit maximization, uncertainty, coordination failure

## JEL Classification: G31

## Introduction

The model for project evaluationwepropose has two money holders who must decide how to invest their money in two investment funds that, in turn, will use the money to bid to acquire ownership in two projects. Importantly, the profitability of each project depends on the specific joint ownership structure that results from the money which each MIF receives, as the funds are assumed to have different management capabilities.

We start by assuming that $N>0$ money points are owned by the population which consists of two individuals, $I_{1}$ and $I_{2}$. Money holder $I_{l}, l=1,2$, has $V_{l}>0$ money points where $V_{1}+V_{2}=N$. The number of money points held by each individual may

[^0]differ to allow the possibility of pre-auction trading. Each $I_{l}$ must decide independently on the number of money points to invest in each of two money funds, $F_{j} j=1,2$. The number of money points that $I_{l}$, chooses to allocate to $F_{1}$ is denoted by $x, x \in\left[0, V_{1}\right]$ with the remaining $V_{1}-x$ money points being allocated to $F_{2}$.

Similarly, we denote by $y, y \in\left[0, V_{2}\right]$, the money points investment of $I_{1}$ in $F_{2}$, with $V_{2}-y$ being invested in $F_{2}$. As a consequence of investing its money points in this manner $I_{1}$ acquires the proportion $\frac{x}{x+y}$ of the profit of $F_{1}$, and $\frac{V_{1}-x}{\left(V_{1}-x\right)+\left(V_{2}-y\right)}$ of the profit of $F_{2}$. Correspondingly, $I_{2}$ acquires the proportion $\frac{y}{x+y}$ of the profit of $F_{1}$ and $\frac{V_{2}-y}{\left(V_{1}-x\right)+\left(V_{2}-y\right)}$ of the profit of $F_{2}$. The general case when the number of money holders, the number of funds, and the number of investments are arbitrary may be handled in a similar manner to the development below, but at a cost of greater complexity.

At the outset, neither $F_{j}$ has any money. In order to attract money from the $I_{1}$, each $F_{j}$ reveals information useful to the $I_{l}$. We assume that this information relates to the cost structure of the $F_{j}$. Specifically,weassume that each $F_{j}$ announces that its costs will be a fixed proportion of the revenues it will earn by investing the money points that it will acquire. This assumption is equivalent to the assumption that the profit of the $F_{j}$ is equal to $\sigma_{j} R_{j}(x+y)$ where $\sigma_{j}$ is constant, $\sigma_{j} \in[0,1], j=1,2$, and $R_{j}: \mathfrak{R}_{+} \rightarrow \mathfrak{R}_{+}$is the revenue received by $F_{j}$ as a result of the bidding game in which, using money points acquired from the $I_{l}, \stackrel{F}{F}_{1}$ and $F_{2}$, compete to acquire share in the projects offered for financing. The $\sigma_{i}$ can be thought of as the proportion of revenue that the $F_{j}$ promise to distribute to the share holders. $R_{1}(x+y)$ depends on $x+y$ since this is the number of money points available to $F_{1}$ for investment in projects. Similarly, $R_{2}(x+y)$ has the same dependence since the total number of money points, $N$, is fixed.

Thus $I_{1}$, receives $m_{1}$ : where $\mathfrak{R}_{+}^{2} \rightarrow \mathfrak{R}_{+}$

$$
m_{1}(x, y)=\frac{x}{x+y} \sigma_{1} R_{1}(x+y)+\frac{V_{1}-x}{\left(V_{1}-x\right)+\left(V_{2}-y\right)} \sigma_{2} R_{2}(x+y)
$$

Thus $I_{2}$, receives $m_{2}$ : where $\mathfrak{R}_{+}^{2} \rightarrow \mathfrak{R}_{+}$
$m_{2}(x, y)=\frac{y}{x+y} \sigma_{1} R_{1}(x+y)+\frac{V_{2}-y}{\left(V_{1}-x\right)+\left(V_{2}-y\right)} \sigma_{2} R_{2}(x+y)$
$I_{1}$ chooses $x$ to maximize $m_{1}$ and $I_{2}$ chooses $y$ to maximize $m_{2}$. We refer to the problem of simultaneously maximizing $m_{1}$ and $m_{2}$ as the money investment problem (MIP). In what follows, we take $x$ and $y$ to be continuous over their respective ranges.

