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THE FOURTH DEGREE PARABOLA (BI-QUADRATIC PARABOLA) AS A TRANSITION CURVE

Atınç Pırtı, M. Ali Yücel

Original scientific paper

The transition curves in the modern road and railway construction are route elements equally crucial as alignment and curve (circular). In order to prevent a sudden change of the centrifugal force, the transition curve must be applied due to the impact of the motion in a sharp curve. Over the years the application of the clothoid has become widespread in many countries in the world. However, in this study, in order to eliminate the problems concerning the road dynamics, created by clothoid for vehicles at high speed, the fourth degree parabola (Bi-quadratic parabola) is examined. The possibility of using bi-quadratic parabola in defining transition curves during design of transportation facilities is analyzed in the paper. Basic properties of bi-quadratic parabolas, their fundamental mathematical expression, calculation of point coordinates, driving-dynamic characteristics of bi-quadratic parabolas are described, and the function of change in curvature and lateral impact along the bi-quadratic parabola is presented.

Keywords: *fourth degree parabola, highway and railway route, clothoid, transition curve*

Parabola četvrtog stupnja (bi-kvadratna parabola) kao prijelazna krivulja

Izvorni znanstveni članak

Prijelazne krivulje su u konstrukciji modernih cesta i željezničkih pruga elementi jednako bitni kod poravnanja kao i zakrivljenja (kružnog). Kako bi se spriječila iznenadna promjena centrifugalne sile, prijelazna se krivulja mora primijeniti zbog djelovanja sile gibanja u oštroj krivulji. Tijekom godina proširila se primjena klotoida na mnoge zemlje svijeta. Međutim, u ovom istraživanju, da bi se eliminirali problemi povezani s prometnom dinamikom, stvoreni klotoidom kod vozila pri velikim brzinama, ispituje se parabola četvrtog stupnja (bi-kvadratna parabola). U ovom se radu analizira mogućnost uporabe bi-kvadratnih parabola pri definiranju prijelaznih krivulja tijekom projektiranja prijevoznih sredstava. Opisuju se osnovna svojstva bi-kvadratnih parabola, njihovi osnovni matematički izrazi, računanje koordinata točke te pogonsko-dinamičke karakteristike bi-kvadratnih parabola. Daje se funkcija promjene zakrivljenosti i bočnog udarnog opterećenja duž bi-kvadratne parabole.

Ključne riječi: *parabola četvrtog stupnja, cestovni i željeznički pravac, klotoida, prijelazna krivulja*

1 Introduction

The most important function of a highway is to provide a means to move people and goods from one region to another as fast and safe as possible. Connection to access roads, effects on surrounding communities and nature, and the resultant shape are repeatedly examined in route planning. Route planning begins by searching feasible routes to connect two cities at both ends that pass through important cities. Then, various route possibilities are studied intensively until the best route is chosen. At this preliminary stage, the following issues have to be taken into consideration:

1) Interchange location
2) Land use
3) City planning
4) Topographic and geological features
5) Meteorological conditions
6) Technical capabilities
7) Environmental concerns
8) Construction and maintenance costs.

During the highway design process, the horizontal and vertical alignment, the road width, and other primary dimensions are determined at an early stage in accordance with the maximum design speed and design traffic volume. Designers try to find the optimal alignment considering terrain features and many factors such as geological conditions and land use (Topographic, Geological and Meteorological Conditions; wherever possible, the route should be planned so as not to pass through unfavourable topographic and geologic ground conditions such as steep mountains, wide rivers, and soft soil sites. Areas where the local climate is so unstable that it will adversely affect driving conditions are also avoided. In unavoidable cases, proper technical measures need to be taken). In the geometric design of highways, railways, pipelines, etc., the design and setting out of curves is an important aspect of the engineer's work. The initial design is usually based on a series of straight sections whose positions are defined largely by the topography of the area. The intersections of pairs of straights are then connected by horizontal curves. In the vertical design, intersecting gradients are connected by curves in the vertical plane [5, 8].

1.1 Transition curve

Transition curves were first utilized by railways as early as 1880th to provide easement between tangents and circular curves. Curves used in horizontal planes to connect two straight tangent sections are called horizontal curves. Two types are used: circular arcs and spirals. Easement curves are desirable, especially for railroads and rapid-transit systems, to lessen the sudden change in curvature at the junction of a tangent and a circular curve. A transition curve makes an excellent easement curve because its radius decreases uniformly from infinity at the tangent to that of curve it meets. Transition curves are used to connect a tangent with a circular curve, a tangent with a tangent (double spiral), and a circular curve with a circular curve [8, 1, 2, 3].

