Zagreb International Review of Economics & Business, Vol. 9, No. 2, pp. 1-11, 2006

ISSN 1331-5609; UDC: 33+65

Capacity Constraints and Investment Decisions under Cournot Competition

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Abstract: This paper analyses how the equilibrium is affected when adding investment decisions and capacity constraints to the traditional Cournot duopoly model. Authors investigate a multiperiod setting with two firms taking investment decisions in every period. We prove that under these circumstances the Cournot equilibrium is unstable and the tendency is to a cartel structure in the industry. However, this behavior is not necessarily cooperative or subject to a tacit agreement. It is optimising for the duopolists to cut down the amount produced in spite of the behavior of the other firm until they reach the monopoly equilibrium.

Key words: cartel, Cournot, capacity, duopoly, investment

JEL Classification: L10-13, L19

Introduction

This paper analyses how the equilibrium is affected when adding investment decisions and capacity constraints to the traditional Cournot duopoly model.[1] We investigate a multiperiod setting (infinite number of periods) with two firms taking investment decisions in every period. Firms produce a homogeneous good and are profit maximisers taking into account the quantity produced by the other firm when deciding how much to produce and invest. The only cost of production that firms face is the investment cost. There are no direct production costs such as labor or material costs. The investment cost is variable in the period when it takes place, but sunk for the following ones.

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The demand is completely known before the investment takes place and the production capacity is constrained by the investment incurred in the present and all past periods minus the depreciation of the capital. The price in the market is cleared after both firms observe the total amount of production.

We prove that under these circumstances the Cournot equilibrium is unstable and the tendency is to a cartel structure in the industry. However, this behavior is not necessarily cooperative or subject to a tacit agreement. It is optimising for the duopolists to cut down the amount produced in spite of the behavior of the other firm until they reach the monopoly equilibrium.

The Model

Suppose that we have two identical firms, with the same cost structure, facing a known linear demand of the form

$$p_t = 1 - q_t \tag{1}$$

The marginal cost is constant for quantities up to a certain production limit and equal to infinity (i.e. vertical) at that point (inverted L-shape).

Let the production function be

$$q = K \tag{2}$$

where one unit of investment produces one unit of output.

The depreciation rate, δ, is constant and equal in every period, $0 < \delta < 1$. $\alpha = 1 - \delta$ is the percentage of capital invested on previous periods that has not been depreciated. The price of one unit of capital is \$r. Then, the cost function for the t period is:

$$TC_t = rK_t \tag{3}$$

and the marginal cost is:

$$\begin{array}{lll} 0 & if & q_t \leq \sum_{i=0}^{t} \alpha^{(i-t)} K_i \\ 4 & if & q_t > \sum_{i=0}^{t} \alpha^{(i-t)} K_i \end{array} \tag{4}$$

Each duopolist maximise its discounted profits value over an infinite horizon, where d denotes the discount factor and i the real interest rate, given by the following relation:

$$d = 1/(1+i) \tag{5}$$

The interest rate is known and constant for every period. The firms' maximisation problem can be given by the following Lagragians functions for firms one and two respectively:

$$L_1 = \sum_{t=1}^{\infty}\left[(1-q_{1t}-q_{2t})q_{1t} - rK_{1t}\right]d^{t-1} - \sum_{t=1}^{\infty}\lambda_{1t}\left[q_{1t} - \sum_{j=1}^{t}\alpha^{t-j}K_j\right] \tag{6}$$

$$L_1 = \sum_{t=1}^{\infty}\left[(1-q_{2t}-q_{1t})q_{2t} - rK_{2t}\right]d^{t-1} - \sum_{t=1}^{\infty}\lambda_{2t}\left[q_{2t} - \sum_{j=1}^{t}\alpha^{t-j}K_j\right] \tag{7}$$

The capacity constraint holds for each period. λ denotes the Lagrange multiplier.

The choice variables in each period are the quantity produced q_{it} and the amount of capital to invest K_{it}. We show the model for duopolist one only, as both firms are identical.

The first-order conditions for firm 1 are:

$$L_{q11}: (1-2q_{11}-q_{21}) - \lambda_{11} = 0 \quad \text{for firm 1, period 1} \tag{8}$$
$$L_{q12}: d(1-2q_{12}-q_{22}) - \lambda_{12} = 0 \quad \text{for firm 1, period 2} \tag{9}$$
$$L_{q1t}: d^{t-1}(1-2q_{1t}-q_{2t}) - \lambda_{1t} = 0 \quad \text{for firm 1, period t} \tag{10}$$

Equations (8), (9) and (10) are the familiar conditions where marginal revenue equals marginal cost. In our model we are dealing with the discounted value of marginal cost (the lagrange multiplier λ_t) and the discounted value of marginal revenue for period t.

