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Approximate systems of hyperspaces

In the present paper we give some partial answers to the following question. **QUESTION.** *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an an approximate inverse system of normal spaces. Under what conditions is the collection* $2^{\mathbf{X}} = \{2^{X_a}, 2^{p_{ab}}, A\}$ *an approximate inverse system, i.e., under what conditions does this collection satisfy condition (A2)?*

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1 Introduction

Let X be a topological space. The closure of A⊆X is denoted by $Cl_X A$ or ClA.

For any topological space the set of all non-empty closed subsets of X is denoted by 2^X . The Vietoris topology on 2^X is the topology with a base

$$\langle U_1, U_2, ..., U_n \rangle = \{F : F \in 2^X, F \subseteq \bigcup_{i=1}^{n} U_i, F \bigcap U_i \neq \emptyset, i = 1, ..., n\}, \quad (1)$$

where U_1, ..., U_n are open subsets of X.

In the sequel we use the notion of *normal cover* [2, p. 379]. An open cover $\mathcal{U}$ of a space X is *normal* if there exists a sequence $\mathcal{U}_1$, $\mathcal{U}_2$, ... , $\mathcal{U}_n$, ... of open covers of X such that $\mathcal{U}_1 = \mathcal{U}$ and $\mathcal{U}_{n+1}$ is a star refinement of $\mathcal{U}_n$ for n=1, 2, ... We denote by Cov(X) the set of all normal coverings of X. A T_1 space X is paracompact iff each open cover of X is normal [2, Theorem 5.1.12.]. A T_1 space X is normal iff each locally finite open cover of X is normal [2, p. 379].

If $\mathcal{U}, \mathcal{V} \in$Cov(X) and $\mathcal{V}$ refines $\mathcal{U}$,we write $\mathcal{V} \prec \mathcal{U}$.If f,g:Y→X are *$\mathcal{U}$-near* mappings,i.e. if for any y∈Y there exists U∈ $\mathcal{U}$ with f(y),g(y)∈U,we write (f,g)≺ $\mathcal{U}$.

If X is a subspace of Y,and if $\mathcal{U}$ is a cover of Y, then by the *trace* of $\mathcal{U}$ on X is meant the cover {U⋂X:U∈ $\mathcal{U}$}. The trace of $\mathcal{U}$ on X is denoted by $\mathcal{U}|X$. If f:X→Y is a continuous mapping and if $\mathcal{U}$ is a normal (respectively,locally finite) cover of Y,then $f^{-1}(\mathcal{U})=\{f^{-1}(U):U\in \mathcal{U}\}$ is a normal (respectively,locally finite) cover of X [1, 1.21. Proposition.]. If X⊆Y and if $\mathcal{U}$ is a normal (respectively,locally finite) cover of Y,then $\mathcal{U}|X$ is a normal (respectively,locally finite) cover of X [1, 1.22. Proposition.].

In this paper we study the approximate inverse system in the sense of S. Mardešić [8].

DEFINITION 1.1 *An approximate inverse system is a collection* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *,where* $(A,\leq)$ *is a directed preordered set,*$X_a, a\in A$,*is a topological space and* $p_{ab}:X_b \to X_a$,
$a\leq b$,*are mappings such that* $p_{aa}=id$ *and the following condition (A2) is satisfied:*

(A2) For each $a\in A$ *and each normal cover* $\mathcal{U}\in Cov(X_a)$ *there is an index* $b\geq a$ *such that* $(p_{ac}p_{cd},p_{ad})\prec \mathcal{U}$,*whenever* $b\leq c\leq d$.

We will call an inverse system in the sense of [2, p. 135.] a *usual inverse system.*

Other basic notions,including approximate mapping, the limit of an approximate system and approximate resolution, are defined as in [8] and [10].