The $R_{j}(x+y)$ are determined by the following process. With $N_{1}=x+y$ and $N_{2}=N-N_{1}$ respectively, $F_{1}$ and $F_{2}$ play a non-cooperative game in which they submit bids to acquire shares in company $i$, we $=1,2$. Each $F_{j}$ submits a money point bid of $a_{i j}$ in company where $a_{i j} \geq 0$ and $\sum_{i} a_{i j}=N_{j}$. As a consequence of the bidding, each $F_{j}$ receives the proportion of ${ }^{i} p_{i j}=\frac{a_{i j}}{\sum_{j} a_{i j}}$ of $\pi_{i}$, the profit of project $i$.weassume that the $\pi_{i}: \mathfrak{R}_{+}^{2} \rightarrow \mathfrak{R}_{+}$, we $=1,2$, depend on $p_{i j}, j=1,2$, that is,weassume that the $F_{j}$ have different skills in managing and restructuring the projects in which they have acquired share, and that the impact of their skills on the profit of a given project depends on the proportion of ownership that they achieve in the project as a result of the bidding game.

Furthermore, we assume, for tractability, that the profit functions $\pi_{i}\left(p_{i 1}, p_{i 2}\right)$ can be reasonably approximated by a first-order Taylor expansion. It follows, since $p_{i 1}+p_{i 1}=1$ for $w e=1,2$, that:

$$
\begin{aligned}
& \pi_{1}\left(p_{11}, p_{12}=\pi_{1}(0,1)+p_{11}\left[\frac{\partial \pi_{1}(0,1)}{\partial p_{11}}-\frac{\partial \pi_{1}(0,1)}{\partial p_{12}}\right]\right. \\
& \pi_{2}\left(p_{21}, p_{22}\right)=\pi_{2}(1,0)+p_{22}\left[\frac{\partial \pi_{2}(1,0)}{\partial p_{22}}-\frac{\partial \pi_{2}(1,0)}{\partial p_{21}}\right]
\end{aligned}
$$

In the remainder of the paper we use the notation $k_{12}=\pi_{1}(0,1) ; k_{21}=\pi_{2}(1,0)$;

$$
\Delta_{1}=\frac{\partial \pi_{1}(0,1)}{\partial p_{11}}-\frac{\partial \pi_{1}(0,1)}{\partial p_{1,2}} ; \Delta_{2}=\frac{\partial \pi_{2}(1,0)}{\partial p_{22}}-\frac{\partial \pi_{2}(1,0)}{\partial p_{21}} ; \text { and } r_{i}=\frac{\Delta_{i}}{k_{i j}} \text { for } j \neq i \text {. }
$$

In summary, we assume that the profit function can be written as;

$$
\pi_{i}=k_{i j}+p_{i i} \Delta_{i}=k_{i j}\left(1+p_{i i} r_{i}\right) \text { for } i \neq j, i, j=1,2
$$

The parameter $k_{12}$ represents the profit that project 1 would make if it were totally purchased by $F_{2}$. The parameter $k_{21}$ has a similar interpretation. The parameter $\Delta_{1}$ represents the difference in the differential advantage (disadvantage) that $F_{1}$ has over $F_{2}$, in managing project 1 . The parameter $\Delta_{2}$ has a similar interpretation. Thus, $\pi_{1}$ is modeled as the sum of the value that would occur if $F_{2}$ were to manage project 1 exclusively plus the improvement, (deterioration) when ownership is shared with $F_{1}$. The profit $\pi_{2}$ has a similar interpretation. Notice that if $F_{1}$, and $F_{2}$, have the same differential impact on $\pi_{1}$, the value of the profit function would be the same regardless of how ownership were shared.

We note that since the $p_{i i}$ depend on $x+y$, the $\pi_{i}$ depend on $x+y$ also. For subsequent use, we define $\pi_{i}(z)=k_{i j}+z \Delta_{i}$. Thus, after having submitted their bids, $F_{j}$ receives the revenue $p_{1 j} \pi_{1}\left(p_{11}\right)+p_{2 j} \pi_{2}\left(p_{22}\right)$.

The revenue accruing to $F_{j}$ at the Nash equilibrium of the bidding game is what we call $R_{j}\left(N_{1}\right)$ and thus the profit available for distribution to the $I_{l}$ by $F_{j}$ is where $N_{1}=x+y$.

