DETECTING THE NUMBER OF COMPONENTS IN A NON-STATIONARY SIGNAL USING THE RÉNYI ENTROPY OF ITS TIME-FREQUENCY DISTRIBUTIONS

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Abstract: *A time-frequency distribution provides many advantages in the analysis of multicomponent non-stationary signals. The simultaneous signal representation with respect to the time and frequency axis defines the signal amplitude, frequency, bandwidth, and the number of components at each time moment. The Rényi entropy, applied to a time-frequency distribution, is shown to be a valuable indicator of the signal complexity. The aim of this paper is to determine which of the treated time-frequency distributions (TFDs) (namely, the Wigner-Ville distribution, the Choi-Williams distribution, and the spectrogram) has the best properties for estimation of the number of components when there is no prior knowledge of the signal. The optimal Rényi entropy parameter α is determined for each TFD. Accordingly, the effects of different time durations, bandwidths and amplitudes of the signal components on the Rényi entropy have been analysed. The concept of a class, when the Rényi entropy is applied to TFDs, is also introduced.*

Keywords: – nonstationary signals
– time-frequency distributions
– Rényi entropy

1. INTRODUCTION

Time-frequency distributions (TFDs) show non-stationary signals simultaneously in the time and frequency domain, indicating the presence of individual components [1, 2]. A fundamental tool for measuring the information content of a given probability distribution is the entropy function. The marginal properties exhibited by quadratic TFDs [1] are the same as those of probability density functions, therefore the generalized Rényi entropy (RE) can be applied to quadratic TFDs as a tool for detecting the number of components in the signal [3,4,5]. Information measures used in probability theory have been also applied to the time-frequency (TF) plane in [3], by treating TFDs, $\rho_z(t,f)$ as density functions [5]. The α^{th} order RE of the normalized TFD is defined as [4, 5, 6, 7]:

$$R_\alpha = \frac{1}{1-\alpha}\log_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{\rho_z(t,f)}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho_z(u,v)\,dudv}\right)^{\alpha} dtdf. \quad (1)$$

The RE is well-defined as long as the integral in Eq. (1) is greater than zero [4]. As α approaches 1, Eq. (1) approaches the Shannon entropy function [4] (due to the logarithm in the definition, $\alpha = 1$ is not recommended for time-frequency distributions with negative values). Since the cross-terms oscillatory structure cancels under the integration with odd powers, α should be an odd integer value for a general quadratic TFD [3, 4].

An important property of the RE is its ability to count the number of signal components in a TFD [4]:

$$R_\alpha(A_n) - R_\alpha(A_1) = \log_2 n \quad (2)$$

where $R_\alpha(A_1)$ is the entropy value of one component of the signal, and $R_\alpha(A_n)$ is the entropy value of n components. In the case of a two component signal, Eq. (2) becomes:

$$R_\alpha(A_2) - R_\alpha(A_1) = \log_2 2 = 1. \quad (3)$$

It is worth noting that shifting the signal into the TF plane doesn't affect the value of its RE [4].

