

# THE INTEGRATED AGENT IN MULTI-AGENT SYSTEMS

Mirko Maleković, Mirko Čubrilo

University of Zagreb, Faculty of Organization and Informatics, Varaždin, Croatia  
E- mail: {mmalekov|mcubrilo}@foi.hr

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*In this paper, we characterize the integrated agent in multi-agent systems. The following result is proved: if a multi-agent system is reflexive (symmetric, transitive, Euclidean) then the integrated agent of the multi-agent system is reflexive (symmetric, transitive, Euclidean), respectively. We also prove that the analogous result does not hold for multi-agent system's serialness. A knowledge relationship between the integrated agent and agents in a multi-agent system is presented.*

**Keywords:** integrated agent, knowledge operators, multi-agent systems, reasoning about knowledge.

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## 1. INTRODUCTION

The theory of knowledge investigates reasoning about knowledge, in particular, reasoning about the knowledge of agents who reason about the world and each other's knowledge. This theory is given in [1], [2], [3], and [4].

A multi-agent system (MAS) is any collection (group) of interacting agents, [1]. Incorporating knowledge and time into MAS is described in [6]. Some important relationships between temporal operators and knowledge operators are given in [5] and [7].

In this paper, we define an integrated agent in MASs. We prove that the integrated agent for an MAS is reflexive (symmetric, transitive, and Euclidean) if the MAS is reflexive (symmetric, transitive, and Euclidean), respectively. We also show that the analogous result does not hold for being serial. Because the fundamental property of knowledge, i.e. an agent knows only the true propositions, does not hold if the agent is not serial, we say that such an agent is a believer (not a knowledger). Well, if we want to build the integrated agent (for an MAS) who is a knowledger, then it is not sufficient for the MAS to be serial. In addition, we present a knowledge relationship between the integrated agent and the agents in an MAS. The relationship states that the integrated agent is more knowledgeable than the agents in the MAS).

The paper consists of four sections and an Appendix containing the proofs. In Section 2, we introduce the basic notions of reasoning about knowledge. The main results, described above, are given in Section 3. Section 4 contains our conclusions.

## 2. THE BASIC NOTIONS

In this section, we introduce, in accordance with [7], some basic concepts and notations.

Suppose we have an MAS consisting of  $m$  agents, named  $1, 2, \dots, m$ . We assume these agents wish to reason about a world that can be described in terms of a non-empty set  $P$  of primitive propositions. A language is just a set of formulas, where the set of formulas  $PLK$  that are of interest to us is defined as follows:

- (1) The primitive propositions in  $P$  are formulas;
- (2) If  $F$  and  $G$  are formulas, then so are  $\neg F$ ,  $(F \wedge G)$ ,  $(F \vee G)$ ,  $(F \Rightarrow G)$ ,  $(F \Leftrightarrow G)$ , and  $K_i(F)$  for all  $i \in \{1, 2, \dots, m\}$ , where  $K_i$  is a modal operator.

A Kripke structure  $M$  for an MAS =  $\{1, 2, \dots, m\}$  over  $P$  is a  $(m + 2)$ -tuple  $M = (S, I, k_1, k_2, \dots, k_m)$ , where  $S$  is a set of possible worlds (states),  $I$  is an interpretation that associates with each world in  $S$  a truth assignment to the primitive propositions in  $P$ , and  $k_1, k_2, \dots, k_m$  are binary relations on  $S$ , called the possibility relations for agents  $1, 2, \dots, m$ , respectively.

Given  $p \in P$ , the expression  $I[w](p) = \text{true}$  means that  $p$  is true in a world  $w$  in the structure  $M$ . The fact that  $p$  is false, in a world  $v$  of the structure  $M$ , is indicated by the expression  $I[v](p) = \text{false}$ .

The expression  $(u, v) \in k_i$  means that an agent  $i$  considers a world  $v$  possible, given his information in a world  $u$ . Since  $k_i$  defines what worlds an agent  $i$  considers possible in any given world,  $k_i$  will be called the possibility relation of the agent  $i$ .

We now define what it means for a formula to be true at a given world in a structure.

