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THE POSSIBILITY OF APPLYING THE CALCULUS OF FUNCTIONAL DEPENDENCES TO A KNOWLEDGE BASE

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The calculus of functional dependencies has proven very efficient in designing databases. This work illustrates the possibility of applying the calculus of functional dependence to the calculus of proposition. The calculus of propositions expanded in propositions, when expanded in this way, provide the possibility of significantly shortening the forming of implications, thus substantially speeding up the operations within the knowledge base. This paper shows the possibility of expanding the calculus of propositions with the calculus of functional dependencies. It also shows the possibility of using the calculus of functional dependencies in the forming of implications within the knowledge base. The contribution of this work is the incorporation of the calculus of functional dependencies into the calculus of proposition. The insertion of the calculus of functional dependencies into the calculus of propositions opens up the possibility of a much shorter forming of the implications, thus speeding up the operations within the knowledge base.

Keywords: calculus of functional dependences, implications forming, knowledge base.

INTRODUCTION

In the calculus of propositions, as a part of mathematical logic, the proposition is defined as an assertion which can be either true (symbol **T**) or false (symbol $\perp$). Every proposition has the form a of statement and it cannot be true and false simultaneously. Propositions are denoted by the alphabet symbols. Operations with propositions are defined (E. Mendelsohn, 1964) as follows: Operation of negation (symbol $\neg$), conjunction (symbol $\wedge$), disjunction (symbol $\vee$), implication (symbol $\Rightarrow$), equivalence (symbol $\Leftrightarrow$). In this paper, capital letters from the alphabet (**A,B,C,...**). will be used to denote propositions. If **A** is a proposition, then $\neg$**A** is its negation. Simple propositions and their negations are called literals. By using simple propositions and their operations, complex propositions can be formed, such as Conjunction $\mathbf{A \wedge B}$, Disjunction $\mathbf{A \vee B}$, Implication $\mathbf{A \Rightarrow B}$ etc. Within the calculus of propositions each complex proposition can be presented in the so-called conjunctive normal form, i.e. in the form $\mathbf{A_1 \wedge A_2 \wedge ... \wedge A_n}$, where $\mathbf{A_1,...,A_n}$ are disjunctions composed of simple propositions. Furthemore, in the calculus of propositions each complex proposition can be presented by means of simple propositions and operations of negation, conjunction and disjunction. Disjunction can also be presented by negation, conjunction and implication (e. g $\mathbf{A \vee B \Leftrightarrow \neg A \Rightarrow B}$). We shall assume a knowledge base which consists of simple propositions and implications $\mathbf{X \Rightarrow Y}$, where **X** and **Y** are simple expressions or conjunctions composed of simple expressions (if propositions in the knowledge base are not in the forementioned form, then, by using a forementioned transformations that are used in the calculus of propositions, we can translate them into the given form). We shall assume that our knowledge base does not contain either false propositions, or false implications. This paper will show how the calculus of functional dependencies can be expanded into logic (or more precisely, into the calculus of propositions). Such an expansion will make it possible to introduce notions and calculating methods from the calculus of functional dependencies to the calculus of propositions. Since the calculus of functional dependencies has proven to be efficient in the logical design of databases, this paper will present the possibility of organising a knowledge base using notions and methods from the calculus of functional dependencies. The definitions and examples have been produced according to the model of definitions and examples from the book written by S. Tkalac (1993).

This work shows the possibility of expanding the calculus of propositions with the calculus of functional dependencies. It also shows the possibility of using the calculus of functional dependencies in the forming of implications in the knowledge base. A part of this work includes is the incorporation of the calculus of functional dependencies into the calculus of proposition. The insertion of the calculus of functional dependencies into the calculus of propositions opens up the possibility of a much shorter forming of the implications thus speeding up the operations within the knowledge base.