We assume that both $I_{l}$ share the same information set concerning the projects and skill levels, as well as the reasoning and characteristics of the $F_{j}$. Since the $R_{j}\left(N_{1}\right)$, the results of the bidding game between the $F_{i}$, are required by the $I_{l}$ to solve their problem, we investigate this bidding game first.

## The Money Fund Problem

We now formalize the non-cooperative bidding game played by the $F_{j}$. Given $N_{1}$ and $N_{2}$, and given the bids of $F_{j^{\prime}}, j^{\prime} \neq j, F_{j}$ must choose its bids to maximize its profit. Since, by earlier assumption, its profit is a fixed multiple of its revenue, $F_{j}$ 's bids must satisfy

$$
\max _{a_{1,}, a_{2 j}} \sum_{i} p_{i j} \pi_{i}\left(p_{i i}\right)
$$

subject to $a_{i j} \geq 0$ and $\sum_{i} a_{i j}=N_{j}$ and where $p_{i j}=\frac{a_{i j}}{\sum_{j} a_{i j}}$. I refer to these programs
as the money fund problem (VFP).
The Lagrangian for $F_{1}$ is:

$$
L_{1}=p_{11} \pi_{1}\left(p_{11}\right)+p_{21} \pi_{2}\left(p_{22}\right)-\lambda_{1}\left(a_{11}+a_{21}-N_{1}\right)
$$

with first-order conditions:
$\frac{\partial L_{1}}{\partial a_{11}}=0=\frac{\left(1-p_{11}\right)}{a_{1} \bullet} \pi_{1}+p_{11} \frac{\left(1-p_{11}\right)}{a_{1} \bullet} \Delta_{1}-\lambda_{1}=\frac{\left(1-p_{11}\right)}{a_{1} \bullet}\left(\pi_{1}+p_{11} \Delta_{1}\right)-\lambda_{1}$
$\frac{\partial L_{1}}{\partial a_{21}}=0=\frac{\left(1-p_{21}\right)}{a_{2 .}} \pi_{2}-p_{21} \frac{p_{22}}{a_{2 .}} \Delta_{2}-\lambda_{1}=\frac{p_{22}}{a_{2 .}}\left(\pi_{2}-\Delta_{2}+p_{22} \Delta_{2}\right)-\lambda_{1}$
$\frac{\partial L_{1}}{\partial \lambda_{1}}=0=a_{11}+a_{21}-N_{1}$
where
$a_{i \bullet}=\sum_{j} a_{i j}$

Similarly, the Lagrangian for $F_{2}$ is:

$$
L_{2}=p_{12} \pi_{1}\left(p_{11}\right)+p_{22} \pi_{2}\left(p_{22}\right)-\lambda_{2}\left(a_{12}+a_{22}-N_{2}\right)
$$

with first-order conditions:

$$
\begin{align*}
& \frac{\partial L_{2}}{\partial a_{12}}=0=\frac{\left(1-p_{12}\right)}{a_{1}} \pi_{1}-p_{12} \frac{p_{11}}{a_{1}} \Delta_{1}-\lambda_{2}=\frac{\left.p_{11}\right)}{a_{1} \bullet}\left(\pi_{1}-\Delta_{1}+p_{11} \Delta_{1}\right)-\lambda_{2}  \tag{4}\\
& \frac{\partial L_{2}}{\partial a_{22}}=0=\frac{\left(1-p_{22}\right)}{a_{2}} \pi_{2}-p_{22} \frac{\left(1-p_{22}\right)}{a_{2 \bullet}} \Delta_{2}-\lambda_{2}=\frac{\left(1-p_{22}\right)}{a_{2}}\left(\pi_{2}+p_{22} \Delta_{2}\right)-\lambda_{2}  \tag{5}\\
& \frac{\partial L_{2}}{\partial \lambda_{2}}=0=a_{12}+a_{22}-N_{2} \tag{6}
\end{align*}
$$

Before presenting the solution to the VFP, we provide the following lemma. Recall that

$$
r_{i}=\frac{\Delta_{i}}{k_{i j}} \text { and } \pi_{i}(z)=k_{i j}+z \Delta_{i} .
$$

