UDC: 65.012.4:339.6
Professional paper

A TRADING COMPANY'S INVENTORY MANAGEMENT MODEL

Stjepan Vidačić

University of Zagreb, Faculty of Organization and Informatics, Varaždin, Croatia
E-mail: stjepan.vidacic@zg.tel.hr

This paper is a review of some general mathematical models for inventories management and an example of their application in a trading company's inventory management. The first part of this paper is a description of the problem of optimal inventories management in a complex trading company that deals with a wide range of products, and where the function of ordering goods is distributed according to the types of goods among several marketing officers. The second part of the paper is a summary of some existing general mathematical models for inventories management. The third part of the paper is a definition of the basic requirements for the automatic management of goods inventories in a trading company. The fourth part of this work presents a mathematical model for a trading company's inventory management, which is done automatically by a commercial program package for inventory accounting. By using this application a company may make substantial savings.

Keywords: inventories management, mathematical model, trading company, supply, demand, order.

1. INTRODUCTION

The term "inventories" means various types of goods that are stored in order to satisfy market demands in the normal course of business.

The reasons to keep goods as inventories are as follows:

1. The discretion needed for the realization of an order at approximately constant demand
2. The random nature of the level of demand in the period between two orders
3. Anticipated market changes, as a consequence of the seasonal character of production and demand and also the anticipated price increase.

Despite the fact that the above facts usually lead to an increase in the inventories volume, a series of factors demonstrate the reason for keeping minimal inventories, the most important being the following:

1. Goods storage and maintenance costs
2. Costs of freezing current assets in these inventories
3. Losses in the quantity or the quality of inventories.

The management of inventories within the supply system of a trading company (Fig. 1) consists of two basic quantities: the order volume (x) and the time interval between two orders (t).

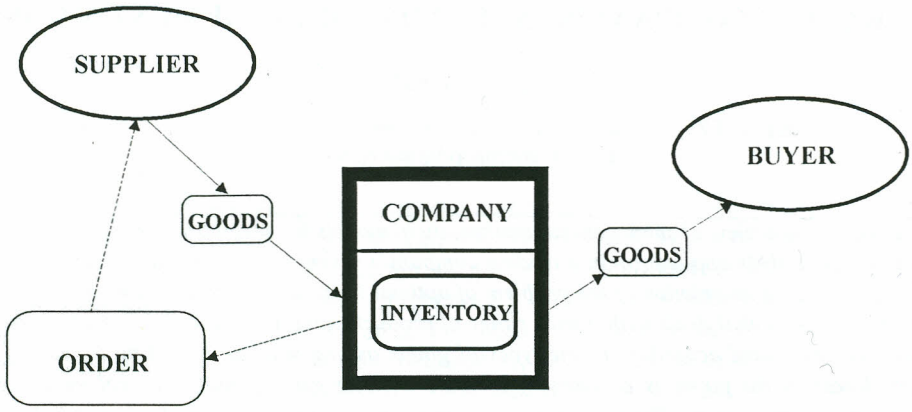

Figure 1. A trading company's supply system

The set of rules according to which (x) and (t) are determined will define the inventories management strategy.

The order volume (x), i.e. the quantity of goods to be ordered from a supplier is in function of:

- the current quantity of goods in stock
- the supplier's delivery terms
- the planned sales within the period of waiting for the supplier's delivery.

However, in order to avoid the risk of an unjustified increase in investments for an unnecessary increase in the inventories or to avoid the risk of negative inventories, it is necessary to determine the trend of growth or a fall in sales in a previous planning period, which should be as long as possible.

Determination of the time interval (t) between two orders is not a major problem. If the planned sales are realized, the time interval is equal to the supplier's delivery term, and if the sales are increased within the term of delivery, the time interval may be shorter, while in the case of a sales decrease, it is longer than the supplier's delivery term.

In this sense, the optimal strategy of inventories management in a trading company is the one that optimally determines (x) and (t) and ensures the uninterrupted optimal coordination of inventories, market supply and demand, along with the constant costs of storage and impermissible negative inventories.