1. AXIOMS AND RULES OF INFERENCE

By using the Armstrong's axioms model from the calculus of functional dependencies for propositions **X,Y,Z** and **W**, axioms **(a1)-(a3)** are formulated:

(a1) $X \wedge Y \Rightarrow Y$ **Reflexivity**

(a2) $((X \Rightarrow Y) \wedge W) \Rightarrow (X \wedge W \Rightarrow Y \wedge W)$ **Increase**

(a3) $((X \Rightarrow Y) \wedge (Y \Rightarrow Z)) \Rightarrow (X \Rightarrow Z)$ **Transitivity**

By using this model the rules of inference from the calculus of functional dependence for the propositions **X,Y,Z** and **W**, the rules of inference **(p1)-(p3)** are formulated:

(p1) $((A \Rightarrow B) \wedge (A \Rightarrow C)) \Rightarrow (A \Rightarrow B \wedge C)$ **Union or additivity**

(p2) $(A \Rightarrow B \wedge C) \Rightarrow ((A \Rightarrow B) \wedge (A \Rightarrow C))$ **Rule of decomposition**

(p3) $((X \Rightarrow Y) \wedge (W \wedge Y \Rightarrow Z)) \Rightarrow (X \wedge W \Rightarrow Z)$ **Rule of pseudo-transitivity**

According to the B-axioms for deriving functional dependence for the statements **X,Y,Z,C** and **W** the derivation rules **(b1)-(b3)** are formed:

(b1) $X \wedge Y \Rightarrow Y$ **Reflexivity**

(b2) $((X \Rightarrow Y \wedge Z) \wedge (Z \Rightarrow C \wedge W)) \Rightarrow (X \Rightarrow Y \wedge Z \wedge C)$ **Accumulation**

(b3) $(X \Rightarrow Y \wedge Z) \Rightarrow (X \Rightarrow Y)$ **Projectivity**

In propositions **(a1)-(a3)**, **(p1)-(p3)** and **(b1)-(b3)** we can claim that the right side of implications are the logical consequences of their left side. The truth of propositions can be shown, e.g. by truth tables that are used in the calculus of propositions.

Furthermore, in the text we will introduce some notions into the calculus of propositions, using as a model the notions used in the calculus of functional dependencies. In our considerations, we will assume that the left and right sides of the implications are conjunctions composed of simple expressions, i.e. that the implications are of the form $\mathbf{A_1 \wedge A_2 \wedge ... \wedge A_n \Rightarrow B_1 \wedge B_2 \wedge ... \wedge B_m}$. The set of literals contained in the set **S** will be denoted as $\mathbf{L_s}$. The number of mutually different literals in conjunction X will be denoted as $|\mathbf{X}|$, e.g. if **A,B** and **C** are simple and mutually different expressions then $|\mathbf{A \wedge \neg B \wedge C}|$**=3**.

2. CLOSURE OF A SET OF IMPLICATIONS

By starting with a set of implications S we can obtain a set **S'**. The elements of the set **S'** are not contained in the set **S** but they are implied by the inference rules (p1-p3) or (b1-b3) and axioms. A union of the sets **S** and **S'** is called the closure of the set S (denoted as S^+). Closure of the set S is defined as follows:

Closure of the set S of implications is a set of implications S^+ which satisfies the following conditions:

- **the given set of implications S is a subset of this closure ($S \subseteq S^+$)**
- **by applying the axiom (a1)-(a3) on implications in S^+ and literals in S, no implication can be made that is not already contained within S.**

The set of literals contained in S, is significant for determining S^+ because S^+ also includes all the implications obtained from the axiom of reflexivity. Thus, e.g. set S^+ for every set of implications specified on $\mathbf{L_S}$**={A,B}** will consist of, among other things, the implications **A⇒A, B⇒B, A∧B⇒A, A∧B⇒B, A∧B⇒A∧B**. For any set of implications specified on $\mathbf{L_s}$**={A,C}**, these implications will not be contained. If the set S is specified, we shall assume that the elements of set $\mathbf{L_S}$ are literals which are contained in the left and right sides of the implications in $\mathbf{S^+}$.

3. THE CLOSURE OF A CONJUNCTION

The closure of conjunction is defined in the following way:

Let **S** be a set of implications, let $\mathbf{W \subseteq L_S}$ and let **X** be a conjunction composed of the elements of the set **W**. In S^+ there is a subset of implications whose left sides equal the conjunction **X**. Within this subset of implications there is the implication **X⇒Y** whose right side is conjunction with the maximum number of literals. For any other element of this subset **X⇒Z** holds $|\mathbf{Z}| \leq |\mathbf{Y}|$. The right side them of implication **X⇒Y** is called a closure of the conjunction **X** (symbol $\mathbf{X^+}$). Due to the axiom of reflexivity, the closure of **X** must always contain **X**.