## Lemma 1

Let $k_{i j}>0$ for $i \neq j$ and let $r_{i} \in(-1,1]$. For any $\alpha \in[0,1]$, there exists a unique set of values $z_{1}^{*}, z_{2}^{*}, \Theta^{*} \in[0,1]$ that simultaneously satisfy.
(i) $1-z_{1}=\Theta \frac{1+r_{1} z_{1}}{1+2 r_{1} z_{1}}$
(ii) $1-z_{2}=(1-\Theta) \frac{1+r_{2} z_{2}}{1+2 r_{2} z_{2}}$
(iii) $z_{1} \pi_{1}\left(z_{1}\right)+\left(1-z_{2}\right) \pi_{2}\left(z_{2}\right)=\alpha\left[\pi_{1}\left(z_{1}\right)+\pi_{2}\left(z_{2}\right)\right]$

## Theorem 1

When $k_{i j}>0$ for $i \neq j$, and when $r_{i} \in(-1,1]$, there exists a Nash equilibrium of the VFP and it is unique. In particular, let $z_{1}^{*}, z_{2}^{*}$, and $\Theta^{*}$ be the solutions to the equations of lemma 1 corresponding to $\alpha=N_{1} / N$. Then, under the stated conditions, the
unique solution to equations (1)-(6), i.e., the Nash equilibrium of the VFP for $i, j=$ 1,2 and $j \neq i$ is:

$$
a_{i i}^{*}=K^{-1} z_{i}^{*} \pi_{i}\left(z_{i}^{*}\right), \quad a_{i j}^{*}=K^{-1}\left(1-z_{i}^{*}\right) \pi_{i}\left(z_{i}^{*}\right), \quad \lambda_{1}=K \Theta^{*}, \quad \lambda_{2}=K\left(1-\Theta^{*}\right)
$$

where $K=\frac{\pi_{1}\left(z_{1}^{*}\right)+\pi_{2}\left(z_{2}^{*}\right)}{N}$
It is useful to highlight a result established in the proof of Theorem 1 signifying the proportion of each project owned by each fund.wedo this in the next corollary. In what follows, an asterisk above any function denotes that function evaluated at the solution to the VFP presented in Theorem 1.

## Corollary 1

The solution to the VFP yields $p_{i i}^{*}=z_{i}^{*}$.
Interpreting $\pi_{i}^{*} / a_{i}^{*}$ as the resulting value per money in project $I$, Theorem 1 establishes that these values are the same for both projects at the Nash equilibrium of the bidding game. Furthermore, this common value is equal to the economy-wide value of a money given by $\left(\pi_{1}^{*}+\pi_{2}^{*}\right) / N$. This common value of a money is also equal to the sum of the two shadow prices that is denoted by $K$ in Theorem 1. An additional money to the system, yielding approximately the value $K$, would be divided between $F_{1}$ and $F_{2}$ in the amounts $\lambda_{1}$ and $\lambda_{2}$. Thus, $F_{1}$, would receive $\Theta^{*}$ percent of this additional amount, and $F_{2}$ the remainder, $1-\Theta^{*}$ where $\Theta^{*}$ incorporates, among other things, the relative skill levels of $F_{1}$ and $F_{2}$.

At the Nash equilibrium, a total of $K^{-1} \pi_{j}^{*}=N\left(\pi_{j}^{*} /\left(\pi_{1}^{*}+\pi_{2}^{*}\right)\right.$ money points are invested in project $j, j=1$, 2 , with $F_{j}$ contributing $z_{j}^{*}$ percent of these money points.wecan interpret this total either as the part of the outstanding number of money points acquired by company $j$ being proportional to $\pi_{j}^{*}$, or as the profit of project $j$ denominated in units of economy-wide value per money. Although the money investment in projectwedepends on $\pi_{j}{ }_{j}$, this profit cannot be known in advance since it depends on the composition of ownership resulting from the bidding game itself. Finally, the ratio $a_{i i}^{*} / a_{i j}^{*}=p_{i i}^{*} / p_{i j}^{*}=z_{i}^{*} /\left(1-z_{i}^{*}\right), j \neq i$ depends on all the parameters of the problem including the skill levels of the $F_{j}$. We next establish the revenue that receives as a consequence of the solution to the VFP. Let $\prod^{*}=\pi_{1}^{*}+\pi_{2}^{*}$.

Corollary 2
At the Nash equilibrium of the VFP, the revenue to $F_{j}$ is equal to $\left(N_{j} / N\right) \prod^{*}$.