The basic features of trading companies dealing in the mass sale of goods in wholesale and retail are as follows:

1. Dealing with a wide range of goods
2. Large sales within a short period

3. The inadmissibility of negative inventories
4. The size of inventories with regard to storage space
5. The maintenance of minimal inventories
6. Random character of demand for goods in the coming period
7. The tendency for sales to continue to increase
8. Determining quantities to be ordered according to previous sales, the current quantities available in stock and the supplier's delivery term
9. The function of ordering goods is distributed among several marketing officers according to the type of goods.

This shows, and the experience from the practice of trading companies support this standpoint, that the automatization of the complex process of monitoring and maintaining the optimal level of inventories is necessary, as in this way the time needed for the analysis of sales and formation of optimal order is minimized, which in the case of the application of the quality commercial program packages for inventories accounting is not a particular problem.

2. SOME GENERAL MATHEMATICAL MODELS OF INVENTORIES MANAGEMENT

General mathematical models of inventories management are characterized with the objective function and a set of constraints.

The objective function is mostly used to keep costs at a minimum, with an appreciation of the relations in the supply and demand system, possibility to complete the inventories, inventories management strategy and other limiting factors.

Mathematical models of inventories management may have a very complex character and for that reason they are simplified in order to describe approximately certain management tasks.

Optimal inventories management by means of simplified mathematical models consists in solving the following two tasks:

1) It is necessary to determine total demand, with fixed time periods to submit supply requests.
2) It is necessary to determine total demand and the time needed to submit supply requests.

The management task consists of determing an optimal solution for both tasks, the optimal solution being the one that enables them to achieve minimum expenditures incurred in the process of inventories organization and management.

While considering the fact that the basic purpose of supply is to have goods available at the right time and in adequate quantities with as small costs as possible, it is necessary to organize the inventories management with this very important objective mind.

The possibility of purchasing the goods may not be strictly adapted to the time of their demand, i.e. the needs for particular products are often complex and uncertain.

Inventories management consists of the optimal coordination of mutually adverse aspirations for a higher level of supply and for lower costs.

There are two categories of inventories: the market category and the production category. Market inventories must satisfy the customers' demand and do not necessary need to ensure their own uninterrupted production. Some warehouses may be a combination of both categories of inventories and it is therefore difficult to provide a unique interpretation of all of the categories of inventories, as some of them differ so much from others that it would be illusory to look for general solutions.

In this sense, the necessary inventories must be forecast. Forecasts may be based on information from the past, from orders, on cost analysis, etc., or any combination of these parameters.

When determining the levels of inventories and other parameters important to the management of inventories, the specific circumstances of each particular case must be taken into account.

2.1. Inventories management models with known demand and the infinite costs of negative inventories

The first definition of an inventories management mathematical model with constant demand starts with the following known data [2]:

- A – the volume of demand for a product within a given time interval (T)
- C_1 – the costs of storage per unit of the product and per unit of time are known and proportional to the quantity of goods in stock
- C_2 – the costs of failure to satisfy demand, if such a failure is permitted
- C – the costs of a single order.

A solution to the inventories management problem, in this case, usually consists of finding values for the following:

F(x) – the objective function $F(x) = \frac{TC_1}{2}x + \frac{CA}{x}$

x* – the optimal order size $x^* = \sqrt{2\frac{AC}{TC_1}}$

t* – the optimal time interval between two orders $t^* = \frac{T}{A}\sqrt{2\frac{AC}{TC_1}}$

F(x*) – the amount of total costs when the optimal policy for inventories management is applied $F(x^*) = \sqrt{2ATC_1}$.

2.2. Inventories management models with known demand and the finite costs of negative inventories

The inventory model may be defined by allowing supply to be higher than the level of inventories in a certain period of time.

Because of the untimely satisfaction of demand, there are costs (C_2), and these are expressed per unit of the goods that haven't been supplied over a particular period of time.