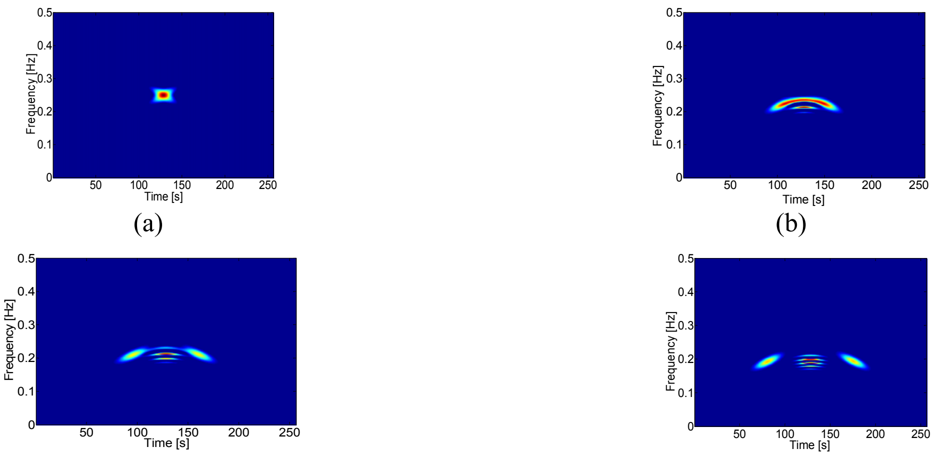
2. SIGNAL FEATURES AND THE RE

2.1. Components Separation in Time and the RE

The effects of different choices of the parameter α have been tested on a signal whose components present variable distances in time. The components are two Gabor logons whose time distance increases from 0 to 100 s. It is shown in [4] that, for a quadratic TFD, if the first component is supported on the time interval $[0,\varepsilon]$ and the second one is supported on $[\Delta t, \varepsilon + \Delta t]$, then

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X^{\alpha}{}_{1,2}(t,f)\, dt\, df = 0 \tag{4}$$

holds when $\Delta t > 1/2(\alpha+1)\varepsilon$, for odd α and if the components are located on the same frequency $(\Delta f = 0)$, where $X^{\alpha}{}_{1,2}(t,f)$ is the TFD of the cross-component [7, 8, 9] and Δt is the time separation between the components. This means that for odd α, the oscillatory structure of $X^{\alpha}{}_{1,2}(t,f)$ [8, 9, 10], cancels under the integration for Δt sufficiently large [6]. This constraint does not refer to the spectrogram, which is always positive. For $\alpha = 3$, the right side of Eq. (5) has minimal value, thus Eq. (3) holds for close components in time. Fig. 1 shows the Wigner-Ville distribution (WVD) of two Gabor logons with increasing time separation, while the RE for different values of α w.r.t. the time distance between the Gabor logons is shown in Fig. 2. It is also worth pointing out that for even α, Eq. (5) is not valid, because the cross-components are not annulated in the integration. Table 1 summarizes the effect of time separation Δt on the RE for the signal in Fig. 1.



(a) (b) (c) (d)

Figure 1. Separation of two Gabor logons in the TF plane: Δt=0 s (a), Δt=35 s (b), Δt=65 s (c), Δt=95 s (d)

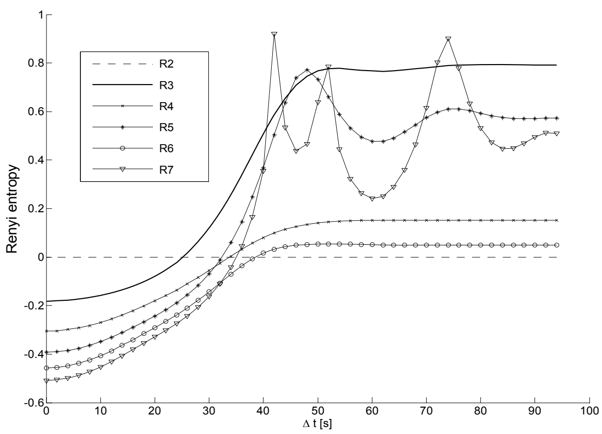


Figure 2. RE for different α w.r.t. the time separation between the two Gabor logons

Table 1. The transition of the RE in Fig. 2

$\Delta t \in [0, 10]$	only one component is detected.
$\Delta t \in [10, 40]$	the components are beginning to separate, the entropy rises exponentially.
$\Delta t \in [40, 50]$	the entropy reaches its maximum.
$\Delta t \in \left[50, \frac{1}{2}(\alpha+1)\varepsilon\right]$	the maximum entropy starts to oscillate; the oscillation are reduced as Δt approaches $\frac{1}{2}(\alpha+1)\varepsilon$.
$\Delta t \in \left[\frac{1}{2}(\alpha+1)\varepsilon, \infty\right]$	the oscillations stop, and the components are completely separated. For R_3 the oscillations are the smallest and the settling time is the shortest.