The first-order conditions, when differentiating with respect to capital, are:

$$L_{K11}: -r + \sum_{t=1}^{\infty}\lambda_{1t}\alpha^{t-1} = 0 \quad \text{for firm 1, period 1} \tag{11}$$

$$L_{K12}: -dr + \sum_{t=2}^{\infty}\lambda_{1t}\alpha^{t-2} = 0 \quad \text{for firm 1, period 2} \tag{12}$$

$$L_{K11}: -d^{t-1}r + \sum_{i=t}^{\infty}\lambda_{1i}\alpha^{i-t} = 0 \quad \text{for firm 1, period t} \tag{13}$$

From equation (11), we see that the cost of one unit of capital in period one (r) is spread into each of the subsequent periods where this capital unit can effectively produce output. From equations (11), (12) and (13) and after some algebraic manipulation we get the marginal cost function:

$$\lambda_1 = r(1-\alpha d) = d\left[r(\delta + i)\right] \tag{14}$$

$$\lambda_2 = d\lambda_1 = dr(1-\alpha d) = d^2\left[r(\delta+i)\right] \tag{15}$$
$$\lambda_n = d^{n-1}\lambda_1 = d^{n-1}r(1-\alpha d) = d^n\left[r(\delta+i)\right] \tag{16}$$

From equation (14) we see that for each dollar spent on capital investment (r) there is an opportunity cost involved, that is equal to the interest rate (i) and a depreciation rate (δ), with the discount factor d applied. The discounted value of the opportunity cost plus the depreciation rate is equivalent to the marginal cost of the period ($d\left[r(\delta+i)\right]$). We assume that the term $(d\left[r(\delta+i)\right]) < 1$ to ensure concavity of the profit function. This guarantees that the point chosen by the firms is at a maximum and not a minimum and avoids dealing with the second-order conditions.

Equations (15) and (16) show that the marginal costs are equal in each period and the value of the marginal revenue in each period is the same as well. Thus, under this scenario, the tangency conditions for this problem are:

$$\frac{MR_2}{MR_1} = \frac{MR_{t+1}}{MR_t} = \frac{MC_2}{MC_1} = \frac{MC_{t+1}}{MC1} = d \tag{17}$$

Given that MC and MR are equal in every period, in equilibrium these ratios are equal to the discount factor d, which is assumed to be constant throughout time.

The derivative of the Lagrangean with respect to the multiplier is the output constraint as seen in equation (18).

$$L_{\lambda 1t} : q_{1t} - \sum_{j=1}^{t} \alpha^{t-j} K_j = 0 \tag{18}$$

Replacing (16) and (17) into (18) and after some algebraic manipulation we get:

$$q_{11} = q_{12} = q_{1t} = q_1 \tag{19}$$
$$K_{11} = q_1, K_{12} = \delta q_1 \text{, in general} \tag{20}$$
$$K_{1t} = \delta^{t-1} q_1 \tag{21}$$

From these equations it is clear that the duopolists always produces the same amount in every period, and they purchase only an amount of capital enough to replace depreciation. Thus, in period one, the investment amount is equal to the production, as seen in equation (20), and for the subsequent periods the firms invest the amount that has been depreciated in the previous period. See equation (21).

This applies to firm two as well. Thus, the reaction functions are obtained from equating both duopolists' first-order conditions:

$$R_1 ; q_1 = \frac{1 - q_2 - r(1-d\alpha)}{2} \tag{22}$$
$$R_2 ; q_2 = \frac{1 - q_1 - r(1-d\alpha)}{2} \tag{23}$$

From (22) and (23) the following Cournot equilibrium is obtained:

$$q_1 = q_2 = q_c = \frac{1 - r(1 - d\alpha)}{3}, \; p_c = \frac{1 + 2r(1 - d\alpha)}{3} \tag{24}$$

Replacing these amounts in the profit functions and after some algebraic manipulation the indirect profit functions are obtained:

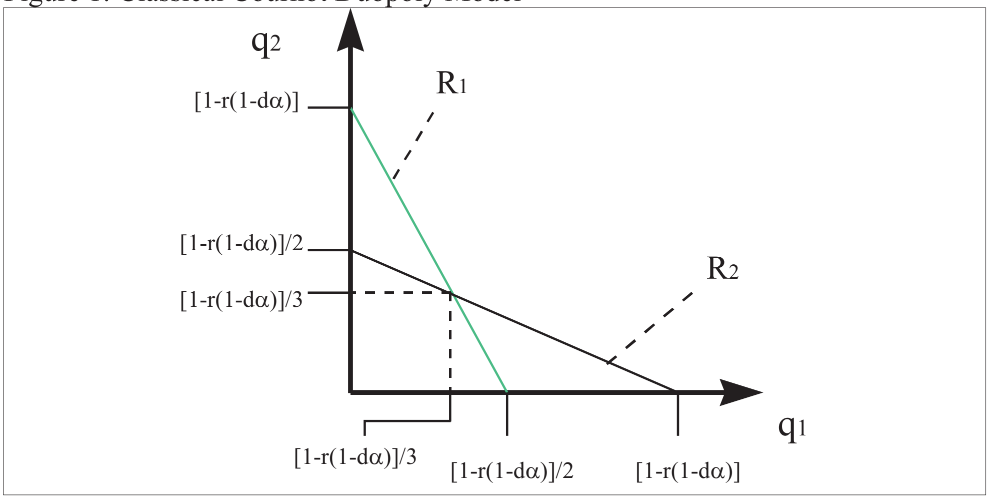
$$\Pi_1 = \Pi_2 = \Pi c = \left[\frac{1 - r(1 - d\alpha)}{3}\right]^2 \left[\frac{1}{1-d}\right] = q_c^2 \left[\frac{1}{1-d}\right] = \sum_{t=0}^{\infty} \frac{q_c^2}{\left[1+i\right]^t} \tag{25}$$

Equation (25) indicates that the present value of the profit is equal to the perpetuity of the square of the amount produced from time period zero to infinity. This means that in each period every duopolist has a profit equal to the square of the amount produced, (q_c^2).

The second-order conditions are needed to ensure that the above point is indeed a maximum and not a minimum. The assumption that $d\left[r(\delta + i)\right] < 1$ and the linearity of the demand function gives the needed global concavity of the profit function.

Equations (22) and (23), the reaction functions, are plotted in Figure 1.

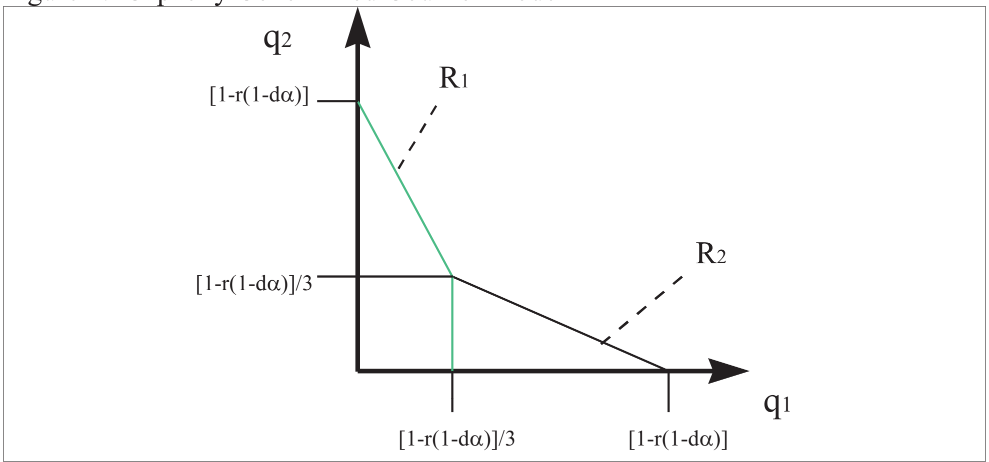
Figure 1: Classical Cournot Duopoly Model



In the classical Cournot duopoly model the reaction functions are continuous, differentiable and smooth as seen in Figure 1. These regularity conditions allow that any time that one firm's output changes, the other firm will react. In fact, assuming perfect information, one duopolist will always know the other duopolist actions, and thus it will always respond. In equilibrium, the conjectural variation is zero, which means that each firm thinks that its change in production will not have a response from the other firm.

However, the model developed here is quite different. The reaction functions are neither differentiable nor smooth as shown in Figure 2.

Figure 2: Capacity Constrained Cournot Model



This model assumes that firms take the decision about their production capacity at the beginning of each period. Therefore, the duopolist does not have a complete freedom to react to the other firm's actions because of its capacity constraint.

In fact, each firm's production must lie between zero and the level of capital obtained in the period. In terms of the reaction functions, if for any reason firm two decides to reduce his level of production, firm one cannot consequently increase its output because its production capacity is constrained. This is one of the main differences with respect to the original Cournot model of Figure 1. The kinks in the reaction functions are the key in giving unstability to the traditional Cournot equilibrium under the assumptions of this model, and lead to the conclusion that a cartel equilibrium is optimal for this structure.