The model is defined by the following objective function of the total costs of supply and inventories management [2]:

$$F(x,y) = \frac{y^2 C_1 T}{2x} + \frac{(x-y)^2 C_2 T}{2x} + \frac{CA}{x},$$

where:

A – the total demand over a period of time (T)
C_1 – the costs of keeping inventories for a unit of goods over a unit of time
C – the fixed costs of the realization of a single order
x – the volume of demand for goods within a given time interval (t)
y – the volume of inventories during the period (t).

The optimal values of the supply system variables that correspond to the minimum of the function F(x,y) are determined by the following expressions:

$$x^* = \sqrt{\frac{2AC}{TC_1}} \cdot \sqrt{\frac{C_1 + C_2}{C_2}}, \quad y^* = \sqrt{\frac{2AC}{TC_1}} \cdot \sqrt{\frac{C_2}{C_1 + C_2}},$$

$$t^* = \frac{T}{A} x^* = \sqrt{\frac{2TC}{AC_1}} \cdot \sqrt{\frac{C_1 + C_2}{C_2}},$$

$$t_1^* = \frac{y^*}{x^*} t^* = \sqrt{\frac{2TC}{AC_1}} \cdot \sqrt{\frac{C_2}{C_1 + C_2}},$$

where:

t_1 – the time interval within which the customer is normally supplied with inventories
t_2 – the time interval in which demand may not be satisfied
t – the time between two sequential orders
n – the number of orders during a period of time (T).

$$t_2^* = t^* - t_1^* = \sqrt{\frac{2TC}{AC_1}} \cdot \sqrt{\frac{C_1 + C_2}{C_2}} \cdot \left(1 - \frac{C_2}{C_1 + C_2}\right),$$

$$n^* = \frac{T}{t^*} = \sqrt{\frac{ATC_1}{2C}} \cdot \sqrt{\frac{C_2}{C_1 + C_2}}.$$

The minimal costs are:

$$F(x^*, y^*) = \sqrt{2ACTC_1} \cdot \sqrt{\frac{C_2}{C_1 + C_2}}.$$

2.3. An inventories management mathematical model with random demand

Demand in the supply system for a certain product has the characteristic of being of random size and is determined with the probability law p(x). If the demand (x) is lower than the inventories level (y), then the costs of supply will be (C_1) per unit of the product.

Where (x) is higher than the inventory level (y), a deficiency of a product creates the costs of an intervention purchase (C_2) per unit of the product.

As storage costs are small in relation to (C_1) and (C_2) amounts, the time period (T) may be neglected and the management process may be regarded as independent of time.

The total costs function has the form [2]:

$$F(y) = C_1 \sum_{x=0}^{y} (y - x)p(x) + C_2 \sum_{x=y+1}^{\infty} (y - x)p(x).$$

The minimum F(y*) is reached for the value (y*) while satisfying the inequality:

$$\sum_{x=0}^{y^*-1} p(x) < \frac{C_2}{C_1 + C_2} < \sum_{x=0}^{y^*} p(x).$$

Such a supply system is typical, for instance, when the problem of the supply of spare parts for complex devices is considered. Some of the probabilistic models can be found in [3].

2.4. Models of inventories with several different products

One problem with inventories management is to find an optimal level of inventories for several different products that are simultaneously kept in stock.

Generally, it can be said that in the supply system there are n various products and that their demand in a particular period is known, namely being A_1, A_2, ..., A_j, ..., A_n of units.

Then three types of costs may be defined in the supply system for these products, namely [2]:

C_{0j} – the direct costs of production-purchase per unit of product (j)

C_{1j} – the costs of preparation and organization of production-purchase of a series for (j) type of product

C_{2j} – the costs of storage per unit of stored product (j), that are given as percentage (p_j) of the value of the goods in stock.

It is necessary to determine the optimal quantities of a production-supply series (x_j), for each of the products, provided that total costs in the supply system are minimal within the examined period.