2 P - embedded inverse systems

Let f : X→Y be a continuous mapping onto a normal space Y. We define a mapping $2^f : 2^X \to 2^Y$ by $2^f(K) = Cl_Y(f(K))$, $K\in 2^X$. This mapping is continuous [6, Lemma 1.10]). If $\mathbf{X} = \{X_a, p_{ab}, A\}$ is an approximate inverse system of normal spaces, then we have the collection $2^{\mathbf{X}} = \{2^{X_a}, 2^{p_{ab}}, A\}$. It is natural to ask the following question.

QUESTION. Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system of normal spaces. Under what conditions is the collection $2^{\mathbf{X}} = \{2^{X_a}, 2^{p_{ab}}, A\}$ an approximate inverse system, i.e., under what conditions does this collection satisfy condition (A2)?

In this section we give some partial answer to this question. It is known that the collection $\beta\mathbf{X} = \{\beta X_a, \beta p_{ab}, A\}$ is an approximate inverse system [7, Lemma 2.9] . Thus, the collection $2^{\beta\mathbf{X}} = \{2^{\beta X_a}, 2^{\beta p_{ab}}, A\}$ is an approximate inverse system [14, Lemma (9.4)]. In the sequel we consider approximate inverse systems of normal spaces. If $\mathbf{X}$ is an approximate inverse system of normal spaces, then there exists an embedding $E_a : 2^{X_a} \to 2^{\beta X_a}$ defined by $E_a(K) = Cl_{\beta X_a} K$, $a\in A$, [5, p. 764, Lemma.].

LEMMA 2.1 *The diagram*

$$\begin{array}{ccc} 2^{X_a} & \stackrel{2^{p_{ab}}}{\longleftarrow} & 2^{X_b} \\ \downarrow E_a & & \downarrow E_b \\ 2^{\beta X_a} & \stackrel{2^{\beta p_{ab}}}{\longleftarrow} & 2^{\beta X_b} \end{array} \qquad (D2)$$

$a, b \in A$, commutes.

Proof. Lemma follows from the equation

$$\beta p_{ab}[Cl_{\beta X_b}(F)] = Cl_{\beta X_a} Cl_{X_a} p_{ab}(F), \qquad F \in 2^{X_b} \tag{2}$$

We need to prove that

$$\beta p_{ab}[Cl_{\beta X_b}(F)] \supseteq Cl_{\beta X_a} Cl_{X_a} p_{ab}(F), \qquad F \in 2^{X_b} \tag{3}$$

and

$$\beta p_{ab}[Cl_{\beta X_b}(F)] \subseteq Cl_{\beta X_a} Cl_{X_a} p_{ab}(F), \qquad F \in 2^{X_b} \tag{4}$$

First, let us prove (3). From $F \subseteq Cl_{\beta X_b}(F)$ it follows that $\beta p_{ab}(F) \subseteq \beta p_{ab} Cl_{\beta X_b}(F)$. Thus, $p_{ab}(F) \subseteq \beta p_{ab} Cl_{\beta X_b}(F)$. We infer that $Cl_{\beta X_a} p_{ab}(F) \subseteq \beta p_{ab} Cl_{\beta X_b}(F)$. Hence, $Cl_{X_a} p_{ab}(F) \subseteq \beta p_{ab} Cl_{\beta X_b}(F)$ since $Cl_{X_a} p_{ab}(F) \subseteq Cl_{\beta X_a} p_{ab}(F)$. This means that
$Cl_{\beta X_a} Cl_{X_a} p_{ab}(F) \subseteq \beta p_{ab} Cl_{\beta X_b}(F)$ since $\beta p_{ab} Cl_{\beta X_b}(F)$ is closed.

Let us prove (4). Suppose, on the contrary, that there exists a point $x \in \beta p_{ab}[Cl_{\beta X_b}(F)] \setminus Cl_{\beta X_a} Cl_{X_a} p_{ab}(F)$. This means that there exists a point $y \in Cl_{\beta X_b}(F)$ such that $\beta p_{ab}(y) = x$ and a neighbourhood U (in βX_a) such that $U \cap Cl_{X_a} p_{ab}(F) = \emptyset$. Thus, $V = (\beta p_{ab})^{-1}(U)$ and F are disjoint and V is a neighbourhood of y. Hence, $y \notin Cl_{\beta X_b}(F)$, a contradiction. Thus, (4) is proved. Consequently, Lemma 2.1 is proved. ■