Before analyzing the stability of the equilibrium of this model, we show the solution of this problem under the assumption of a monopoly structure (for the details see Appendix A).

$$q_m = \frac{1 - r(1 - d\alpha)}{2}, \; p_m = \frac{1 + r(1 - d\alpha)}{2} \text{ for every period.} \tag{26}$$

and

$$\Pi_m = \left[\frac{1-r(1-d\alpha)}{2}\right]^2 \left[\frac{1}{1-d}\right] = q_m^2 \left[\frac{1}{1-d}\right] = \sum_{t=0}^{\infty} \frac{q_m^2}{[1+i]^t} \tag{27}$$

This solution involves a smaller total amount produced and a higher price.

Recalling equation (24), the duopoly strategy given the maximisation problem is to set each firm's level of production in

$q_c = \dfrac{1-r(1-d\alpha)}{3}$ for every period.

Suppose that firm one decides to reduce its level of production for one period of time only, given that it knows that it is not possible for the other firm to react by increasing its production because of its capacity constraints. Suppose further, that firm one doesn't give any signals to the other one about its intentions. Let q_0 be the first firm's reduced output:

$$q_0 = \frac{1-r(1-d\alpha)}{6} \tag{28}$$

Given q_0, the industry production is the monopoly amount for this period. q_0 is one half of q_c and one fourth of the total Cournot amount. Firm one sets the Cournot equilibrium amount for all the subsequent periods. Therefore, the profit function for firm one is:

$$\Pi_0 = q_m^2 + d\Pi_c + drq_0 a \tag{29}$$

The first term of this expression, is the monopoly profit for period one times its market share (p). The second term is the discounted value of the Cournot profit from period two to infinity. The last term is the discounted value of the savings in the cost of investment for period two, which is equal to the investment of period one minus depreciation.

After some algebraic manipulations, we get the profit equation as a function of the Cournot equilibrium amounts:

$$\Pi_0 = \frac{1}{3}\frac{9}{4} q_c^2 + d\,\frac{q_c^2}{1-d} + \frac{q_c}{2} rd\alpha \tag{30}$$

The first term is the profit in period one, where its market share (p) is one third given that it decides to produce the amount such that the industry total output is the what the monopolist would produce, and where q_m is equal to three-halves of q_c.

The second term is the discounted value of the Cournot equilibrium profits from period two to infinity. The last term is the discounted value of the total cost savings in period two.

The fundamental question is whether ϑ_0, the present discounted value of one firm's profits under the scenario that it behaves strategically reducing output in one period, is lower or greater than ϑ_c, the present discounted value of the regular Cournot equilibrium. Appendix B proves that ϑ_0 is greater than ϑ_c.

Hence, if duopolist one believes that duopolist two is going to set the equilibrium Cournot amount, firm one has an incentive to reduce the amount produced to q_0. The same analysis can be made for the other duopolist. If firm two thinks that firm one is going to set the amount in Cournot equilibrium, it sets his production in q_0.

When both duopolists simultaneously realise that it is convenient to reduce the production, the total output for the industry becomes $2 * q_0$. This amount is equivalent to one half of the Cournot equilibrium and to two thirds of the monopoly equilibrium. This is a curious result, as both firms together produce even less than the monopoly output!

The problem arises when both firms realise the situation, which is not optimal. When this happens, they are willing to increase their output marginally so as to increase their profits. We hypothesise that after some periods of learning, the duopolists should achieve a cartel equilibrium. This is the Nash equilibrium for this particular model because by holding the strategy of the other firm constant, no duopolist can achieve a higher profit by choosing a different strategy.

Suppose now, that firm one correctly believes that firm two is going to set its amount in q_0. Will the firm react by setting its production equal to three fourths of the total Cournot market equilibrium, and thus have a reaction capacity equivalent to the one experienced in the traditional Cournot model. The firm prefers to set its amount at q_c, receiving a monopoly price for more units and having a market share of two thirds.

If both firms have the same belief—that the rival is going to set the amount q_0—each produces the Cournot equilibrium to take advantage of the lower amount produced by the other firm. However, when they realise this is a suboptimum situation, either one of them, and eventually both, will decide to reduce output to one half of Cournot, because this is better than Cournot, in spite of the other one's behavior. The final scenario is a Nash cartel equilibrium.

