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THE PAST TEMPORAL OPERATORS IN MULTI-AGENT SYSTEMS

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In this paper, we consider the past temporal operators in multi-agent systems. Three temporal operators po (previous), p♦ (once), and p♥ (has-always-been) are defined.

These past temporal operators can be used for reasoning about events that happen along a single run r (in the past) in the system R, where R models the possible behaviors of the system being modeled. Some important properties of agents (expressed by the formulas : Ki ⇒ poKj, Ki ⇒ p♦Kj, and Ki ⇒ p♥Kj) are characterized..

Keywords: knowledge bases, knowledge operators, multi-agent systems, past temporal operators, reasoning about knowledge.

1. INTRODUCTION

The theory of multi-agent systems is described in [1]. In [2], we have described incorporating knowledge (knowledge operators Ki, i = 1,.., m) and time (the future temporal operators). An excellent introduction to temporal logic can be found in [3].

The future temporal operators can talk about events that happen only in the present or future, not events that happen in the past.

In this paper, we define the past temporal operators: po (previous), p♦ (once), and p♥ (has-always been). By using these temporal operators and knowledge operators, we can build the knowledge-temporal formulas that express the properties of agents in multi-agent systems. Some new results regarding the properties that relate the knowledge of two agents will be proved. These results can be very useful in analyzing the respective multi-agent system.

The paper consists of four sections and an appendix containing proof of the past propositions. In Section 2, we introduce the basic notions of multi-agent systems, knowledge operators, and the past temporal operators. In Section 3, we state the past propositions:

Proposition (Ki ⇒ poKj) [agent j previously knew F if agent i knows F, under the condition that some premise U1 (defined later) holds]; Proposition (Ki ⇒ p♦Kj) [agent j has known F if agent i knows F, under the condition that some premise U2 (defined later) holds]; and Proposition (Ki ⇒ p♥Kj) [agent j has always known F if agent i knows F, under the condition that some premise U3 (defined later) holds]. Conclusions are given in Section 4. The Appendix contains the proofs of all the propositions mentioned above.

2. BASIC NOTIONS

In this section, we introduce the basic concepts and notations.

Suppose we have a group consisting of m agents, named 1, 2,.., m . We assume these agents wish to reason about a world that can be described in terms of a nonempty set P of primitive propositions. A language is just a set of formulas, where the set of formulas PLK, of interest to us, is defined as follows:

(1) The primitive propositions in P are formulas;
(2) If F and G are formulas, then so are $\neg F$, $(F \vee G)$, and Ki(F) for all $i \in \{1, 2, .., m\}$, where Ki is a modal operator.

We omit the parentheses in formulas such as $(F \vee G)$ whenever it does not lead to confusion. We also use standard abbreviations from prepositional logic, such as $F \wedge G$ for

$\neg(\neg F \vee \neg G)$, $F \Rightarrow G$ for $\neg F \vee G$, and $F \Leftrightarrow G$ for $(F \Rightarrow G) \wedge (G \Rightarrow F)$.

A Kripke structure M for an agent group {1, 2,.., m} over P is a (m + 2)-tuple

M = (S, I, k1, k2,.., km), where S is a set of possible worlds, I is an interpretation that associates with each world in S a truth assignment to the primitive propositions in P, and k1, k2,.., km are binary relations on S, called the possibility relations for agents 1, 2,.., m, respectively.

Given $p \in P$, the expression I[w](p) = true means that p is true in a world w in a structure M. The fact that p is false, in a world v of a structure M, is indicated by the expression I[v](p) = false.

The expression $(u, v) \in ki$ means that an agent i considers a world v possible, given his information in a world u. Since ki defines what worlds an agent i considers possible in any given world, ki will be called the possibility relation of the agent i.

We now define what it means for a formula to be true at a given world in a structure.

Let $(M, w) \models F$ mean that F holds or is true at (M, w). The definition for $\models$ is as follows:

(a) $(M, w) \models p$ iff I[w](p) = true, where $p \in P$;
(b) $(M, w) \models F \vee G$ iff $(M, w) \models F$ or $(M, w) \models G$;
(c) $(M, w) \models \neg F$ iff $(M,w) \not\models F$, that is, $(M, w) \models F$ does not hold;
(d) $M \models F$ iff $(M, w) \models F$ for all $w \in S$.