Example:

Let the set of implications be specified by **S={A⇒D, A∧B⇒E, B∧F⇒E, C∧D⇒F, E⇒C}**

We have: $\mathbf{L_S}$**={A,B,C,D,E,F}**

Let $\mathbf{W=\{A,E\}\subseteq L_S}$ and $\mathbf{X=A\wedge E}$

1. $\mathbf{A\wedge E \Rightarrow A\wedge E}$ reflexivity
2. $\mathbf{A\Rightarrow D}$ specified implication
3. $\mathbf{A\wedge E\Rightarrow A\wedge E\wedge D}$ accumulation 1st and 2nd
4. $\mathbf{E\Rightarrow C}$ specified implication
5. $\mathbf{A\wedge E\Rightarrow A\wedge E\wedge D\wedge C}$ accumulation 3rd and 4th
6. $\mathbf{C\wedge D\Rightarrow F}$ specified implication
7. $\mathbf{A\wedge E\Rightarrow A\wedge E\wedge D\wedge C\wedge F}$ accumulation 5th and 6th
8. $\mathbf{(A\wedge E)^+=A\wedge C\wedge D\wedge E\wedge F}$

4. CLOSURE ELEMENT OF THE SET OF IMPLICATIONS

Using the algorithm for determining the closure of a conjunction in the specified set of implications S, we can determine whether an arbitrary implication $\mathbf{W\Rightarrow Z}$ is an element of the set $\mathbf{S^+}$. Namely, if the literals of conjunction $\mathbf{Z}$ are contained among the literals of $\mathbf{W^+}$ in S, then $\mathbf{W\Rightarrow Z\in S^+}$ holds according to the axiom of projectivity.

Example:

Let the set of implications be $\mathbf{S=\{A\Rightarrow D,\ A\wedge B\Rightarrow E,\ B\wedge F\Rightarrow E,\ C\wedge D\Rightarrow F,\ E\Rightarrow C\}}$. It should now be determined whether implication $\mathbf{A\wedge B\Rightarrow F}$ logically proceeds from S.

We have:

$\mathbf{(A\wedge B)^+=A\wedge D\wedge B\wedge E\wedge C\wedge F}$

Therefore $\mathbf{A\wedge B\Rightarrow A\wedge B\wedge C\wedge D\wedge F}$ from which, according to the axiom of projectivity, follows $\mathbf{A\wedge B\Rightarrow F}$. Therefore, $\mathbf{A\wedge B\Rightarrow F}$ follows logically from S.

5. EQUIVALENT SETS OF IMPLICATIONS

Let **S** and **V** be two sets of implications. We assume that **S** and **V** are mutually equivalent (denoted as $\mathbf{S\equiv V}$) if $\mathbf{S^+=V^+}$.

The set of implications **S** is considered to be a covering of the set of implications **V** if $\mathbf{S\equiv V}$ (that is $\mathbf{S^+=V^+}$).

Equivalence is a symmetric notion. Equivalence holds only if $\mathbf{S^+=V^+}$. Because of the symmetric property of equivalence, if **S** is the covering of the set **V**, then **V** also is the covering of the set **S**. Every implication in **S** must be logically inferred from **V** and vice versa. We can say that the set **S** logically follows on from the set **V** and vice versa.

If we find out that every implication from **S** logically follows on from the set **V** and vice versa, then we have determined that **S** and **V** are equivalent sets of implications.

6. NON-REDUNDANT COVERING OF THE SET OF IMPLICATIONS

The set of implications S is equivalent to its closure S^+. S^+ can contain implications that logically follow on and may be inferred from the rest of implications in the set S. Such implications are regarded as redundant implications. A set that contains redundant implications is a redundant set. Non-redundant covering of the set of implications will be defined in the following way:

The set of implications S is non-redundant if there is no proper subset S' of S for which $S' \equiv S$. A set of implications is a non-redundant covering of V if $S \equiv V$ and S is non-redundant.