2.2. Components Separation in Frequency and the RE

To show the effects of different choices of the parameter α on the RE, for a signal whose components have a variable frequency separation, a signal with two linear frequency modulated (LFM) components has been chosen. Fig. 3 shows the WVD of the test signal for the component frequency separation Δf varying from 0 to 0.2 Hz.

Fig. 4 illustrates the effects of different parameters α on the RE of the signal whose frequency separation between the components increases and it can be seen that for even α, Eq. (3) is not valid. The best results are obtained for α = 3 since the oscillations (that are evident for α = 5 and α = 7) are almost non-existent and the shortest transition period is achieved. As test distributions we have chosen the popular WVD, the intuitive and non-negative spectrogram (SP), and the Choi-Williams distribution (CWD), which presents reduced interferences. Our extensive simulations have shown that for the SP and the CWD, the choice of the parameter α brings minimal differences in the behavior of the RE for variable time and frequency separations of the signal components. In the case of α = 7, a slightly shorter transition period has been observed for both the SP and the CWD, and thus in the rest of the paper, the simulations including the RE of the SP and the CWD will be performed for $\alpha = 7$. All previous works in the literature referring to the components counting using the RE imply the knowledge of the RE of at least one component [4].

This information however, is often unavailable in practical situations. We have afterwards examined the effects of time duration and frequency bandwidth of signals on the RE of different TFDs in order to find out if the number of signal components can be obtained from the RE of the TFD with no a priori information of the signal.

2.3. The Components Frequency Bandwidth and the RE

The effects of different frequency bandwidths on the RE is illustrated on the examples of two signals A_1 and A_2, shown in Fig. 5 and 6, respectively. The signal A_1 is the one-component LFM whose frequency changes from 0.0 to 0.5 Hz with the time duration of 256 s. The signal A_2 is a cosine signal whose frequency is 0.25 Hz with the time duration of 256 s. The RE values for the two signals TFDs are shown in Table 2.