Finally, we shall define a modal operator Ki, where Ki(F) is read: Agent i knows F.

(e) $(M, w) \models Ki(F)$ iff $(M, t) \models F$ for all $t \in S$ such that $(w, t) \in ki$.

In (e) we have that an agent i knows F in a world w of a structure M exactly if F holds at all worlds t that the agent i considers possible in w.

Multi-agent systems

A multi-agent system is any collection of interacting agents. Our key assumption is that if we look at a system at any point in time, each of the agents is in some state. We refer to this as the agent's local state. We assume that an agent's local state encapsulates all the information to which the agent has access. As each agent has a local state, it is very natural to think of the whole system as being in some (global) state. The global state includes the local states of the agents and the local state of an environment. Accordingly, we divide a system into two components: the environment and the agents, where we view the environment as everything else that is relevant. Also, the environment can be viewed as just another agent. We need to say that a given system can be modeled in many ways. How to divide the system into agents and environment depends on the system being analyzed.

Let Le be a set of possible local states for the environment and let Li be a set of possible local states for agent i , $i = 1, .., n$. We define $G = Le \times L1 \times .. \times Ln$ to be the set of global states. A global state describes the system at a given point in time. Since a system constantly changes (it is not a static entity), we are interested in how systems change over time. We take time to range over the natural numbers, that is, the time domain is the set of the natural numbers, N.

A run over G is a function $r : N \to G$.

Thus, a run over G can be identified by a sequence of global states in G . The run r represents a complete description of how the system's global state evolves over time. Thus, r(0) describes the initial global state of the system in a possible execution, r(1) describes the next global state, and so on.

If r(m)=(se, s1,.., sn), then we define r[e](m)=se and r[i](m)=si, for i=1,.., n .

Note that r[i](m) = si is the local state of agent i at the (global) state r(m) .

A system R over G is a set of runs over G. The system R models the possible behaviors of the system being modeled. At the end of this section, let us point out that the authors in [1] showed how to model a knowledge base as a multi-agent system. The result is very interesting because in doing so we can now talk about what the knowledge base knows about its own knowledge. Some problems in this approach will be studied in a forthcoming paper.

Knowledge in multi-agent systems

We assume that we have a set P of primitive propositions, which we can think of as describing basic facts about a system R. Let I be an interpretation for the propositions in P over G , which assigns truth values to the primitive propositions at the global states. Thus, for every $p \in P$ and $s \in G$, $I[s](p) \in \{true, false\}$. An interpreted system IS is a pair (R, I).

Now, we define knowledge in interpreted systems.

Let IS = (R, I) be an interpreted system. A Kripke structure for IS, denoted

M(IS) = (S, I, k1, .., kn), is defined in a straightforward way.

$S = \{r(m) \mid r \in R, m \in N\}$, that is, S is the set of the global states at the points (r, m) in the system R.

The possibility relations k1, k2,.., kn are defined as follows.

Let r(m) = (se, s1,.., sn), r'(m') = (se', s1',.., sn') be two global states in S. We say that r(m) and r'(m') are indistinguishable to an agent i iff si = si'.

Thus, the agent i has the same local state in both r(m) and r'(m'). We define $ki = \{(r(m), r'(m')) \in S \times S \mid$ r(m) and r'(m') are indistinguishable to the agent i$\}$, i = 1, 2,.., n.

Accordingly, $(r(m), r'(m')) \in ki$ iff si = si', i = 1, 2,.., n .

There is no possibility relation ke for the environment because we are not usually interested in what the environment knows.

Now, it is evident what it means for a formula F in PLK to be true at a state r(m) in an interpreted system IS. For instance, we have

$(IS, r(m)) \models p$ iff $I[r(m)](p) = true$, for all $p \in P$.

$(IS, r(m)) \models Ki(F)$ iff $(IS, r'(m')) \models F$ for all $r'(m') \in S$ such that $(r(m), r'(m')) \in ki$.