We can prove that a set of implications S is not-redundant, if we show that every single implication $X \Rightarrow Y$ in S does not logically follow on from the rest of the set S (that is from the set $S \setminus \{X \Rightarrow Y\}$). If any implication $X \Rightarrow Y$ can be inferred from the rest of the set S, then set S is redundant.

7. REDUNDANT LITERALS IN IMPLICATION

There is a possibility to reduce the number of literals in conjunctions on the left and right side of the implications in the set S, and this doesn't result in changing the equivalence of the set S.

Let S be a set of implications and let $X \Rightarrow Y \in S$. The literal A of implication conjunct is redundant if:

1. **$X = A \wedge Z$, $X \neq Z$, i $(S \setminus \{ X \Rightarrow Y\}) \cup \{Z \Rightarrow Y\} \equiv S$, or**
2. **$Y = A \wedge W$, $Y \neq Z$, i $(S \setminus \{ X \Rightarrow Y\}) \cup \{X \Rightarrow W\} \equiv S$.**

A literal in the conjunction of the left or right side of the implication is redundant if it can be removed from the left or right side of the implication in S, and in that way the obtained set S' is equivalent to the set S.

Let S be a set of implications and let $X \Rightarrow Y \in S$. We can assume that the implication $X \Rightarrow Y$ is left reduced if X contains no redundant literals. The implication $X \Rightarrow Y$ is regarded as right reduced if Y does not contain any redundant literals. The implication $X \Rightarrow Y$ is reduced if it is left reduced and right reduced.

The set of implications S is left reduced if every implication from S is left reduced. The set of implications S is right reduced if every implication from S is right reduced. The set of implications S is reduced if every implication from S is reduced.

8. CANONICAL COVERING

The set of implications S is canonical if every dependency in S has the form $X \Rightarrow A$ and S is non-redundant and left reduced.

Since the canonical set S is non-redundant and every right side of an implication has only one literal, canonical covering is right reduced, i.e. reduced. By applying the rule of decomposition to the implication $X \Rightarrow A_1 \wedge A_2 \wedge \ldots \wedge A_n$ the set of implications

{$X \Rightarrow A_1, X \Rightarrow A_2, ..., X \Rightarrow A_n$} will be obtained. According to this, for every set of implications from **S**, it follows an that there is a covering of the set **V** in which every implication has the form **$X \Rightarrow A$**. If the set of implications **S** is reduced, by applying the rule of decomposition we will obtain its canonical covering. The opposite is also true. If **G** is a canonical set of implications, by applying the rule of union to the implication with the equal left side in **G**, we will obtain a reduced covering **S** from **V**.

9. EQUIVALENT CONJUNCTIONS

With regard to the closure of the conjunction specified in the set of implications, the notion of equivalent conjunctions may be introduced as well.

Let S be a set of implications and let X and Y be conjunctions. The conjunctions X and Y are considered mutually equivalent (denoted $X \equiv Y$) if the implications $X \Rightarrow Y$ and $Y \Rightarrow X$ are elements of the set S^+.

Along with the definition of equivalence of conjunctions **X** and **Y**, it follows on that: if **$X \equiv Y$**, then the following must hold too

- The set of literals from **X** is a subset of the set of literals from **Y^+** and the set of literals from **Y** is a subset of the set of literals from **X^+**
- **$X^+ \equiv Y^+$**.

The set of conjunctions of all the implications in S can be divided. This is based on the equivalence into subsets, so that the elements of every subset are mutually equivalent. **$E_S(X)$** will denote a subset of the implications in **S** whose left sides are mutually equivalent. **$e_S(X)$** will denote a set of all the left sides of the implications in **$E_S(X)$**, and **E_S** the set of all subsets **$E_S(X)$** in **S**. Since no conjunct **Z** from **S** can be an element of the two different subsets **$e_S(X)$**, no implication **$Z \Rightarrow W$** in **S** can be an element of the two different subsets **$E_S(X)$**, so that **E_S** is a partition of the set **S**.