Let  $(M, w) \models F$  mean that  $F$  holds or is true at  $(M, w)$ . The definition of  $\models$  is as follows:

- (a)  $(M, w) \models p$  iff  $I[w](p) = \text{true}$ , where  $p \in P$ ;
- (b)  $(M, w) \models F \wedge G$  iff  $(M, w) \models F$  and  $(M, w) \models G$ ;
- (c)  $(M, w) \models F \vee G$  iff  $(M, w) \models F$  or  $(M, w) \models G$ ;
- (d)  $(M, w) \models F \Rightarrow G$  iff  $(M, w) \models F$  implies  $(M, w) \models G$ ;
- (e)  $(M, w) \models F \Leftrightarrow G$  iff  $(M, w) \models F \Rightarrow G$  and  $(M, w) \models G \Rightarrow F$ ;
- (f)  $(M, w) \models \neg F$  iff  $(M, w) \not\models F$ , that is,  $(M, w) \models F$  does not hold;
- (g)  $M \models F$  iff  $(M, w) \models F$  for all  $w \in S$ .

Finally, we shall define a modal operator  $K_i$ , where  $K_i(F)$  is read: Agent  $i$  knows  $F$ .

- (h)  $(M, w) \models K_i(F)$  iff  $(M, t) \models F$  for all  $t \in S$  such that  $(w, t) \in k_i$ .

In (h) we can see that agent  $i$  knows  $F$  in a world  $w$  of a structure  $M$  exactly if  $F$  holds at all worlds  $t$  that the agent  $i$  considers possible in  $w$ .

### 3. THE INTEGRATED AGENT

This section comprises our main results (propositions) regarding the integrated agents.

Let  $MAS = \{1, 2, \dots, m\}$  be a multi-agent system. Additionally, let  $M = (S, I, k_1, k_2, \dots, k_m)$  be a Kripke structure for the MAS. The integrated agent for the MAS, denoted  $ia(MAS)$ , is defined by  $k_{ia} = k_1 \cap k_2 \cap \dots \cap k_m$ .

What an agent knows is a consequence of the properties of the associated possibility relation, [5].

Let  $k_j \subseteq S \times S$  be a possibility relation of an agent  $j$ .

(Ref)  $k_j$  is reflexive iff (for all  $t \in S$ ) $[(t, t) \in k_j]$ ;

(Symm)  $k_j$  is symmetric iff (for all  $u, v \in S$ ) $[(u, v) \in k_j$  implies  $(v, u) \in k_j]$ ;

(Tra)  $k_j$  is transitive iff (for all  $t, u, v \in S$ ) $[(t, u) \in k_j$  and  $(u, v) \in k_j$  implies  $(t, v) \in k_j]$ ;

(Euc)  $k_j$  is Euclidean iff (for all  $t, u, v \in S$ ) $[(t, u) \in k_j$  and  $(t, v) \in k_j$  implies  $(u, v) \in k_j]$ ;

(Ser)  $k_j$  is serial iff (for all  $t \in S$ )(for some  $u \in S$ ) $[(t, u) \in k_j]$ .

An agent  $j \in MAS$  is reflexive (symmetric, transitive, Euclidean, serial,) iff his possibility relation  $k_j$  is reflexive (symmetric, transitive, Euclidean, serial,) respectively. An MAS is reflexive (symmetric, transitive, Euclidean, serial) iff every agent in the MAS is reflexive (symmetric, transitive, Euclidean, serial), respectively.

#### Proposition1

If MAS is reflexive (symmetric, transitive, Euclidean), then  $ia(MAS)$  is reflexive (symmetric, transitive, Euclidean), respectively.

Proposition1 says that the integrated agent  $ia(MAS)$  for a multi-agent system MAS is reflexive if the MAS is reflexive. This very fact also holds for symmetric, transitive, and Euclidean MASs.

#### Proposition2

Implication: If MAS is serial, then  $ia(MAS)$  is serial does not hold.

Proposition2 states that the fact given in Proposition1 does not hold for the serial property, that is, there exists a multi-agent system MAS that is serial and its integrated agent  $ia(MAS)$  is not serial.

From now on we suppose that  $F$  is an arbitrary formula in PLK. We also write  $ia$  instead of  $ia(MAS)$ .