DEFINITION 2.2 Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system of normal spaces. We say that $2^{\mathbf{X}}$ is *P - embedded* in $2^{\beta \mathbf{X}}$ if each E_a, $a \in A$, is a P - embedding, i.e., for each normal cover $\mathcal{U}$ of $E_a(2^{X_a})$ there exists a normal cover $\mathcal{V}$ of $2^{\beta X_a}$ such that the trace $\mathcal{V}|E_a(2^{X_a})$ is a refinement of $\mathcal{U}$.

In the sequel we identify $2^{\mathbf{X}} = \{2^{X_a}, 2^{p_{ab}}, A\}$ with $E(2^{\mathbf{X}}) = \{E_a(2^{X_a}), 2^{\beta p_{ab}} | E_b(2^{X_b}), A\}$. In this case we shall consider $2^{p_{ab}}$ as the restriction $2^{\beta p_{ab}}|E_b(2^{X_b})$ since the diagram 2.1 commutes. Moreover, 2^{X_a} is a subset of $2^{\beta X_a}$.

THEOREM 2.3 *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system of normal spaces such that $2^{\mathbf{X}}$ is P - embedded in $2^{\beta \mathbf{X}}$. Then $2^{\mathbf{X}} = \{2^{X_a}, 2^{p_{ab}}, A\}$ is an approximate inverse system.*

Theorem follows from the following simple lemma.

LEMMA 2.4 *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system. If $\{Y_a : a \in A\}$ is a collection of subsets Y_a of X_a such that $p_{ab}(Y_b) \subseteq Y_a$ and each Y_a is P - embedded in X_a, then $\mathbf{Y} = \{Y_a, p_{ab}|Y_b, A\}$ is an approximate inverse system.*

Proof. Let $\mathcal{U}$ be a normal cover of Y_a. There exists a normal cover $\mathcal{V}$ of X_a such that $\mathcal{V}|Y_a$ refines $\mathcal{U}$. By virtue of (A2) for **X** there exists a b≥a such that $(p_{ad}, p_{ac}p_{cd}) \prec \mathcal{V}$, b≤c≤d. We infer that $(p_{ad}|Y_d, p_{ac}|Y_c\, p_{cd}|Y_d) \prec \mathcal{V}|Y_a \prec \mathcal{U}$, b≤c≤d. It follows that the collection **Y** satisfies (A2), i.e., **Y** is an approximate inverse system. The proof is completed. ■

The main theorem of this section is the following theorem.

THEOREM 2.5 *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an approximate inverse system of normal spaces . Then* $2^{\mathbf{X}}$ *is P - embedded in* $2^{\beta\mathbf{X}}$ *if and only if* $\beta(2^{X_a}) = 2^{\beta X_a}$, *a∈A.*

Proof. If $2^{\mathbf{X}}$ is P - embedded in $2^{\beta\mathbf{X}}$, then 2^{X_a} is P-embedded in $2^{\beta X_a}$, a∈A. Hence, 2^{X_a} is C* - embedded in $2^{\beta X_a}$ and 2^{X_a} is pseudocompact [1, Theorem 15.16]. We infer that $\beta(2^{X_a}) = 2^{\beta X_a}$ since $2^{\beta X_a}$ is a compactification in which 2^{X_a} is C* - embedded [2, 3.6.3. Corollary.].

Conversely, if $\beta(2^{X_a}) = 2^{\beta X_a}$, then X_a and 2^{X_a} are pseudocompact [3, Theorem 2.1 and its proof]. Moreover, 2^{X_a} is C* - embedded in $\beta(2^{X_a}) = 2^{\beta X_a}$. This means that 2^{X_a} is P - embedded in $\beta(2^{X_a}) = 2^{\beta X_a}$ [1, Theorem 15.16]. The proof is complete.■

Let m be an infinite cardinal. We say that a Tychonoff space X is *m - bounded* if each subset S of X with |S| ≤m has a compact closure. Each m - bounded space X is m - compact. The property of being *m - bounded* is productive, closed hereditary and preserved under continuous mapping.