We say that a formula F in LK is valid in an interpreted system IS, denoted $IS \models F$, iff $(IS, r(m)) \models F$ for all $r(m) \in S$.

To be able to make temporal statements, we extend our language PLK by adding temporal operators, which are new modal operators for talking about time. In [2] we described the future temporal operators (the operators talk about events that happen only in the future). In this paper, we characterize the past temporal operators for reasoning about the past. The language (PLK + the past temporal operators) will be denoted by PLKPT., and will be used for reasoning about events that happen along a single run r (in the past) in the system R.

The past temporal operators

A past formula (includes at least one past temporal operator) describes a property of a prefix of the state, lying to the left of the current position, that is, a past formula at the state r(m) describes a property of the states r(0), r(1),.., r(m – 1), r(m).

The Previous Operator po If $F \in$ PLKPT, then so is poF, read previously F. Its semantics is defined by $(IS, r(m)) \models poF$ iff $m > 0$ and $(IS, r(m-1)) \models F$.

Thus, poF holds at state r(m) iff r(m) is not the first state in the run r and F holds at state r(m – 1). In particular, poF is false at state r(0). This operator makes sense because our notion of time is discrete. All the other past temporal operators make perfect sense even for continuous notions of time.

The Once Operator p♦
If F ∈ LKPT, then so is p♦F, read once F. Its semantics is defined by
$(IS, r(m)) \models p♦F$ iff $(\text{for some } m', 0 \le m' \le m)[(IS, r(m')) \models F]$.

Accordingly, p♦F holds at state r(m) iff F holds at state r(m) or some preceding state.

The Has-always-been Operator p♥
p♥F ∈ LKPT if F ∈ LKT. It is read has always been F, and defined by
$(IS, r(m)) \models p♥F$ iff $(\text{for all } m', 0 \le m' \le m)[(IS, r(m')) \models F]$

Thus, p♥F holds at state r(m) iff F holds at state r(m) and all preceding positions.

3. THE PAST PROPOSITIONS

In this section, we give the past propositions that talk about some important agent properties. These properties relate the knowledge of two agents.

In the following propositions, we shall use the set S[j, r](m') defined by

S[j, r](m') = {ri(mi) | (r(m'), ri(mi)) ∈ kj}. Thus, S[j, r](m') is the set of the states in S that agent j considers possible in state r(m'). Also, F is an arbitrary formula in LKPT.

Proposition (Ki ⇒ poKj)
If $S[j, r](m-1) \subseteq S[i, r](m)$, then $(IS, r(m)) \models Ki(F) \Rightarrow poKj(F)$.

The proposition states that at state r(m) agent j previously knew F if agent i knows F, under the condition that U1: $S[j, r](m-1) \subseteq S[i, r](m)$ holds.

Proposition (Ki ⇒ p♦Kj)
If $(\text{for some } m', 0 \le m' \le m)[S[j, r](m') \subseteq S[i, r](m)]$, then $(IS, r(m)) \models Ki(F) \Rightarrow p♦Kj(F)$.

Proposition (Ki ⇒ p♦Kj) says that at state r(m) agent j has known F if agent i knows F, under the condition that U2: $(\text{for some } m', 0 \le m' \le m)[S[j, r](m') \subseteq S[i, r](m)]$ holds.

Proposition (Ki ⇒ p♥Kj)
If $(\text{for all } m', 0 \le m' \le m)[S[j, r](m') \subseteq S[i, r](m)]$, then $(IS, r(m)) \models Ki(F) \Rightarrow p♥Kj(F)$.

Proposition (Ki ⇒ p♥Kj) states that at state r(m) agent j has always known F if agent i knows F, under the condition that U3: (for all m', $0 \le m' \le m$)[S[j, r](m') $\subseteq$ S[i, r](m)] is true.

4. CONCLUSIONS

We have considered the properties of the past temporal operators and the knowledge operators. These properties relate the knowledge of two agents. We have stated the past propositions: Proposition (Ki ⇒ poKj) [agent j previously knew F if agent i knows F, under the condition that the premise U1 holds]; Proposition (Ki ⇒ p♦Kj)) [agent j has known F if agent i knows F, under the condition that the premise U2 holds]; and Proposition (Ki ⇒ p♥Kj) [agent j has always known F if agent i knows F, under the condition that the premise U3 holds]. The Appendix contains the proofs of all the propositions mentioned above.