The solution to the VPF yields each $F_{j}$ the proportion $N_{j} / N$ of the sum of the profits produced by projects 1 and 2 at the Nash solution. This establishes that $R_{j}\left(N_{1}\right)=\left(N_{j} / N\right) \prod^{*}$ and that profit equals $\sigma_{j}\left(N_{j} / N\right) \prod^{*}$. It also follows that at the Nash equilibrium, the revenue per money for each of the $F_{j}$ is identical. We can now return to the problem facing the $I_{l}$, the original money holders.

## The Money Investment Problem

For the money investment problem (MIP) in which $I_{l}$ wishes to maximize $m_{1}, I_{l}$ must know $R_{1}(x+y)$ and $R_{2}(x+y)$. From Corollary 2 and the remarks following it, $R_{j}=p_{1 j}^{*} \pi_{1}^{*}+p_{2 j}^{*} \pi_{2}^{*}=\left(N_{j} / N\right) \prod^{*}$ where $\prod^{*}=\pi_{1}^{*}+\pi_{2}^{*}$. Having assumed that each $I_{l}$ has the same information concerning the bidding game played by $F_{1}$ and $F_{2}$ conditional on the funds having $N_{1}=x+y$ money points, and $N_{2}=N-N_{1}$ money points, respectively, it follows that each $I_{l}$ also knows the Nash equilibrium of the VFP as presented in Theorem 1. Consequently, the respective objective functions of the $I_{l}$ can be restated as:

$$
m_{1}(x, y)=\frac{x}{x+y} \sigma_{1} \frac{N_{1}}{N} \prod^{*}+\frac{V_{1}-x}{\left(V_{1}-x\right)+\left(V_{2}-y\right)} \sigma_{2} \frac{N_{2}}{N} \prod^{*}
$$

and

$$
m_{2}(x, y)=\frac{y}{x+y} \sigma_{1} \frac{N_{1}}{N} \prod^{*}+\frac{V_{2}-y}{\left(V_{1}-x\right)+\left(V_{2}-y\right)} \sigma_{2} \frac{N_{2}}{N} \prod^{*}
$$

Since $N_{1}=x+y$ and $N_{2}=N-N_{1}$, the last expressions can be reduced to:

$$
m_{1}(x, y)=\left[\sigma_{1} \frac{x}{N}+\sigma_{2} \frac{V_{1}-x}{N}\right] \prod^{*}(x+y)
$$

and

$$
m_{2}(x, y)=\left[\sigma_{1} \frac{y}{N}+\sigma_{2} \frac{V_{2}-y}{N}\right] \prod^{*}(x+y)
$$

Thus, in the money investment problem (MIP), investor $I_{1}$ seeks where

$$
x^{*}=\underset{x \in\left[0, V_{1}\right]}{\arg \max } m_{1}(x, y) \text { subject to } y \in\left[0, V_{2}\right]
$$

and investor $I_{2}$, seeks where

$$
y^{*}=\underset{y \in\left[0, V_{2}\right]}{\arg \max } m_{2}(x, y) \text { subject to } x \in\left[0, V_{1}\right]
$$

We next define an efficient allocation of money points. Let $N_{1}^{*}=\arg \max _{N_{1} \in[0, N]} \prod^{*}\left(N_{1}\right)$. Note that $N_{1}^{*}$ is an apportionment of money points to the VPFs that achieves the maximum total profit.

Definition 1
An allocation of money points $(x, y), x \in\left[0, V_{1}\right], y \in\left[0, V_{2}\right]$, is an efficient allocation if $x+y=N_{1}^{*}$.

The case when $\sigma_{1}=\sigma_{2}=\sigma$. We continue by investigating the case in which the $F_{j}$ pay out the same proportion of their revenues to the $I_{i}$; that is the case when $\sigma_{1}=\sigma_{2}=\sigma$. In this situation, $m_{1}=\sigma\left(V_{1} / N\right) \prod^{*}(x+y)$. Since increasing $\prod^{*}(x+y)$ benefits both $I_{l}$ it is in their joint interest to achieve the largest possible

* by their respective money investments. It follows that it is in the interest of the $I_{l}$ to choose their money investments $x^{*}$ and $y^{*}$, respectively, such that $x^{*}+y^{*}=N_{1}^{*}$ i.e., to choose their investments to be efficient. It also follows that there exists an infinity of equilibria to the VIP of the form $\left(x^{*}, y^{*}\right)$ where $x^{*}=N_{1}^{*}-y^{*}$ for $x^{*} \in\left[0, V_{2}\right]$ and for $y^{*} \in\left[0, V_{2}\right]$. We summarize the previous remarks in the following theorem.