The optimal level of inventories for a certain period is given by the expression:

$$x_j^* = \sqrt{\frac{2C_{1j}A_j}{p_j C_{0j}}}.$$

Minimum costs are determined by the expression:

$$(\min)F(x) = F(x^*) = \sum_{j=1}^{n}\left[C_{0j}A_j + \frac{C_{1j}}{x_j^*}A_j + \frac{p_j}{2} + \left(C_{0j}x_j^* + C_{1j}\right)\right].$$

2.5. Inventories models based on other operations research methods

For a solution to the complex problems of inventories management and modelling supply other methods of operations research are also used, such as linear programming, dynamic programming and other similar methods.

Application of these methods within the business program package is gaining importance in the fields of the production, the distribution and the consumption of material goods.

The complexity of the inventories management problem is most frequently shown in two ways:

1) The presence of limitations in the supply system
2) The dynamic character of the supply system functioning with a substantial influence of the inventories maintenance costs on the total costs of system maintenance.

2.5.1. *Inventories models with limitations*

When solving the problem of optimal inventories management, one can often see that the optimal volume of an order may not be realized because of a limitation of resources, such as storage space and the like. Therefore, within the mathematical inventories model there are one or more limitations, along with the cost function. Generally speaking, the task will be reduced to a search for the extreme condition of the costs function.

For instance, a company dealing with the sales of goods, by purchasing a certain product wholesale, and keeping inventories of these goods and selling them wholesale or retail upon the customer's demand, faces the problem of determining the optimal sales policy with the aim of realizing maximum profit in a particular time period, and this is examined as a planning period divided into n intervals.

The conditions under which a company sells goods may be defined as having the following quantities [2]:

B – a limited warehouse capacity
A – an initial level of inventories in stock
x_j – the quantity of goods to be purchased in interval (j), where j = 1,2,...,n

y_j – the quantity of goods to be sold in interval (j)
p_j – the selling price per unit of goods in interval (j)
c_j – the storage costs per unit of goods.

The profit mode by a company in a planning period is defined by the following objective function:

$$D = \sum_{j=1}^{n} p_j y_j - \sum_{j=1}^{n} c_j x_j .$$

Constraints related to the sales of goods are defined by the following inequality system:

$$-\sum_{j=1}^{i-1} x_j + \sum_{j=1}^{i} y_j \le A$$

$$\sum_{j=1}^{i} x_j - \sum_{j=1}^{i} y_j \le B - A$$

where: $x_j >= 0$ i $y_j >= 0$, for $i = 1,2,\ldots,n$
$j = 1,2,\ldots,n$

A model formed in this way is a static linear model of inventories management for a single type of goods. The solution to the problem for each interval of the planning period gives optimal quantities of goods to be sold in that period in order to make maximum profit.

2.5.2. *Dynamic inventories management models*

The problems of inventories management belong to the area of applied operational research where the dynamic nature of the problem and the temporal dimension are the most common elements that need to be taken into account when setting and forming a mathematical model.

Generally, the problem is examined in a limited planning period which is divided into intervals.

The functioning of the supply system and inventories management in an interval depends on parameters like the following:

- the level of inventories at the beginning and at the end of the interval
- the volume and change in demand within these intervals
- the nature and volume of costs within these intervals
- the possibility to produce or purchase products and the conditions for storing them during the interval.

For instance, if a company wants to determine a production plan for a product for a certain time period consisting of n intervals, with the volume of demand alredy known for each time period, the mathematical model used to show the optimal production plan may be defined as containing the following variables:

x_i – the production volume within the interval
u_i – the level of inventories at the end of the interval
b_i – the volume of product demand known at the beginning of interval i.

The objective function may be written in the following form [2]:

$$(\min)F(x,u) = \sum_{i=1}^{n} C_i(x_i, u_i).$$

Variables (x_i) and (u_i) must satisfy the following limitations:

$$u_{i-1} + x_i - u_i = b_i \quad (i = 1,2, \ldots, n), \text{ where}$$

- u_0 is the given level of demand at the beginning of the planning period that must be satisfied
- $u_n = 0$, i.e. inventories at the end of the last interval must be equal to zero
- x_i , u_i are integers.

If the objective function is linear, the task may be solved as a task of integer programming. However, this function is most frequently linear, i.e. it is given in a discrete form.