APPENDIX

Proof (Proposition (Ki ⇒ poKj)

Assume U1: S[j, r](m – 1) $\subseteq$ S[i, r](m). We would like to show (IS, r(m)) ⊨ Ki(F) ⇒ poKj(F). Assume V1: (IS, r(m)) ⊨ Ki(F). We have to show (IS, r(m)) ⊨ poKj(F), that is, (IS, r(m – 1)) ⊨ Kj(F).
Let ri(mi) ∈ S be an arbitrary state such that (r(m – 1), ri(mi)) ∈ kj. It follows that ri(mi) ∈ S[j, r](m–1). We have (by the assumption U1) ri(mi) ∈ S[i, r](m). Therefore, (r(m), ri(mi))∈ki. Because V1 holds, we have (IS, ri(mi)) ⊨ F. Consequently, we have (IS, r(m – 1)) ⊨ Kj(F), as we wanted to show.

Proof (Proposition (Ki ⇒ p♦Kj)

Assume U2: (for some m', $0 \le m' \le m$)[S[j, r](m')$\subseteq$S[i, r](m)]. We would like to show (IS, r(m)) ⊨ Ki(F) ⇒ p♦Kj(F). Assume V2: (IS, r(m)) ⊨ Ki(F). We have to show (IS, r(m)) ⊨ p♦Kj(F). Let m', $0 \le m' \le m$, be such a point that (by the assumption U2) U2': S[j, r](m') $\subseteq$ S[i, r](m) holds. We shall prove (IS, r(m')) ⊨ Kj(F).
Let ri(mi) ∈ S be an arbitrary state such that (r(m'), ri(mi)) ∈ kj, that is, ri(mi)∈S[j, r](m'). It follows (by the assumption U2) that ri(mi)∈S[i, r](m). Therefore, (r(m), ri(mi)) ∈ ki. We obtain (by the assumption V2) (IS,ri(mi)) ⊨ F. Consequently, we have (IS, r(m')) ⊨ Kj(F), that is, (IS, r(m)) ⊨ p♦Kj(F)., as desired.

Proof (Proposition (Ki ⇒ p♥Kj)

Assume U3: (for all m', $0 \le m' \le m$)[S[j, r](m') $\subseteq$ S[i, r](m)]. We would like to show (IS, r(m)) ⊨ Ki(F) ⇒ p♥Kj(F). Assume V3: (IS, r(m)) ⊨ Ki(F). We have to show (IS, r(m)) ⊨ p♥Kj(F). Let m' be an arbitrary point such that $0 \le m' \le m$. We need to prove (IS, r(m')) ⊨ Kj(F). Let ri(mi) ∈ S be an arbitrary state such that (r(m'), ri(mi)) ∈ kj. We have to prove (IS, ri(mi)) ⊨ F. From (r(m'), ri(mi) ∈ kj, it follows that ri(mi) ∈ S[j, r](m'). We have (by U3) ri(mi) ∈ S[i, r](m). Therefore, (r(m), ri(mi)) ∈ ki. We have (by V3) (IS, ri(mi)) ⊨ F. Consequently, we obtain (IS, r(m')) ⊨ Kj(F), that is, (IS, r(m)) ⊨ p♥Kj(F), as we wanted to show.

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TEMPORALNI OPERATORI O PROŠLOSTI U VIŠEAGENTNIM SUSTAVIMA

Sažetak

U ovom članku razmotreni su temporalni operatori o prošlosti u višeagentnim sustavima. Definirana su tri temporalna operatora: po *(prethodno),* p♦ *(bilo je barem jedanput) i* p♥ *(uvijek je bilo) Temeljem navedenih operatora i operatora znanja karakterizirana su neka važna svojstva agenata u više agentnim sustavima.*

Ključne riječi: baze znanja, operatori znanja, višeagentni sustavi, temporalni operatori za prošlost, rezoniranje o znanju.