Theorem 2
When $\sigma_{1}=\sigma_{2}=\sigma$ there exists an infinity of equilibria to the MIP consisting of the set of efficient allocations.

But despite the fact that the $I_{l}$ find it in their interest to have $x^{*}+y^{*}=N_{1}^{*}$, the non-cooperative nature of the Nash game offers no mechanism to cause the target $N_{1}^{*}$ to be met. Since the target represents the division of the total number of money points in the system between the $F_{j}$ that maximizes economy-wide profit, there is consequently no mechanism to achieve this efficient outcome. Thus, the failure to achieve efficiency is the result of the absence of coordination between the money holders.

Notice that this coordination failure is present even in the case in which the money point holders have identical and full information, and have as their goal the wish to allocate their money points in a manner consistent with the maximization of economy-wide profit.wenow show that the introduction of uncertainty exacerbates the situation since it creates a situation in which the goal of the money holders is no longer one of maximizing total economy-wide profit; in fact,weshow that the goal differs for the different money point holders.

When uncertainty is present,wemust consider the investors' attitudes toward risk. To this end, we let $u_{l}: \mathfrak{R}_{+} \rightarrow \Re_{+}$with $u_{l}\left(m_{l}\right)=1-\exp \left(-\gamma_{l} m_{l}\right), \gamma_{l}>0$ be the utility function of $I_{l}$. We assume that all information is known to the money point holders as before, with one exception: $\Delta_{1}$ is known imperfectly. We assume that both money point holders perceive $\widetilde{\Delta}_{1}$ as a random variable, distributed normally with mean $\Delta_{1}$ (as before) and variance $\sigma^{2}$. We denote this density as $\phi\left(\Delta_{1}, \sigma^{2}\right)$. It follows that $\widetilde{\Pi}^{*}$ is random since $\widetilde{\Pi}^{*}=k_{12}+k_{21}+p_{11} \Delta_{1}+p_{22} \Delta_{2}$. The expectation of any function of $\widetilde{\Delta}_{1}$ with respect to $\phi$ is denoted by $E_{\phi}$. Thus $E_{\phi} \widetilde{\Pi}^{*}=\Pi^{*}$ with $\Pi^{*}$ as before. Let $N^{*}(\phi)=\max _{N 1} E_{\phi} \Pi^{*}=\max _{N 1} \Pi^{*}\left(N_{1}\right)$. We define Assumption A to be made up of the following statements:

$$
\begin{aligned}
& I_{l} \text { has utility function } u_{l}\left(m_{l}\right)=1-\exp \left(-\gamma_{l} m_{l}\right), \gamma_{l}>0 \text {; } \\
& I_{l} \text { is a von Neumann-Morgenstern expected utility of wealth maximizer; } \\
& \sigma_{1}=\sigma_{2}=\sigma ; \\
& \widetilde{\Delta}_{1} \text { is distributed as } \phi\left(\Delta_{1}, \sigma^{2}\right) ; \\
& \text { All other information is known with certainty; } \\
& \text { Both } I_{l} \text { have the same information; and } \\
& \text { The funds } F_{j} \text { are risk-neutral. } \\
& \text { In what follows, we let } N_{11} \text { be the target of } I_{l} \text { and } N_{12} \text { be the target of } I_{2} \text {. }
\end{aligned}
$$

## Theorem 3

In the presence of uncertainty about the difference in the differential impact of the funds' skills on the profit of company 1 , and if $F_{1}$, is expected, but uncertain, to be more skilled than $F_{2}$ in managing company 1, then risk-averse money holders allocate fewer money points to $F_{2}$ compared to the certainty case, and more money points to resulting in an inefficient allocation of money points among the funds. In particular, let Assumption A hold. Let $\gamma_{1} V_{1} \neq \gamma_{2} V_{2}, \Delta_{1}>0$ and $N_{1}^{*}(\phi) \in(0, N)$. Then there exists a constant $c$ such that for $\sigma^{2} \in(0, c), N_{11} \neq N_{12}, N_{11}<N_{1}^{*}(\phi)$ and $N_{12}<N_{1}^{*}(\phi)$.