Let **S** and **V** be two equivalent sets of implications. Let **X** be a conjunction with literals from **S**. Set **$E_S(X)$** is non-empty only if the set **$E_V(X)$** is also a non-empty set. In other words, the number of elements in the partition of set **S** (in **E_S**) should always be equal to the number of elements in the partition of set **V** (in **E_V**). It follows on that:

Any two mutually equivalent sets of implications must have the same number of equivalence classes.

10. COMBINED IMPLICATION

Let us define a combined implication:

A combined implication on the set of implications **S** has the form **$(X_1,...,X_n) \Rightarrow Y$**. **$(X_1,...,X_n)$** is the left side, and **Y** is the right side of the combined implication. On the set **S**, the combined implication **$(X_1,...,X_n) \Rightarrow Y$** holds if for any two members of the left side of the combined implication **X_i** and **X_j** holds **$X_i \Rightarrow Y_j$** and **$X_i \Rightarrow Y$**.

The combined implication $(X_1,...,X_n)\Rightarrow Y$ on set S is a shorter way of recording the set of implications whose assumptions are mutually equivalent. In other words, for any two members of the left side X_i and X_j the following must hold $X_i^+\equiv X_j^+$.

The set of implications S is a characteristic for the combined implications $(X_1,...,X_n)\Rightarrow Y$ if $S\equiv(X_1,...,X_n)\Rightarrow Y$.

The set of implications S is the natural characteristic set of implications for a combined implication, if for every conjunction X_i on the left side of the combined implication in S there is exactly one implication in which X_i is the left side. The natural characteristic set S for the combined implication $(X_1,...,X_n)\Rightarrow Y$ can be presented as $S=\{X_1\Rightarrow Y_1,...,X_n\Rightarrow Y_n\}$.

A ring-like characteristic set of implications S for the combined implication $(X_1,...,X_n)\Rightarrow Y$ has the form $S=\{X_1\Rightarrow X_2,X_2\Rightarrow X_3,...,X_{n-1}\Rightarrow X_n,X_n\Rightarrow X_1\wedge Y\}$.

A set of combined implications V can be treated as the union of characteristic sets of implications S_i for each combined implication in V, that is, as a set of implications.

11. RING COVERING

The set S is regarded as a covering of the set V if $S\equiv V$. The sets S and V can be sets of implications, sets of combined implications or one of them can be a set of implications and the other can be a set of combined implications.

We consider a set of combined implications as a ring, if for any two mutually equivalent conjunctions X and Y, it can follows on that they cannot be members of the left sides of two different combined implications in S.

The set of combined implications can be redundant and can contain redundant literals. Before defining a non-redundant and reduced ring set, we shall define the notion of movable literal.

Let V be a set of combined implications and let X_i be a member of the left side of one combined implication. For literal A_j hold $A_j\cap X_i=A_j$ (A_j is a part of the conjunction X_i). The literal A_j is considered movable if it can be moved from the left side to the right side of the combined implication, without causing a change in the equivalence of set V. The member X_i is considered movable if all propositions that make up conjunction X_i are movable.

The ring set V is non-redundant if no combined implication from V can be removed, without changing the equivalence of the set G, and if no combined implication in G has a movable member on the left side.

Let G be a redundant set of implications. The combined implication $(X_1,...,X_n)\Rightarrow Y$ is reduced if none of the members on the left side contain no movable literals, and if the right side of the combined implication contains no redundant literals. The set of combined implications V is reduced if all the elements of the set are reduced.

According to the definition of the combined implication $(X_1,...,X_n) \Rightarrow Y$ it follows that the literal contained in **Y** is not contained in any conjunction X_i.

Ring covering of the non-redundant set of implications will have no movable members on the left sides of the combined implications, and in the ring covering of the left reduced implications there will be no movable literals. Instead of determining directly the ring covering of a set of implications, we shall first find the non-redundant left reduced covering of this set, and then we shall search for its ring covering.

The eliminating of redundant atoms from the right side can be done in two steps:

1. We remove from the right side **Y** of the combined implication $(X_1,...,X_n) \Rightarrow Y$, all literals which are contained in at least one left side X_i.
2. We find the natural characteristic covering of ring covering and perform right reduction.