THEOREM 2.6 *Let X be a normal space. Then* 2^X *is m - bounded iff X is m - bounded.*

Proof. The proof is a straightforward modification of the proof of Theorem 5 of [5]. For the sake of completeness we give this proof. If 2^X is m - bounded, then X is also since X, as normal space, is embedded in 2^X as a closed subspace.

Conversely, let us prove that 2^X is m - bounded if X is. Let $E: 2^X \to 2^{\beta X}$ be the embedding defined by $E(K) = Cl_{\beta X}(K)$, $K \in 2^X$. Let $S = \{K_\alpha : \alpha \in A\}$ be a subset of 2^X of the cardinality |A| ≤m. Let $\mathcal{B}$ be the closure of S in $2^{\beta X}$ and let $\mathcal{C}$ be the closure of S in 2^X. We have $E(\mathcal{C}) = \mathcal{B} \cap E(2^X)$. It will be sufficient to prove that $\mathcal{B} \subseteq E(2^X)$, since then $E(\mathcal{C}) = \mathcal{B}$ and then $\mathcal{C}$ will be compact since $\mathcal{B}$ is. Let K* be any point of $\mathcal{B}$ in $2^{\beta X}$, and let $\{Cl_{\beta X}(K_{i_\mu}): i_\mu \in M\}$ be a net converging to K* in $2^{\beta X}$. Let $K = K^* \cap X$. Suppose that $x \in K^* \setminus Cl_{\beta X}(K)$. Then let x∈U with U open in βX with $[Cl_{\beta X}(U)] \cap [Cl_{\beta X}(K)] = \emptyset$. Since $Cl_{\beta X}(K_{i_\mu}) \to K^*$, there exists a $\gamma \in M$ such that, for $\mu \geq \gamma$, $[Cl_{\beta X}(K_{i_\mu})] \cap U \neq \emptyset$. Thus, $K_{i_\mu} \cap U \neq \emptyset$ for $\mu \geq \gamma$. Let $M' = \{\mu : K_\mu \cap U \neq \emptyset\}$ and let $a_\mu \in K_\mu \cap U$, for each $\mu \in M'$.Let $B = Cl_X\{a_\mu : \mu \in M'\}$. Then B is compact since X is m - bounded. Therefore, there exists a subnet of $\{a_{i_\mu}\}$ converging to some a∈B. One can easily show that a∈K* and thus $a \in K^* \cap X = K$. On the other hand, $a \in Cl_{\beta X}(U)$ and $[Cl_{\beta X}(U)] \cap K = \emptyset$. This is impossible. Thus $K^* = Cl_{\beta X}(K)$, i.e., $K^* = E(2^X)$ and $\mathcal{B} \subseteq E(2^X)$. The proof is complete.■

THEOREM 2.7 *Let X be a normal $\aleph_0$ - bounded space. Then $\beta(2^X) = 2^{\beta X}$.*

Proof. By virtue of [3, Theorem 3.8] it follows that $\beta(2^X) = 2^{\beta X}$. The proof is complete.■

THEOREM 2.8 *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system of normal $\aleph_0$ - bounded spaces. Then $2^{\mathbf{X}} = \{2^{X_a}, 2^{p_{ab}}, A\}$ is an approximate inverse system.*

Proof. Apply Theorems 2.7 and 2.5.■

THEOREM 2.9 *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system of Tychonoff $|A|$ - bounded spaces. Then lim$\mathbf{X}$ is a Tychonoff $|A|$ - bounded space. If each X_a is non-empty, then lim$\mathbf{X}$ is non-empty.*

Proof. The non-emptiness of lim**X** follows from Theorem 2.4. [7]. Moreover, lim**X** is $|A|$ - bounded since ΠX_a is $|A|$ - bounded and lim**X** is a closed subset of ΠX_a.■