3. THE REQUIREMENTS FOR AUTOMATIC INVENTORIES MANAGEMENT

As goods inventories management is an organizational-technological problem, automatic inventories management is essentially reduced to an automatic generating of output orders for goods.

The basic requirements for automatic generating of orders for goods are as follows:

1) The data base of inventories accounting within the business information system, with the sales analyses that may be described by the relation SALES:

SALES = { Code, Date, Output, ... }, where

Code – the code of goods
Date – the date of sales
Output – quantity sold.

2) The criteria for generating orders for goods must be flexible within the framework of the company's commercial function so that they are best defined within a corresponding register that may be described by the relation NCRITERIUM:

NCRITERIUM = { Supplier, Group, Term, Period, Factor, ... }, where

Supplier – the supplier code
Group – the goods group indication
Term – the term for delivery - order completion
Period – the planning period of sales analysis
Factor – the sales trend factor.

3) An elaborated mathematical model of inventories management.

Most of the time neither of the general mathematical models as set out in item 2 can be directly applied in practice, but they need to be adapted to the actual production and market conditions in which a company is operating.

In this sense the applied mathematical model of inventories management is most frequently used as a direct function of the algorithm and is made automatic within the actual business program package.

4. AN AUTOMATIC MODEL OF INVENTORIES MANAGEMENT

An automatic model of inventories management in a trading company, along with the constant coordination of supply and demand and the realization of maximum profit, must ensure the following:

1) The minimum time period for determining the quantities of goods to be ordered
2) An automatic generation of ordering documents per supplier and groups of goods.

This study presents a simplified model [1], based on the requirements as set out in item 3, and with the aim of determining the optimal quantity of goods to be purchased $XN_i = F(X_i, XR_i)$, with the costs for the purchase and storage of goods constant (they are not analyzed) and assuming that the purchase of one type of goods is from a single supplier.

The model may be described in the following way:

$$X_i = \frac{I_i.R_i.F_i}{P_i},$$

where:

XN_i – the optimal quantity of goods to be purchased
XR_i – the quantity of goods available in inventories at the moment when the order is generated
X_i – the quantity of goods to be purchased in the delivery term
N_i – the total number of various types of goods
I_i – the sales of goods in the preceding planning period
R_i – the guaranteed term for the delivery of goods by the suppliers (in days)
F_i – the planned coefficient of sales increase in the coming period
P_i – the previous sales analysis period, beginning with the day the order was generated and moving backwards (in days).

The optimal quantity of goods ordered is a function of the current quantities available in inventories and is determined according to the empirical algorithm as presented in Fig. 2.

By applying this model, within the commercial program package for inventories accounting, the system of generating output orders is fully automatic, in the form of a document for standard orders for each group of goods and is automatically generated from the NCRITERIUM register for each supplier, with the optimal quantity ordered in accordance with the given requirements.

The conditions defined in the NCRITERIUM register may be changed and are a function of the supplier and the business policy of a company. Before delivering an order to the supplier, only visual control of the quantities proposed by the algorithm is necessary.