Step 1 is carried out to reduce the number of literals of the natural characteristic ring covering.

12. ANSWERING SCHEMES

The left sides of the combined implications in the ring covering contain mutually equivalent propositions. From the combined implications we can form schemes of possible answers to the questions we ask in the knowledge base. Here is an example of how this can be done:

- **each combined implication Si: $(X_{i1},X_{i2},...,X_{in}) \Rightarrow Y_i$ is assigned the scheme $O_i(\underline{X_{i1}} \wedge \underline{X_{i2}} \wedge ... \wedge \underline{X_{in}} \wedge Y_i)$, with the underlined propositions.**

In the way it is described it can be said that it is possible to treat answering schemes O_i similar to relational schemes in the theory of relational databases. If X_{ik} is a proposition in O_i and if O_j contains X_{ik} or $\neg X_{ik}$, it is obvious that the truth value of X_{ik} in O_i influences the truth value of the answer in O_j. According to the analogy with relational schemes in the theory of relational databases, the proposition X_{ik} from O_i corresponds to the key of the scheme O_i, and X_{ik} or $\neg X_{ik}$ corresponds to the foreign key in O_j. Each class of equivalence will be matched by one answering scheme. This means that the number of possible answering schemes in the knowledge base is determined by the number of equivalence classes that result from the initial set of implications.

An example: (reduced ring covering)

We have to find the reduced ring covering of the given set:
$S=\{B \wedge F \Rightarrow C, B \wedge C \wedge D \Rightarrow E, C \Rightarrow B \wedge D \wedge F, C \wedge D \Rightarrow A \wedge E\}$

<u>Non-redundant covering:</u>

Closure of $B \wedge F$ on the set $S \backslash \{B \wedge F \Rightarrow C\}$ is $(B \wedge F)^+ = B \wedge F$, and it follows on that the implication $B \wedge F \Rightarrow C$ is not redundant in set **S** since its right side **C** is not in the obtained closure.

Closure of $B\wedge C\wedge D$ on set $S\backslash\{B\wedge C\wedge D\Rightarrow E\}$ is $(B\wedge C\wedge D)^{+}=B\wedge C\wedge D\wedge E\wedge A$, and it follows on that the implication $B\wedge C\wedge D\Rightarrow E$ is redundant in set S since its right side E is in the obtained closure. By eliminating the redundant implication, we can obtain the set: $S_1=\{B\wedge F\Rightarrow C, C\Rightarrow B\wedge D\wedge F, C\wedge D\Rightarrow A\wedge E\}$.

The closure of C on the set $S_1\backslash\{C\Rightarrow B\wedge D\wedge F\}$ is $(C)^{+}=C$, and it follows that the implication $C\Rightarrow B\wedge D\wedge F$ is not redundant in set S_1 since its right side $B\wedge D\wedge F$ is not in the obtained closure.

Closure of $C\wedge D$ on the set $S_1\backslash\{C\wedge D\Rightarrow A\wedge E\}$ is $(C\wedge D)^{+}=C\wedge D\wedge B\wedge F$, and it follows on that the implication $C\wedge D\Rightarrow A\wedge E$ is not redundant in set S_1 since its right side $A\wedge E$ is not in the obtained closure.

We have obtained a non-redundant covering $S_1=\{B\wedge F\Rightarrow C, C\Rightarrow B\wedge D\wedge F, C\wedge D\Rightarrow A\wedge E\}$.

Left reduced covering:

The implication $C\Rightarrow A\wedge E$ can be obtained from set $S_2=\{B\wedge F\Rightarrow C, C\Rightarrow B\wedge D\wedge F, C\wedge D \Rightarrow A\wedge E\}$, namely:

$(C)_{S2}^{+}=C\wedge B\wedge D\wedge F\wedge A$ so that there is a redundant literal D in the left side of the implication $C\wedge D \Rightarrow A\wedge E$. Other implications cannot be left reduced.

We have obtained a non-redundant left reduced covering $S_3= \{B\wedge F\Rightarrow C, C\Rightarrow B\wedge D\wedge F, C \Rightarrow A\wedge E\}$, that is $S_4=\{B\wedge F\Rightarrow C, C\Rightarrow A\wedge B\wedge DEF\}$.