We see from Theorem 3 that the immediate impact or the introduction of uncertainty regarding the relative skills of the funds on the profit of company 1 causes a shifting of money points away from $F_{1}$. As a consequence, even if $N_{1}^{*}(\phi)$ were close to $N, F_{2}$ would receive more money points as the uncertainty increases. Earlier we showed that when $\sigma_{1}=\sigma_{2}$ and when all information was known with certainty, each $I_{l}$ strove to achieve the target $N_{1}^{*}$, which, if achieved, would maximize the money holders' respective wealths as well as implement the efficient outcome. That is, the money holders were aiming at the right target; a coordination
failure, however, prevented them from achieving it. This suggested that had a coordination mechanism existed, the efficient allocation would have been implemented. Now, with the introduction of uncertainty into the model, we see that the target at which the $I_{l}$ aim is not the optimal value $N_{1}^{*}(\phi)$ and the $I_{l}$ may have different targets, both unequal to $N_{1}^{*}(\phi)$. Coordination would not resolve this inefficiency. Though we introduced uncertainty only in regard to $\Delta_{1}$, any broader introduction of uncertainty would have further exacerbated the problem. It is not surprising that the introduction of uncertainty results in a sub-optimal solution. However, wenext show that even with certainty and with complete information, when the payouts of the funds to the differ, inefficiency also results.
The case when $\sigma_{1} \neq \sigma_{2}$. We have assumed so far that the $F_{j}$ have identical cost structures. Generally, however, since the $F_{j}$ are not identical, they could have different cost structures, leading them to select different percentages of their revenues to pay out, that is, $\sigma_{1} \neq \sigma_{2}$. When $\sigma_{1} \neq \sigma_{2}$, it is no longer true that the $I_{l}$ will both benefit by seeking to maximize $\Pi^{*}$ since the share of $\Pi^{*}$ that $I_{l}$ receives depends, in this case, on the investments $x^{*}$ and $y^{*}$. Importantly, for the case $\sigma_{1} \neq \sigma_{2}$, the optimal choices of $x^{*}$ and $y^{*}$ by $I_{1}$, and $I_{2}$, respectively, need not always produce a division of the money points consistent with the maximization of economy-wide profit. We show these results to be true in Theorem 4, where we present the solution to the VIP when $\sigma_{1} \neq \sigma_{2}$. To make this point as starkly as possible, we let.

Theorem 4
Even with certainty and even if the money holders start with the same number of money points, when the payouts of the funds differ, the unique Nash equilibrium of the VIP leads to a common inefficient target. In particular, let $\sigma_{1} \neq \sigma_{2}, V_{1}=V_{2}=V$ and let

$$
G\left(N_{1}\right)=\Pi^{*}\left(N_{1}\right)+\frac{N}{2}\left(\frac{N_{1}}{N}+\frac{\sigma_{2}}{\sigma_{1}-\sigma_{2}}\right) \frac{d \Pi^{*}\left(N_{1}\right)}{d N_{1}} \text { for } N_{1} \in(0, N) \text {. }
$$

Then the unique Nash equilibrium of the VIP is:

$$
x^{*}=y^{*}=\frac{N_{1}^{\circ}}{2},
$$

where either $N_{1}^{\circ} \in(0, N)$ and satisfies $G\left(N_{1}^{\circ}\right)=0$ or $N_{1}^{\circ}=0$ or $N$.
If $\Delta_{j} \neq 0$ for at least one value of $j$ and $N_{1}^{\circ} \in(0, N)$, then $N_{1}^{\circ} \neq N_{1}^{*}$.
When payouts are different, each $I_{l}$ will invest $N_{1}^{\circ} / 2$ in $F_{1}$, yielding a total of $N_{1}^{\circ}$ money points to $F_{1}$. Since $N_{1}^{\circ} \neq N_{1}^{*}$, $N_{1}^{\circ}$ will not be the efficient allocation of money
points to $F_{1}$, and thus will not maximize total economy-wide profit. Additionally, whereas a coordination failure between the $I_{l}$ is responsible for inefficient outcomes when $\sigma_{1}=\sigma_{2}$, even permitting coordination when $\sigma_{1} \neq \sigma_{2}$ would not result in an efficient outcome. That is, when $\sigma_{1} \neq \sigma_{2}$, the goal of the money holders is not the goal of maximizing total economy-wide profit, as it was for the case when $\sigma_{1}=\sigma_{2}$.

## SELECTED READINGS

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