Primary functions of transition curves (or easement curves) are [11, 12]:

- To accomplish gradual transition from a straight to a circular curve, so that the curvature changes from zero to a finite value.
- To provide a medium for gradual introduction or change of required superelevation.

- To change the curvature in compound and reverse curve cases, so that a gradual change of curvature is introduced from curve to curve.

To call a spiral between a straight and a curve as a valid transition curve, it has to satisfy the following conditions [11, 12].

- One end of the spiral should be tangential to the straight.
- The other end should be tangential to the curve.
- Spiral's curvature at the intersection point with a circular arc should be equal to the arc curvature.
- Also at the tangent its curvature should be zero.
- The rate of change of curvature along the transition should be the same as that of the increase of the cant.
- Its length should be such that a full cant is attained at the beginning of the circular arc.

Transition curves provide the following advantages [11, 12]:

- A properly designed transition curve allows the vehicle's centripetal acceleration to increase or decrease gradually as the vehicle enters or leaves a circular curve. This transition curve minimizes encroachment on adjoining traffic lanes.
- The transition curve length provides a convenient desirable arrangement for superelevation runoff. The change in the cross fall can be effected along the length of the transition curve in a manner closely fitting the radius-speed relation for the vehicle traversing it.
- A transition facilitates the change in width where the pavement section is to be widened around a circular curve.

In a railway, tramlines, a highway design, it is known that transition curves are placed between components that have different curvatures. The functions of these are to improve the comfort of a travel and reduce the abrasion on the road covering to a minimum due to the vehicles. By means of such curve, the effect of the centrifugal force to the vehicle is distributed along a specific length and also the sudden effect on the spot of the direct entrance of a circle is eliminated. Because of the drawbacks of the clothoid at high speeds, the general properties of the bi-quadratic parabola are investigated in this study [11, 12, 3].

2 Clothoid as a transition curve

2.1 General information

Clothoid (Spiral) curve was first used in the design of the German Autobahn. This name refers to the Greek myth of a Goddess, Clotho; who spun threads. A clothoid curve gradually gets tighter as the line progresses from the starting point, resembling a thread winding to a bobbin. The clothoid curve is placed between a straight line and a circular curve, and between a right-turn and a left-turn curve as a transition curve. This curve is drawn by steering a car with a constant turning angle of the steering at a certain fixed speed. During this steering, the car gradually draws a smaller and smaller spiral-shaped curve. If a clothoid curve with a gradually changing radius is used in an alignment, the driver can at high-speed turn the steering wheel little by little with a constant steering angle and operate the car with ease. That keeps the driver moderately stimulated and concentrated on his/her driving. $A^2 = r \cdot L$ is a parameter of the clothoid curve, which determines the length of the transition curve and the radius of the connecting circular curves. The curve chosen for the purpose is a clothoid, in which the curvature varies inversely to the radius and increases linearly from zero, at the tangent to spiral, to the degree of curvature of the simple curve at the point where the spiral is tangent to the curve [8, 9].

Fig. 1 shows the curvature and the superelevation graphic of a clothoid. The curvature of a clothoid is obtained at any point in Eq. (1):

$$k = \frac{k_E}{L_E} \cdot L, \; k = \frac{1}{r}, \; k_E = \frac{1}{R},$$

$$\frac{1}{r} = \frac{1}{R \cdot L_E} \cdot L, \tag{1}$$

where:
r – the radius value of the transition curve at any point
R – the radius value of circle arc that is being connected by the transition curve
k – the value of the curvature at any point on the transition curve,
k_E – the curvature value of the clothoid's final point
L – transition curve length at any point from the starting point
L_E – the length of transition curve (clothoid).

General equation of the clothoid is obtained by means of Eq. (1).

$$r \cdot L = R \cdot L_E = A^2. \tag{2}$$

The value in Eq. (2) is the parameter of the clothoid and it is constant. Although in the diagrams the changes of the curvature at the clothoid (transition curve), superelevation and the lateral acceleration get smaller, functions are continuous at the start (O) and the final (E) points. Drawbacks related to the security of the roads at high speeds and comfort of the road appear with clothoid. These can be classified as:

- Along the transition curve, rotation movement starts suddenly at the starting point and it ends suddenly at the final point.
- Superelevation diagram forms breaks at the starting and final points (Fig. 1). At these points vertical acceleration shows sudden changes. This situation causes the deterioration of the functional relation between curvature and superelevation. In this case, it is anticipated to round the ramp break points.
- As a result of the straight-line increase of the lateral acceleration, constant size (r_s) rate of change of radial acceleration is obtained along the transition curve. Lateral acceleration's derivative in a unit time is

  $$r_s = \frac{da_s}{dt} \; (\mathrm{m/s^3}),$$

  and as for the constant speed

  $$r_s = \frac{da_s}{dX(dL)} \cdot V \; (\mathrm{m/s^3})$$

is derived from

$$dt = \frac{dL}{V} = \frac{dX}{V}.$$

At the clothoid, the rate of change of radial acceleration appears suddenly at the starting point and again it becomes 0 (zero) suddenly at the final point (Fig. 1). Despite this, the continuity of the rate of change of radial acceleration is required for a comfortable travel [3, 6, 7, 1, 8, 2].

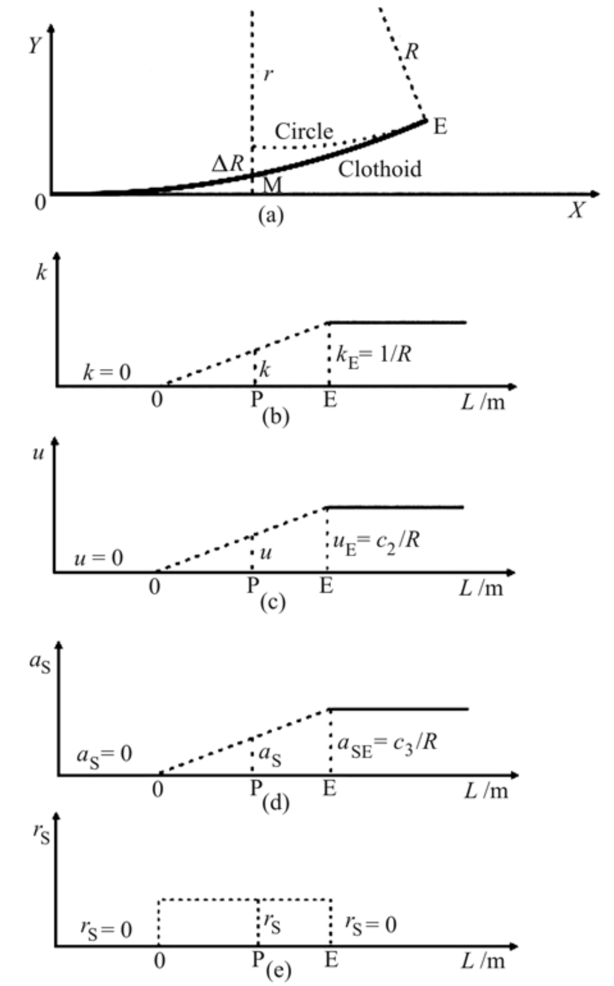

Figure 1 Diagrams of the clothoid: a) situation, b) curvature, c) superelevation, d) lateral acceleration, e) the rate of change of radial acceleration

Considering the drawbacks of the clothoid at high speeds and level of the technique at the present day, the fourth degree parabola takes place among the prior transition curves in road constructions requiring high speed [10, 3].

2.1 The fourth degree parabola (bi-quadratic parabola) as a transition curve

The Fourth Degree Parabola is a transition curve that has some advantages compared to the clothoid. They contain two-second degree parabolas whose radius varies as a function of curve length. The Fourth Degree Parabola has low values of vertical acceleration (Fig. 2), [3, 6, 10].

The curvature of the first part (Parabola I) of the transition curve is obtained

$$k = a \cdot x^2 \tag{3}$$

$$x = L_E.$$

M middle point of the transition curve,

$$x_M = \frac{L_E}{2} \tag{4}$$

and the curvature of the middle point

$$k_M = \frac{1}{2R}. \tag{5}$$

If Eq. (5) is placed in Eq. (3):

$$\frac{1}{2R} = a\frac{L_E^2}{4}, \tag{6}$$

$$a = \frac{2}{RL_E^2}. \tag{7}$$

The curvature equation of Parabola I is obtained:

$$k_I = \frac{2x^2}{RL_E^2}. \tag{8}$$

As for Parabola II, $x = L_E$ and $k = 1/R$ the curvature is defined as

$$k_{II} = \frac{1}{R} - \frac{2(L_E - x)^2}{RL_E^2}, \tag{9}$$

$$L_E - x = x^I. \tag{10}$$

The coordinates and equations of Parabola I

$$\tau = \int_0^L k dx \approx \tan\tau = \frac{dy}{dx}, \tag{11}$$

$$Y = \int_0^L \int_0^L k dx^2 + C. \tag{12}$$

$$Y_I = \int_{x=0}^{L_E/2} \int_{x=0}^{L_E/2} \frac{2x^2}{RL_E^2} dx^2 + C.$$

General equation of the fourth degree parabola can be expressed as

$$Y_I = \frac{x^4}{6RL_E^2}, x \approx L. \tag{13}$$

The coordinate equation of Parabola II can be derived

$$Y_{II} = \int_{x=0}^{L_E/2} \int_{x=0}^{L_E/2} \left(\frac{1}{R} - \frac{2x^2}{RL_E^2}\right) dx^2 + C, \tag{14}$$

$$Y_{\text{II}} = \frac{x^2}{2R} - \frac{x^4}{6RL_{\text{E}}^2} + \text{C}, \tag{15}$$

$$Y_{\text{II}} = Y_{\text{CIRCLE}} - Y_{\text{4.DEG.PARABOLA}}. \tag{16}$$

In Fig. 3, the equation of the two parabolas as the fourth degree appear like that

$$Y_{\text{I}} = Y_{\text{II}} = \frac{x^4}{6RL_{\text{E}}^2}, \; x \approx L. \tag{17}$$

As for Parabola I, X axis is taken as the tangent that is passing through the starting point of the transition curve (O) and the coordinates of Parabola I are calculated. As for Parabola II, by taking the parabola's final point (E) as a start and the backward extension of the circle as X axis, the coordinates of Parabola II are calculated.

For the first part of the fourth degree parabola (Parabola I) necessary Y values can be obtained by applying Eq. (13). For Parabola II, for the P point that is at the length of L_{P} from E point, Y values can be obtained by using the following equations (Fig. 3).

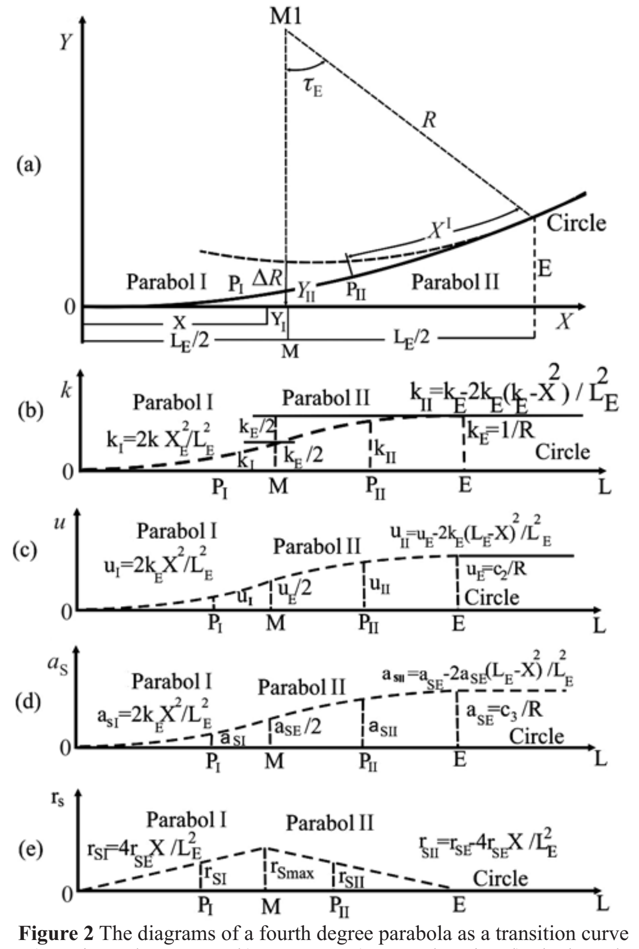

Figure 2 The diagrams of a fourth degree parabola as a transition curve a) Horizontal geometry b) Curvature c) Superelevation d) The lateral acceleration e) The rate of change of radial acceleration [6]

$$\tau_{\text{S}} = \tau_{\text{E}} - \tau_{\text{P}} \quad \text{(Spiral angle of P Point)} \tag{18}$$

$$Y_{\text{P}} = \frac{X^4}{6RL_{\text{E}}^2}, \quad x' = L_{\text{P}} \tag{19}$$