Left reduced ring covering:

From S_4 we obtain the left reduced ring covering $S_5=\{(B\wedge F,C)\Rightarrow A\wedge B\wedge C\wedge D\wedge E\wedge F\}$

Right reduction:

First we eliminate from the right side of the combined implications, the literals B,F and C since they are also in its left side, so that we can obtain a left reduced ring covering $S_6=\{(B\wedge F,C)\Rightarrow A\wedge D\wedge E\}$.

A natural characteristic covering of S_6 is $S_7=\{B\wedge F\Rightarrow A\wedge C\wedge D\wedge E, C\Rightarrow A\wedge B\wedge D\wedge E\wedge F\}$. Right reduction in this example does not change the set S_7, so that the reduced ring covering is $S_6=\{(B\wedge F,C)\Rightarrow A\wedge D\wedge E\}$.

Example: (D.Blanuša, Viša matematika II/1, Tehnička knjiga,Zagreb,1966,p.347)

A bridegroom says to his wife after the wedding: "We will get along well if you fulfil three conditions regarding dinner:

1. If you don't put bread on the table, you have to put ice cream.
2. If you put bread and ice cream, you must not put cucumbers.
3. If you put cucumbers or do not put bread, then you must not put ice cream."

It has to be seen as to whether all these conditions are feasible, and if they are affirmative, how they can be simplified so as to make it easier for the young housewife.

Let us denote these propositions as **A,B, C**:

A .... The wife puts bread on the table
B .... The wife puts cucumbers on the table
C .... The wife puts ice cream on the table

The conditions from this example may be written as:

1. $\neg A \Rightarrow C$
2. $A \wedge C \Rightarrow \neg B$
3. $B \vee \neg A \Rightarrow \neg C$

From 1. it can be noted that it follows the implication $\neg C \Rightarrow A$

From 2. we have:

$(A \wedge C \Rightarrow \neg B) \equiv \neg(A \wedge C) \vee \neg B \equiv \neg A \vee \neg C \vee \neg B$, resulting in the following implications:
$(A \wedge C) \Rightarrow \neg B$
$(A \wedge B) \Rightarrow \neg C$
$(B \wedge C) \Rightarrow \neg A$

From 3. we have:

$(B \vee \neg A \Rightarrow \neg C) \equiv (\neg B \wedge A) \vee \neg C \equiv (\neg B \vee \neg C) \wedge (A \vee \neg C)$, resulting in the following implications:

$B \Rightarrow \neg C$ **and** $C \Rightarrow \neg B$ **and** $\neg A \Rightarrow \neg C$ **and** $C \Rightarrow A$

According to the above, we now have a set of implications:
$S=\{\neg A \Rightarrow C, \neg C \Rightarrow A, A \wedge C \Rightarrow \neg B, A \wedge B \Rightarrow \neg C, B \wedge C \Rightarrow \neg A, B \Rightarrow \neg C, C \Rightarrow \neg B, \neg A \Rightarrow \neg C, C \Rightarrow A\}$.

(a) $(A)^+=A$
(b) $(\neg A)^+=\neg A \wedge C \wedge \neg B \wedge \neg C=\bot$
(c) $(B)^+=B \wedge \neg C \wedge A$
(d) $(\neg B)^+=\neg B$
(e) $(C)^+=C \wedge \neg B \wedge A$
(f) $(\neg C)^+=\neg C \wedge A$
(g) $(A \wedge B)^+=A \wedge B \wedge \neg C$
(h) $(A \wedge C)^+=A \wedge C \wedge \neg B$
(i) $(B \wedge C)^+=B \wedge C \wedge \neg B \neg C \neg A=\bot$
(j) $(A \wedge B \wedge C)^+=\bot$

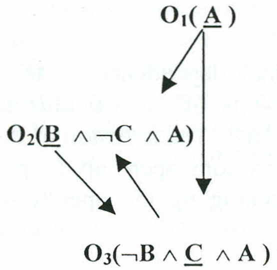
According to (b) we can conclude that from the assumption that the wife does not put bread on the table the logical consequence is false. Therefore, the wife must put bread on the table (according to (a) we can conclude that such an assumption does not lead to a contradiction). From (g) we can conclude that besides bread the wife can also put cucumbers on the table, but not ice cream. From (h) we can conclude that as well as bread, the wife can put ice-cream on the table too, but not cucumbers. Therefore, the wife must serve bread and may (but does not have to) serve cucumbers or ice cream with the bread (but not both at the same time, and this can be deduced by looking at (i)).