LEMMA 2.10 *If $\mathbf{p} = \{p_a : a \in A\} : X \to \mathbf{X} = \{X_a, p_{ab}, A\}$ is an approximate resolution such that all spaces X_a are Tychonoff spaces and X is $|A|$ - compact, then $\mathbf{p}$ is a limit of $\mathbf{X}$.*

Proof. The proof is the same as the proof of Theorem 3.1 [10] in all steps except Step (v). In the case considered here, the Cauchy family C(y) has the non - empty intersection, since C(y) has the finite intersection property, has the cardinality $\leq |A|$ and X is $|A|$ - compact. The proof is complete.■

Let us note that the converse of the above theorem is generally false [10, Example 3.2]. If $\mathbf{X} = \{X_a, p_{ab}, A\}$ is an approximate system of pseudocompact spaces, then we have

THEOREM 2.11 *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system of pseudocompact spaces X_a with limit X and natural projections $p_a : X \to X_a$. Then $\mathbf{p} = \{p_a : a \in A\} : X \to \mathbf{X}$ is an approximate resolution iff X is P - embedded in lim$\beta\mathbf{X}$.*

Proof. If **p** is an approximate resolution, then by (B1), for each normal cover $\mathcal{U}$ of X there is an $a \in A$ and a normal cover $\mathcal{U}_b$, $b \geq a$, of X_b such that $p^{-1}(\mathcal{U}_b)$ refines $\mathcal{U}$. Since each X_b is P-embedded in βX_b [1, Theorem 15.16] (as a pseudocompact C*-embedded subspace) there exists a normal cover $\mathcal{V}_b$ of βX_b such that $\mathcal{V}_b | X_b$ refines $\mathcal{U}_b$. Then $\mathcal{V} = P_b^{-1}\mathcal{V}_b$ is a normal cover of

$\lim\beta\mathbf{X}$, where $P_b : \lim\beta\mathbf{X} \to \beta X_b$, $b\in A$, are the natural projections. It is clear that the trace $\mathcal{V}|X$ refines $\mathcal{U}$. Thus, X is P - embedded in $\lim\beta\mathbf{X}$. Conversely, let X be P - embedded in $\lim\beta\mathbf{X}$. Then for each normal cover $\mathcal{U}$ of X there is a normal cover $\mathcal{V}$ of $\lim\beta\mathbf{X}$ such that $\mathcal{V}|X$ refines $\mathcal{U}$. There is an $a\in A$ such that for each $b\geq a$ there exists a normal cover $\mathcal{V}_b$ with $P_b^{-1}(\mathcal{V}_b)\prec \mathcal{V}$ since $\beta\mathbf{X}$ is an approximate resolution [8, Theorem 9.]. It is clear that $P_b^{-1}(\mathcal{V}_b)|X\prec \mathcal{V}$. Thus, **p** satisfies (B1) [8, p. 252]. In order to complete the proof it suffices to prove that **p** satisfies (B2) [8, p. 252]. Let $\mathcal{W}$ be any normal cover of X_a. There exists a normal cover $\mathcal{Z}$ of βX_a such that $\mathcal{Z}|X_a$ refines $\mathcal{W}$ since X_a is P - embedded in βX_a [1, Theorem 15.16]. By virtue of (B2) for $\mathbf{P} = \{P_a : a\in A\} : \lim\beta\mathbf{X} \to \beta\mathbf{X}$ there exists a $b\in A$, $b\geq a$, such that for each $c\geq b$ $\beta p_{ac}(\beta X_c)\subseteq st(P_a(\lim\beta\mathbf{X}), \mathcal{Z})$. It follows that $p_{ac}(X_c)\subseteq st(p_a(\lim\mathbf{X}, W))$. This means that **p** satisfies (B2). The proof is completed.■

In connection with the last theorem, one can ask under what conditions is $\mathbf{P}|X = \{P_a|X\} : X \to \beta\mathbf{X} = \{\beta X_a, \beta p_{ab}, A\}$ an approximate resolution? We have the following theorem.