Table 2. The RE of A_1 and A_2 for different TFDs

	R_7 (SP)	R_7 (CWD)	R_3 (WVD)
A_1	3.9383	3.0479	0.0672
A_2	2.6686	2.0761	-0.0845

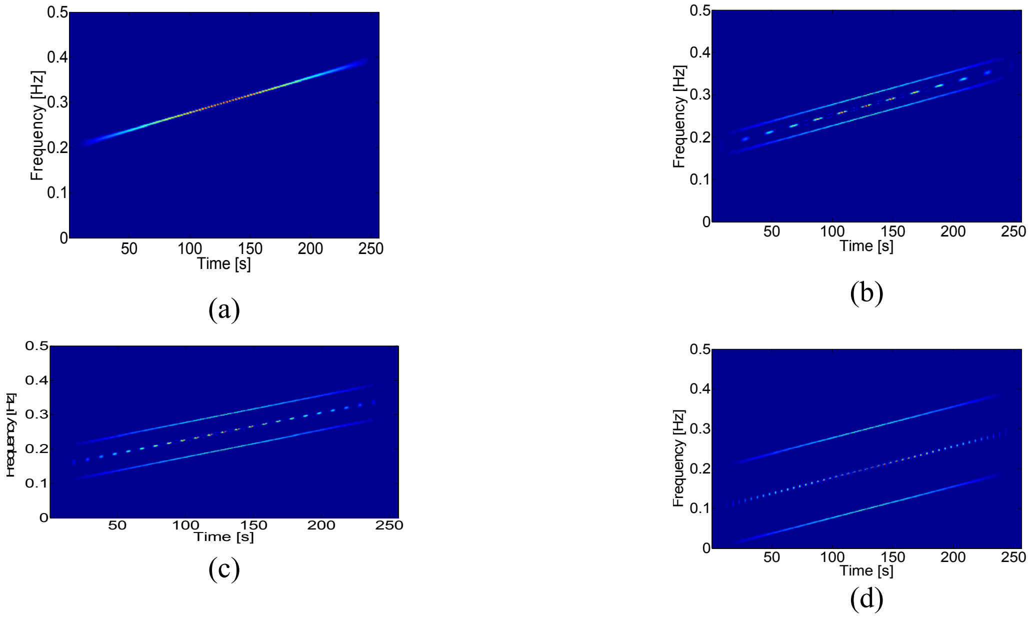


Figure 3. Separation of two LFM components in the TF plane: $\Delta f = 0$ Hz (a), $\Delta f = 0.05$ Hz (b), $\Delta f = 0.1$ Hz (c), $\Delta f = 0.2$ Hz (d)

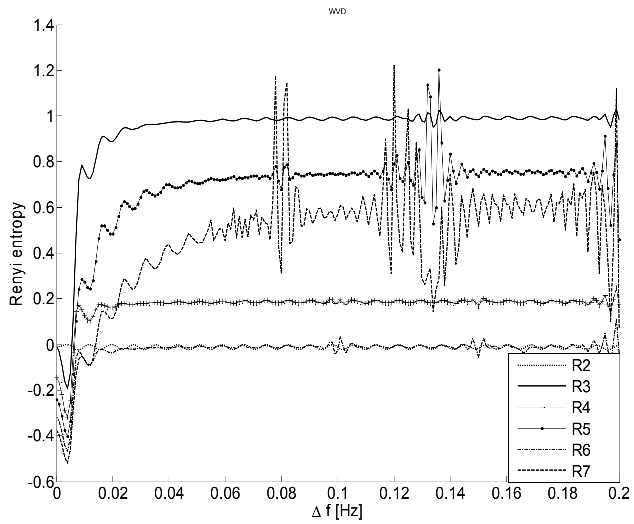


Figure 4. RE for different parameters α with respect to the time separation between the LFM components

Table 2 shows that the bandwidth increase exerts the least influence on the RE of the WVD. Let us now exemplify the result obtained for the RE of the SP of the signal A_1, 3.938: from this value it can't be found out whether the signal has one component with long or two components with short frequency bandwidths since the RE of a signal with two components with constant FMs and with the same time duration achieves the value of 3.669. The CWD shows the same limitation. In the WVD case, however, the RE is close to zero. Therefore, we can say that the signal has one component since in the case of a two-component signal, the RE of the WVD is approximately one.

2.4. The Signal Measurement Time and the RE

Until now, all the considered signals have had the same time axis length of 256 s. Subsequently, we have examined how the changes in the time axis length (the signal measurement time), $M[\mathrm{s}]$, affect the RE.
Providing that our test signal is a Gabor logon with 32 s duration in time and centered at 200 s, the RE is estimated to be over a range of values of M for this test signal and the three TFDs. The results for the WVD, the SP, and the CWD are shown in Fig. 7.
In the case of the WVD, for $\alpha = 3$, the value of the RE stays unchanged, i.e. it is not a function of M. For the SP and the CWD, R_α is a nearly linear function of M. We need to mention that in the case of the CWD instead of $\alpha = 7$, we have used $\alpha = 3$ for faster simulation. The simulations have shown that the WVD is the least sensitive to variations in the signal duration.

2.5. The Signal Time Duration and the RE

Let us next consider the effect of changing the time duration of the signal $T \neq \mathrm{const.}$ on the RE. As a test signal, a one-component LFM (with its frequency increasing from 0.0 to 0.5 Hz) and with the changing time duration from 0 to 2000 s will be used. Fig. 8 shows that among the tested distributions, the RE is independent of the time duration T only in the case of the WVD.
From the results presented in this paper we can conclude that due to its invariance to signal time duration and frequency bandwidth changes in the third order RE of the signal WVD, the WVD is the most suitable TFD for determining the number of components from a single value of the RE.