Conclusion

When a multi-horizon model of Cournot duopoly is combined with investment decisions and capacity constraints, the traditional Cournot equilibrium is no longer stable. Instead, we observe a non-cooperative strategic behavior with the firms producing at the monopoly level. This cartel is not a product of a tacit agreement. It is the outcome of each firm's best strategy. The social welfare outcome involves a deadweight loss larger than the one observed in the traditional duopoly Cournot model.

NOTES

[1] Our modeling draws on Besanko et al, (2004), Carlton and Perloff (2005), Church and Ware (2000), Dixit (1980), Dixit and Skeath (2004), Gibbons (1992), Pepall, Richards, and Norman (1999), Tirole (1988), and Watson (2001).

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APPENDIX A

Having only one firm under the assumptions of the developed model, we can derive equation (27).

(1) $L = \sum_{t=1}^{\infty} \left[(1-q_t)q_t - rK_t \right] d^{t-1} - \sum_{t=1}^{\infty} \lambda_t \left[q_t - \sum_{j=1}^{t} \alpha^{t-j} K_j \right]$

the first-order conditions are:

(2) $L_{q1} : (1-2q_1) - \lambda_1 = 0$

(3) $L_{q2} : d(1-2q_2) - \lambda_2 = 0$

(4) $L_{q1t} : d^{t-1}(1-2q_t) - \lambda_t = 0$

(5) $L_{K1} :- r + \sum_{t=1}^{\infty} \lambda_t \alpha^{t-1} = 0$

(6) $L_{K2} :- dr + \sum_{t=2}^{\infty} \lambda_t \alpha^{t-2} = 0$

(7) $L_{Kt} :- d^{t-1} r + \sum_{i=t}^{\infty} \lambda_i \alpha^{i-t} = 0$

(8) $L_{\lambda t} : q_t - \sum_{j=1}^{t} \alpha^{t-j} K_j = 0$

After some algebraic manipulation we get the tangential condition:

(9) $\frac{MR_2}{MR_1} = \frac{MR_{t+1}}{MR_t} = \frac{d^t(1-2q_{t+1})}{d^{t-1}(1-2q_t)} = \frac{MC_2}{MC_1} = \frac{MC_{t+1}}{MC_t} = \frac{\lambda_{t+1}}{\lambda_t} = d$

The general solution for the problem is:

(10) $\lambda_1 = r(1-\alpha d) = d\left[r(\delta + i)\right]$

(11) $\lambda_2 = d\lambda_1 = dr(1-\alpha d) = d^2\left[r(\delta + i)\right]$

(12) $\lambda_n = d^{n-1}\lambda_1 = d^{n-1} r(1-\alpha d) = d^n\left[r(\delta + i)\right]$

(13) $q_1 = q_2 = q_t = q_m$

(14) $K_1 = q_1, K_2 = \delta q_1$, in general

(15) $K_t = \delta^{t-1} q_m$

Therefore, the final solution after all the substitutions is:

(16) $q_m = \frac{1-r(1-d\alpha)}{2}, \quad p_m = \frac{1+r(1-d\alpha)}{2}$
$= \Pi_m \left[\frac{1-r(1-d\alpha)}{2}\right]^2 \left[\frac{1}{1-d}\right] = q_m^2 \left[\frac{1}{1-d}\right] = \sum_{t=0}^{\infty} \frac{q_m^2}{\left[1+i\right]^t}$ for every period.

The interpretation of the first-order condition is the same as in the Cournot model.

APPENDIX B

To prove equation (30) we assume an interest rate of 10%, a depreciation rate of 15% and a price of capital of at least \$ 0.2056, which is a really low value.

(1) $r > 0.2056$

(2) $r > \dfrac{1}{1+5\frac{1}{1.1}.85}$

(3) $r > \dfrac{1}{1+5d\alpha}$

(4) $r(1+5d\alpha) > 1$

(5) $r5d\alpha > 1-r$

(6) $6rd\alpha > 1-r+rd\alpha$

(7) $2rd\alpha > \dfrac{1}{3} - \dfrac{r(1-\alpha d)}{3}$

(8) $\dfrac{rd\alpha}{2} > \dfrac{q_c}{4} s$

(9) $q_c\left[\dfrac{3q_c}{4} + \dfrac{rd\alpha}{2}\right] > q_c^2$

(10) $\dfrac{3}{4}q_c^2 + rd\alpha\dfrac{q_c}{2} > q_c^2$

(11) $\dfrac{3q_c^2}{4} + d\dfrac{q_c^2}{1-d} + \dfrac{q_c}{2}rd\alpha > \dfrac{q_c^2}{1-d}$ which means that

(12) $\Pi_0 > \Pi_c$