Yet another problem solution for the problem:

According to (a),(c) and (e) we have a ring covering:

$\mathbf{P = \{(A),(B) \Rightarrow \neg C \wedge A, (C) \Rightarrow \neg B \wedge A\}}$

We can form the answering schemes $\mathbf{O_1}$, $\mathbf{O_2}$ and $\mathbf{O_3}$, where the left sides of the combined implications from the ring covering **P** are underlined. The schemes contain possible answers to the question in the example.

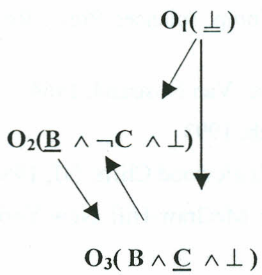


Answering schemes

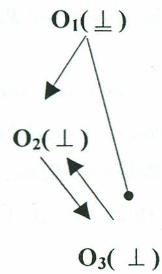
The arrow from $\mathbf{O_1}$ pointing towards $\mathbf{O_2}$ and $\mathbf{O_3}$ shows that the change of proposition in $\mathbf{O_1}$ influences the truthfulness of the answers in $\mathbf{O_2}$ and $\mathbf{O_3}$. Also, the changing of the truth value of the proposition from $\mathbf{O_2}$ influences the truthfulness of the answer $\mathbf{O_3}$ and vice versa.

In accordance with the to the above schemes we can conclude:

The change in the truth value of the proposition **A** in $\mathbf{O_1}$ influences the truth values of the expression in $\mathbf{O_2}$ and $\mathbf{O_3}$. If **A** is false, then we have:

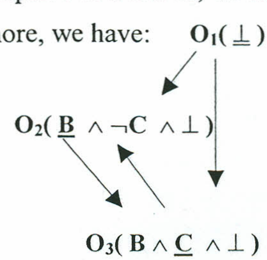


that is:



We can conclude that for the false **A**, since a logical consequence follows a lie in all the schemes, that for a false **A** there are no true answers to the question from the example. For a true **A**, we have a true answer: $\mathbf{O_1}$ **(T)**

Furthermore, we have:



According to the above schemes, we have three possible answers:

1. From $\mathbf{O}_1$ we can conclude: The wife puts bread on the table.
2. From $\mathbf{O}_2$ we can conclude: The wife puts cucumbers and bread on the table and does not put ice cream.
3. From $\mathbf{O}_3$ we can conclude: The wife puts ice cream and bread on the table and does not put cucumbers.

CONCLUSION

The integration of the calculus of functional dependence in the calculus of propositions enables the usage of certain elements of the calculus of functional dependence while working within the knowledge base. The insertion of the calculus of functional dependencies into the calculus of propositions opens up the possibility of a much shorter forming of implications, thus speeding up the operations within the knowledge base.

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MOGUĆNOST PRIMJENE RAČUNA FUNKCIJSKIH ZAVISNOSTI U BAZAMA ZNANJA

Sažetak

Račun funkcijskih zavisnosti pokazao se vrlo efikasnim u oblikovanju baza podataka. U ovom radu pokazuje se mogućnost primjene računa funkcijskih zavisnosti u propozicijskom računu. Također se pokazuje mogućnost upotrebe računa funkcijskih zavisnosti u oblikovanju implikacija u bazi znanja. Doprinos ovog rada je u ugradnji računa funkcijskih zavisnosti u propozicijski račun. Uključivanje računa funkcijskih zavisnosti u propozicijski račun otvara mogućnost mnogo bržeg izvođenja zaključaka, te zbog toga uštedu na operacijama unutar baze znanja.

Ključne riječi: račun funkcijskih zavisnosti, izvođenje zaključaka, baza znanja.