THEOREM 2.12 *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system of Tychonoff spaces X_a. Then $\mathbf{P}|X = \{P_a|X : a\in A\} = \{p_a : a\in A\} : X \to \beta\mathbf{X} = \{\beta X_a, \beta p_{ab}, A\}$ is an approximate resolution if and only if $X = \lim\mathbf{X}$ is a dense subset of $\lim\beta\mathbf{X}$, P - embedded in $\lim\beta\mathbf{X}$.*

Proof. Theorem follows from Proposition 2.2. of [11]. ■

THEOREM 2.13 *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system such that $2^{\mathbf{X}}$ is P - embedded in $2^{\beta\mathbf{X}}$ and let $P_a : \lim 2^{\mathbf{X}} \to 2^{X_a}$ be a natural projection, $a\in A$. Then $\mathbf{P} = \{P_a : a\in A\} : \lim 2^{\mathbf{X}} \to 2^{\mathbf{X}}$ is an approximate resolution iff $\lim 2^{\mathbf{X}}$ is P - embedded in $\lim 2^{\beta\mathbf{X}}$.*

THEOREM 2.14 *Let $\mathbf{X} = \{X_a, p_{ab}, A\}$ be an approximate inverse system of pseudocompact spaces X_a with limit X and surjective projections $p_a : X \to X_a$. Then $\mathbf{p} = \{p_a : a\in A\} : X \to \mathbf{X}$ is an approximate resolution iff X is pseudocompact and $\lim\beta\mathbf{X} = \beta\lim\mathbf{X}$.*

Proof. If X is pseudocompact and $\lim\beta\mathbf{X} = \beta\lim\mathbf{X}$, then X is P - embedded in $\lim\beta\mathbf{X}$ since X is pseudocompact and C^* - embedded in $\beta X = \beta\lim\mathbf{X}$ [1, Theorem 15.16]. By virtue of Theorem 2.11 we infer that **p** is a resolution. Conversely, if **p** is an approximate resolution, then X is P - embedded in $\lim\beta\mathbf{X}$ (Theorem 2.11). From Theorem 15.16 of [1] it follows that X is pseudocompact and C^* - embedded in $\lim\beta\mathbf{X}$. We infer that $\lim\beta\mathbf{X} = \beta\lim\mathbf{X}$ since $\lim\beta\mathbf{X}$ is a compactification of X in which X is C^* - embedded. The proof is complete.■

THEOREM 2.15 *Let* $\mathbf{X} = \{X_a, p_{ab}, A\}$ *be an approximate inverse system such that* $2^{\mathbf{X}}$ *is* P *- embedded in* $2^{\beta\mathbf{X}}$ *and let* $P_a : lim2^{\mathbf{X}} \rightarrow 2^{X_a}$ *be a natural projection which is surjective for each* $a \in A$. *Then* $\mathbf{P} = \{P_a : a \in A\} : lim2^{\mathbf{X}} \rightarrow 2^{\mathbf{X}}$ *is an approximate resolution iff* $lim2^{\beta\mathbf{X}} = \beta(lim\ 2^{\mathbf{X}})$.

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Lončar I. Hiperprostor aproksimativnog limesa

SAŽETAK

U radu su izučavani aproksimativni inverzni sistemi $\mathbf{X} = \{X_a, p_{ab}, A\}$ kod kojih je svaki 2^{X_a} P - smješten u $2^{\beta X_a}$. Tada kažemo da je $2^{\mathbf{X}}$ P - smješten u $2^{\beta\mathbf{X}}$ (definicija 2.2).

Osnovni teorem 2.5 tvrdi da je $2^{\mathbf{X}} = \{2^{X_a}, 2^{p_{ab}}, A\}$ P - smješten u aproksimativni inverzni sistem $2^{\beta\mathbf{X}} = \{2^{\beta X_a}, 2^{\beta p_{ab}}, A\}$ onda i samo onda kada je $\beta(2^{X_a}) = 2^{\beta X_a}$ za svaki $a \in A$.

Teoremi 2.13 i 2.15 daju nužne i dovoljne uvjete da bi P - smješteni sistem bio aproksimativna rezolventa.