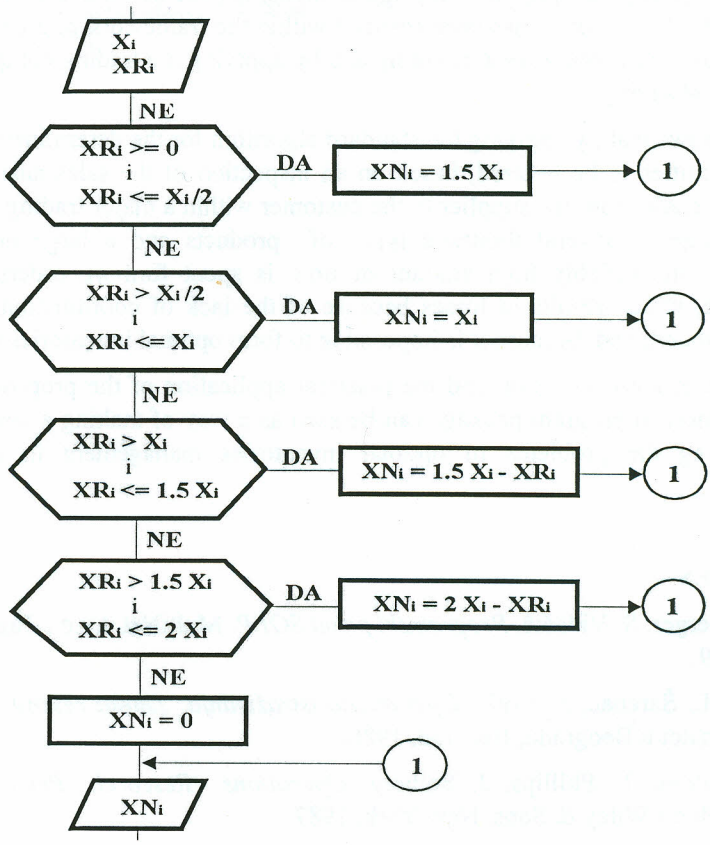

Figure 2. A segment block of the algorithm diagram for the determination of the actual quantity of goods to be purchased.

5. CONCLUSION

The optimal management of trading companies' inventories is a fundamental problem and its solution would have a direct influence on the total efficiency of businesses and the position of a company in a market.

The problem is becoming more complex as the range of products is widening and their sales figures are growing. The fundamental aim of solving this problem is a continuous and a complete satisfying of demand, with minimum costs for inventories formation and storage.

This paper shows a series of general models that, if adapted properly, could help solve the problems with optimal inventories management in trading companies.

By using these models and starting from an assumption that the automation of inventories management can essentially be reduced to an automatic generating of goods ordering, this paper presents an original model for the automatic generating of an optimal order for goods. It has been realized within the framework of a commercial program package for inventories accounting and by applying it a trading company may make substantial savings.

Practice shows that by applying the standard algorithm for the determination of the necessary quantities to be ordered, based on an inspection of the sales analyses in a dynamic trade chain from the supplier to the customer within a major trading company that has a range of several thousand types of products and a large number of customers, an unjustifiably high amount of time is spent forming orders in each ordering cycle, which results in losses because of the lack of coordination between supply and demand, and because it is impossible to form optimal inventories in time.

In this sense, automatization and the practical application of the proposed model within a commercial program package can be seen as a way of making a contribution towards solving the problems of optimal inventories management in a trading company.

REFERENCES

[1] Z. Adelsberger, S. Vidačić. *Programski paket SOPP.* MultiNet d.o.o., Zagreb, 1991-1999.

[2] J. Petrić, L. Šarenac, Z. Kojić. *Operaciona istraživanja: Zbirka rešenih zadataka II.* Univerzitet u Beogradu, Beograd, 1980.

[3] A. Ravindran, T. Phillips, J. Solberg. *Operations Research: Principles and Practice.* John Wiley & Sons, New York, 1987.

Received: 15 July 1999
Accepted: 30 November 1999

Stjepan Vidačić

MODEL UPRAVLJANJA ZALIHAMA ROBE TRGOVAČKE TVRTKE

Sažetak

U radu je dat pregled općih matematičkih modela upravljanja zalihama i primjer njihove primjene u upravljanju zalihama trgovačke tvrtke. U prvom dijelu rada opisan je problem optimalnog upravljanja zalihama u složenoj trgovačkoj tvrtki koja trguje širokim asortimanom robe i gdje je funkcija naručivanja robe distribuirana po vrstama robe između više komercijalista. U drugom dijelu rada dat je sažet prikaz postojećih općih matematičkih

modela upravljanja zalihama. U trećem dijelu rada definirane su temeljne pretpostavke za automatizirano upravljanje zalihama robe trgovačke tvrtke. U četvrtom dijelu rada prikazan je matematički model upravljanja zalihama robe trgovačke tvrtke, koji je automatiziran u okviru komercijalnog programskog paketa robnog knjigovodstva i čijom primjenom tvrtka može ostvariti znatne uštede.

Ključne riječi: upravljanje zalihama, matematički model, trgovačka tvrtka, opskrba, potražnja, narudžba.