#### Proposition3

Let  $M=(S,I,k_1,k_2,\dots,k_m,k_{ia})$  be a Kripke structure for  $MAS=\{1,2,\dots,m,ia\}$ . Then,

(true) If MAS is reflexive, then  $M \models K_{ia}(F) \Rightarrow F$ ;

(positive introspection) If MAS is transitive, then  $M \models K_{ia}(F) \Rightarrow K_{ia}(K_{ia}(F))$ ;

- (negative introspection) If MAS is Euclidean, then  $M \models \neg \text{Kia}(F) \Rightarrow \text{Kia}(\neg \text{Kia}(F))$ ;  
 (not) If MAS is symmetric, then  $M \models F \Rightarrow \text{Kia}(\neg \text{Kia}(\neg F))$ ;  
 (contradiction) If MAS is serial, then  $M \models \neg \text{Kia}(\text{False})$ .

The (true) part of Proposition 3 says that if the integrated agent  $ia$  for a reflexive multi-agent system MAS knows  $F$ , then  $F$  is true; the (positive introspection) part states that if the integrated agent  $ia$  for a transitive MAS knows  $F$ , then he knows that he knows  $F$ ; the (negative introspection) part says that if the integrated agent  $ia$  for an Euclidean MAS does not know  $F$ , then he knows that he does not know  $F$ ; the (not) part states that the integrated agent  $ia$  for a symmetric MAS knows that he does not know  $\neg F$  if  $F$  is true; finally, the (contradiction) part says that  $ia$  for a serial MAS does not know a contradiction, named False.

Now we characterize what we mean when we say that one agent is more knowledgeable than another agent.

Let  $i, j$  be two agents in an MAS and let  $M$  be a Kripke structure for the MAS. We say that agent  $j$  is more knowledgeable than agent  $i$ , denoted  $j \triangleright i$ , iff

$$M \models \text{Ki}(F) \Rightarrow \text{Kj}(F), \text{ for each formula } F \text{ in PLK.}$$

Accordingly, for all formulas  $F$  in PLK: if agent  $i$  knows  $F$ , then agent  $j$  knows  $F$ , too.

#### **Proposition 4**

Let  $ia$  be the integrated agent for an MAS. Then, (for all  $k \in \text{MAS}$ ) [ $ia \triangleright k$ ].

This proposition states that the integrated agent  $ia$  knows more than any agent in the MAS. This result can be applied when developing (building) a reliable multi-agent system. The idea is as follows: if we have a multi-agent system MAS, then, based on the desirable goals or problems that need to be solved by the MAS, we decompose the MAS to multi-agent systems, MAS1, MAS2, ..MAS $t$ , where  $\text{MAS} = \text{MAS1} \cup \text{MAS2} \cup \dots \cup \text{MAS}t$ . For each multi-agent system MASK,  $k = 1, 2, \dots, t$ , we build the integrated agent  $ia(\text{MASK})$ , respectively. Now, if the co-operation among MAS1, MAS2, .., and MAS $t$  is jeopardized at some point in time because, for example, some agent in MAS1 does not function properly (a 'dead' agent), then that agent can be replaced by the integrated agent  $ia(\text{MAS1})$ .

## **4. CONCLUSIONS**

We have defined the integrated agent in multi-agent systems. We have stated that the integrated agent is reflexive (symmetric, transitive, Euclidean) if the corresponding multi-agent system is reflexive (symmetric, transitive, Euclidean), respectively.

We have also shown that the analogous result does not hold for multi-agent system's serialness, that is, there exists a multi-agent system that is serial and its integrated agent is not serial. After these results, we stated that: if the integrated agent for a reflexive multi-agent system knows  $F$ , then  $F$  is true; if the integrated agent for a transitive multi-agent system knows  $F$ , then he knows that he knows  $F$ ; if the

integrated agent for an Euclidean multi-agent system does not know  $F$ , then he knows that he does not know  $F$ ; the integrated agent for a symmetric multi-agent system knows that he does not know  $\neg F$  if  $F$  is true; the integrated agent for a serial multi-agent system does not know a contradiction. Lastly we have shown that the integrated agent knows more than any other agent in the multi-agent system. This result can be applied when building a reliable multi-agent system as was described in Section 3.

## APPENDIX

### Proof (Proposition1)

Assume that MAS is reflexive (symmetric, transitive, Euclidean). We would like to show that  $ia(MAS)$  is reflexive (symmetric, transitive, Euclidean), respectively.