$$Y_{\text{S}} = R - (R + Y_{\text{P}}) \cdot \cos\tau_{\text{S}} \tag{20}$$

$$Y_{\text{P}} = Y_{\text{S}} + \Delta R. \tag{21}$$

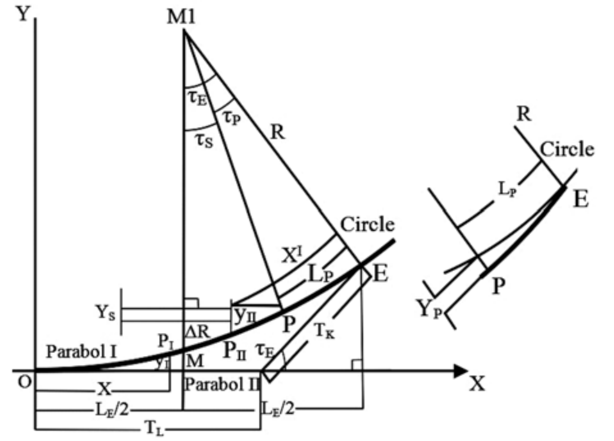

Figure 3 The fourth degree parabola

The shift of the circular curve (ΔR) and the other fundamental components of the transition curve are obtained by means of the following equations (Fig. 3), [3, 7, 10, 6].

$$\Delta R = y_{\text{IM}} + y_{\text{IIM}} \tag{22}$$

$$x = x^{\text{I}} = \frac{L_{\text{E}}}{2}$$

$$y_{\text{IM}} = \frac{L_{\text{E}}^2}{96R}. \tag{23}$$

Table 1 The coordinates obtained for Parabola I (By predicating the tangent at starting point (O))

Parabola I		
Distance, m	X/m	Y/m
0	0	0
50	50,000	0,017
100	99,999	0,267
117	116,998	0,500
118	117,997	0,517
119	118,997	0,535
120	119,997	0,553
121	120,997	0,572
122	121,997	0,591
123	122,997	0,610
124	123,996	0,630
125	124,996	0,651
126	125,996	0,672

Table 2 The coordinates obtained for Parabola II (By predicating the tangent at end point (E))

Parabola II		
Distance, m	X/m	Y/m
0	0	0
50	49,980	1,233
100	99,854	4,730
124	123,741	7,051
125	124,736	7,154
126	125,730	7,258
127	126,725	7,363
128	127,719	7,468
129	128,714	7,574
130	129,708	7,680
131	130,702	7,787
132	131,697	7,894
133	132,691	8,001

The fourth degree parabola shown in Fig. 3, the coordinate values (X, Y) of Parabola I and Parabola II in Tab. 1 and Tab. 2 are calculated by means of the program prepared (L_E = 250 m, R = 1000 m). The horizontal coordinates of Parabola I and Parabola II are calculated via program by using the given value of R radius and L_E (the length of transition curve) with (13) and (15) equations. In the program, the coordinate values are found by dividing the length of the transition curve into equal arc pieces (1 m), (Tab. 1 and Tab. 2). The fourth degree parabola coordinate values are obtained with 50 m intervals in Tab. 3 by using the Helmert coordinate transformation between Parabola I and Parabola II coordinates.

Table 3 The coordinates and the basic component values of the fourth degree parabola ([6] 1987) (R = 1000 m, L_E = 250 m)

Distance, m	The Fourth Degree Parabola	
	X /m	Y /m
0	0,000	0,000
50	50,000	0,017
100	99,999	0,267
150	149,986	1,348
200	199,907	4,096
250	249,651	9,104
τ_E	7,9577 gon	
ΔR	1,302 m	
X_M	124,978 m	
T_K	73,023 m	
T_L	177,200 m	

In this way, in terms of geometric view, the proper transition is obtained from the diagrams of the superelevation and curvature for the alignment and curve components that are connected at the starting and final points of the transition curve (Fig. 2). The advantages below are gained according to the clothoid in terms of the road dynamics: The speed rotation of the wheel axis starts to increase slowly at the starting point of the curve, it reaches its maximum value in the middle of the transition curve and later it again decreases too slowly at the final point. In superelevation diagram, the presence of the broken points prevents the sudden changes at the vertical acceleration. When the lateral acceleration diagram is examined, a systematic and a continuous trend are seen in Fig. 2. Despite these features, at the starting and final points of the transition curve, and also in the middle point of the transition curve with the deterioration of the continuity of the r_s the rate of change of radial acceleration, certain results cannot be obtained about the travel comfort (Fig. 2). As for these explained reasons, the fourth degree parabola is applied at the fast railway lines and road routes by considering the work of the construction [3, 7, 10, 6].