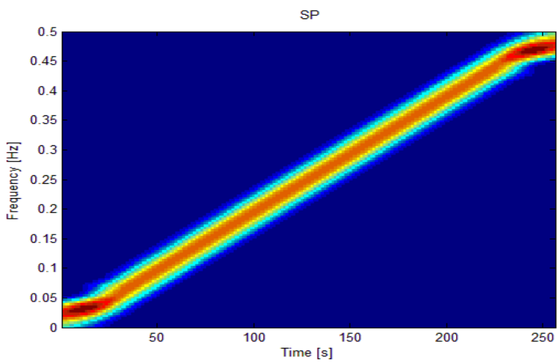


Figure 5. SP of the signal A_1

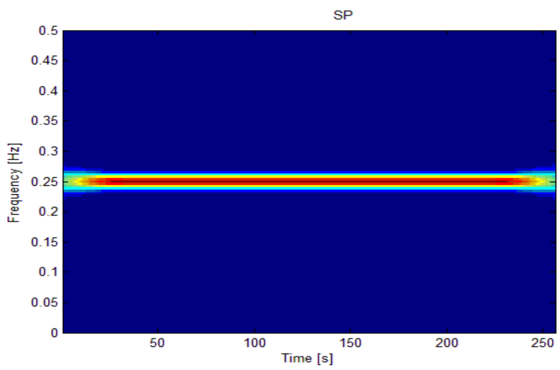


Figure 6. SP of the signal A_2

2.6. The effect of the amplitude on the RE

The aim of this test is to analyze the effect of different amplitudes of the signal components on the RE of the TFD. As a test signal, two Gabor logons will be used, one with the constant amplitude and another with the variable one. The amplitude ratio A is defined in the time domain. Fig. 9 shows the obtained results. When the amplitude of one of the components is larger than the amplitude of the other component by more than three times, the RE will count only the dominant component. Therefore, the use of the RE for estimating the number of components in a signal is restricted only to the signals whose components have similar amplitudes.