It follows, from our assumption, that each agent in  $MAS = \{1, 2, \dots, m\}$  is reflexive (symmetric, transitive, Euclidean), that is, all the possibility relations,  $k_1, k_2, \dots, k_m$ , are reflexive (symmetric, transitive, Euclidean). Therefore,  $k_{ia} = k_1 \cap k_2 \cap \dots \cap k_m$  is reflexive (symmetric, transitive, Euclidean), that is,  $ia(MAS)$  is reflexive (symmetric, transitive, Euclidean), as we wanted to show.

### Proof (Proposition2)

We need to show that A: [if MAS is serial, then  $ia(MAS)$  is serial] does not hold.

Let  $MAS = \{1, 2\}$  be a multi-agent system, where the Kripke structure for the MAS is:  $M = (S, I, k_1, k_2)$ ,  $S = \{w_1, w_2\}$ ,  $I$  is an arbitrary interpretation,  $k_1 = \{(w_1, w_1), (w_1, w_2)\}$ , and  $k_2 = \{(w_2, w_2), (w_1, w_2)\}$ . We can easily see that the MAS is serial.

The integrated agent  $ia(MAS)$  for the MAS is defined by  $k_{ia} = k_1 \cap k_2 = \{(w_1, w_2)\}$ . Because  $k_{ia}$  is not serial, we obtain that  $ia(MAS)$  is not serial, as desired.

In addition, if we define interpretation  $I$  in such a way that  $(M, w_1) \models F$  and  $(M, w_2) \not\models F$ , then we have that  $(M, w_2) \models K_{ia}(F)$  and  $(M, w_2) \not\models F$ . Accordingly,  $ia$  knows  $F$  at  $w_2$  in  $M$  even though  $F$  does not hold at  $w_2$  in  $M$ . Next, we have that  $(M, w_2) \models K_{ia}(F)$  and  $(M, w_2) \models K_{ia}(\neg F)$ , that is,  $(M, w_2) \models K_{ia}(F \wedge \neg F)$ , that is,  $(M, w_2) \models K_{ia}(\text{False})$ . It follows that  $ia$  does not distinguish  $F$  from  $\neg F$ , and therefore  $ia$  is completely useless.

### Proof (Proposition3)

The proof follows from Proposition1 (this paper) and Proposition(Ref), Proposition(Tra), Proposition(Euc), Proposition(Symm), and Proposition(Serial) in [5].

### Proof (Proposition4)

We need to show that (for all  $k \in MAS$ ) [ $ia \triangleright k$ ] holds.

Let  $j \in MAS$  be an arbitrary agent in MAS. We would like to prove that  $ia \triangleright j$ .

It follows, from the integrated agent definition, that  $kia \subseteq kj$ . Suppose that

A:  $M \models Kj(F)$ . We would like to show that B:  $M \models Kia(F)$ . To show B we need to show C:  $(M, w) \models Kia(F)$ , for all  $w \in S$ . Therefore, let  $t \in S$  be an arbitrary world in  $S$ . We prove that D:  $(M, t) \models Kia(F)$ . Let  $v \in S$  be an arbitrary world such that  $(t, v) \in kia$ . We need to prove E:  $(M, v) \models F$ . Because  $(t, v) \in kia$  and  $kia \subseteq kj$ , we obtain  $(t, v) \in kj$ . Now, it follows, from A, that  $(M, t) \models Kj(F)$ , that is,  $(M, v) \models F$ , that is, E holds. Therefore, D holds, that is, C holds, that is, B holds, and finally  $ia \triangleright j$  holds, as we wanted to show.

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Mirko Maleković  
Mirko Čubrilo

## INTEGRIRAJUĆI AGENT U VIŠEAGENTNIM SUSTAVIMA

### Sažetak

*U ovom članku karakteriziran je integrirajući agent u višeagentnim sustavima. Dokazan je sljedeći rezultat: ako je više agentni sustav refleksivan (simetričan, tranzitivan, Euklidov), onda je integrirajući agent danog višeagentnog sustava refleksivan (simetričan, tranzitivan, Euklidov), respektivno. Također, dokazali smo da analogan rezultat ne vrijedi u slučaju serijabilnosti višeagentnog sustava. Konačno, karakteriziran je odnos između znanja integrirajućeg agenta i korespondentnog više agentnog sustava, gdje je dokazano da integrirajući agent zna više od bilo kojeg agenta u višeagentnom sustavu.*

**Ključne riječi:** integrirajući agent, operatori znanja, više agentni sustavi, rezoniranje o znanju.