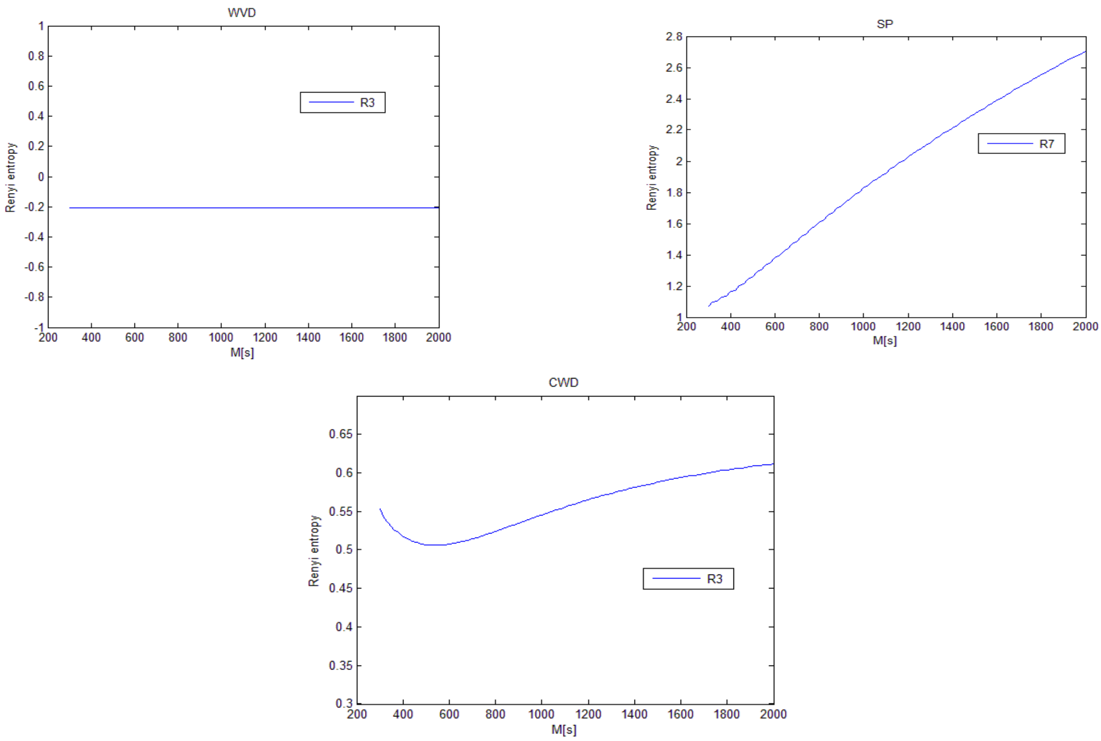


Figure 7. RE as a function of the signal measurement time M for the WVD, SP and CWD od a Gabor logon

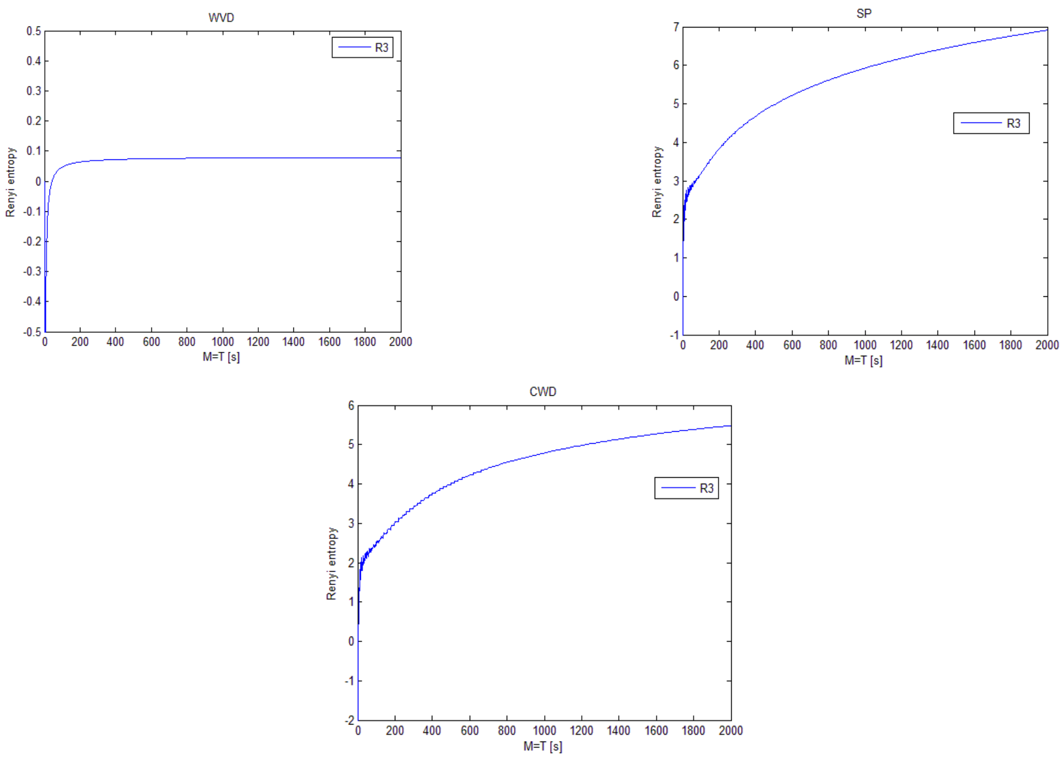


Figure 8. RE as a function of the signal duration for the WVD, SP and CWD of a LFM signal

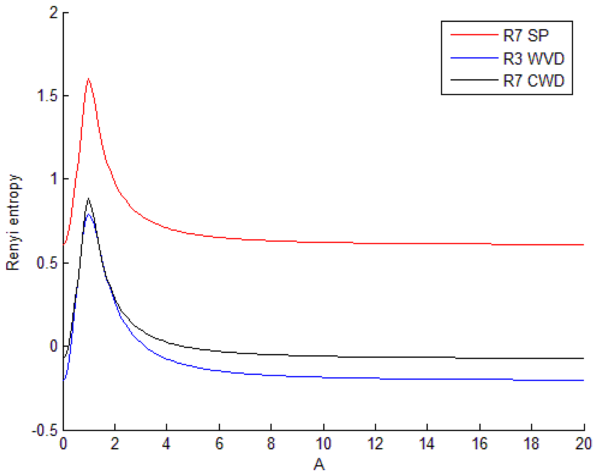


Figure 9. RE of different TFDs w.r.t the amplitude ratio of the two Gabor logon

3. THE RE OF A SIGNAL CLASS

A class represents a group of specific components. Fig. 10 and 11 illustrate a multicomponent signal and a class. If a Gabor logon and a cosine signal are a class, the signal in Fig. 10 is then composed of three such classes.

Let $R_\alpha(K_n)$ be the RE of n classes, $n \in Z^+$, and let $R_\alpha(K_1)$ be the RE of one class. Then the number of identical classes present in the signal is:

$$R_\alpha(K_n) - R_\alpha(K_1) = \log_2 n \qquad (5)$$

where the *class* K_1 can be made of an arbitrary set of components.

For the class in Fig. 11 and the signal in Fig. 10, from Eq. (6) we have:

$$R_\alpha(K_n) - R_\alpha(K_1) = 4.4012 - 2.8116 = \log_2 n \qquad (6)$$

$$n = 3.01 \approx 3 \qquad (7)$$

The number of classes will also be correctly detected if either the WVD or the CWD had been used.

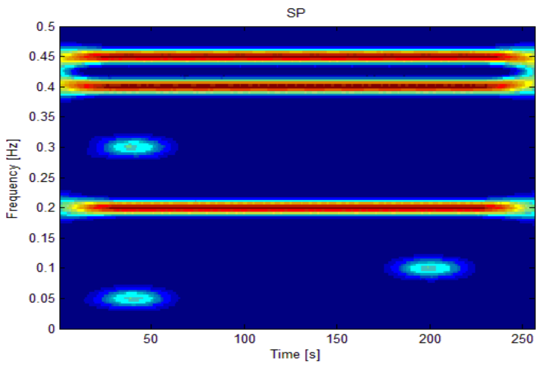


Fig. 10 SP of a multicomponent signal

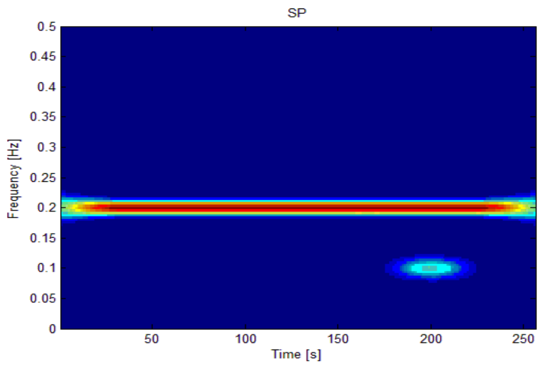


Fig. 11 SP of a signal class composed of two components

4. CONCLUSION

An analysis of the properties of the Rényi entropy is presented as a complexity measure of a non-stationary multicomponent signals when applied to their time-frequency distributions. The parameter α determines the performance of the Rényi entropy as an indicator of the signal complexity. Accordingly, this paper has determined the optimal α for each of the considered time-frequency distributions (the WVD, the SP, and the CWD). Varying the time duration and frequency bandwidth of the signals, the WVD is concluded to be the most suitable among the tested distributions for determining the component number of a signal from a single value of the RE. It is also shown that the counting property of the RE depends on the relative amplitudes of the signal components. Finally, it is shown that the content of a multicomponent signal can be estimated by using the RE not only with respect to single components, but a *class* of components as well.

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