2.3 The fourth degree parabola between two circle arcs at the same way (as an egg curve)

The transition curve as below is applied between a circular curve with R_1 radius and a circular curve with R_2 radius (Fig. 4). The curvature function of the fourth degree parabola between two circle arches at the same way can be expressed in Fig. 4.

The curvature equation of Parabola I is obtained using Eq. (8)

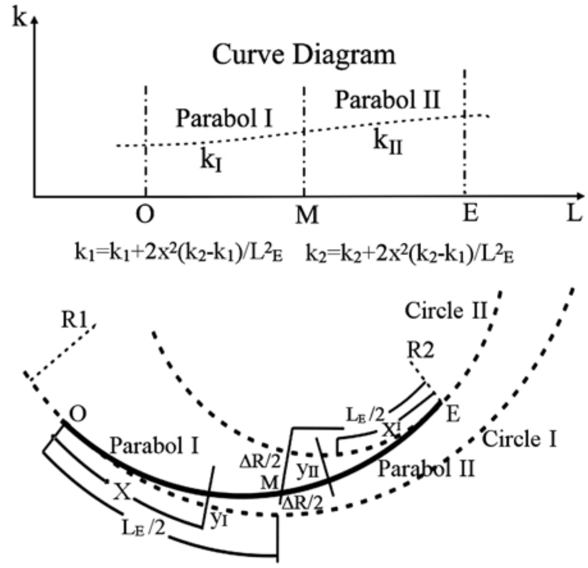

Figure 4 The application of the fourth degree parabola as an egg curve between two circle curves at the same way

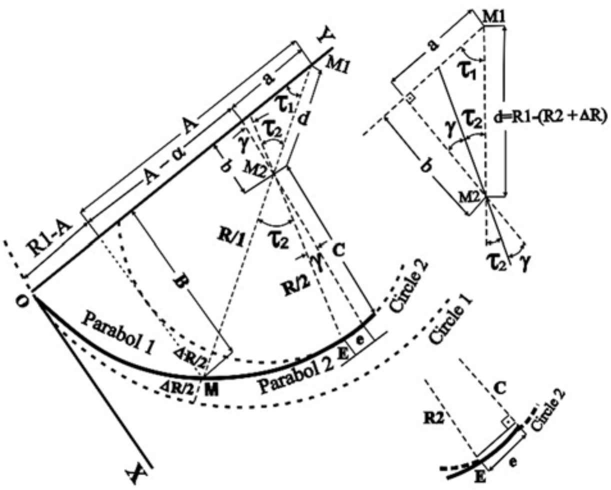

Figure 5 The mathematical calculation diagram of the fourth degree parabola between two circle curves

$$k_I = \frac{1}{R_1} + \frac{2x^2(k_2 - k_1)}{L_E^2}. \tag{24}$$

The curvature equation of Parabola II is obtained like that:

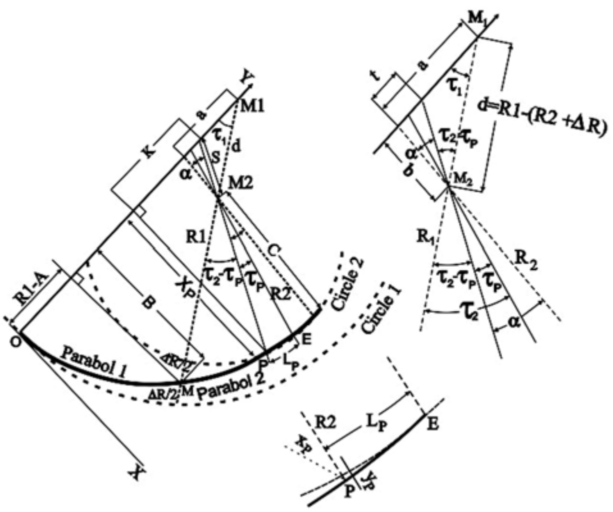

Figure 6 The mathematical calculation of the any point (P) coordinates at the fourth degree parabola used as an egg curve

$$k_{\mathrm{II}} = \frac{1}{R_2} - \frac{2x^2(k_2 - k_1)}{L_{\mathrm{E}}^2}. \tag{25}$$

General equations related to Parabola I and Parabola II are as follows:

$$Y_{\mathrm{I}} = \frac{x^2}{2R_1} + \frac{x^4(R_1 - R_2)}{6L_{\mathrm{E}}^2 \cdot R_1 \cdot R_2} \tag{26}$$

$$Y_{\mathrm{II}} = \frac{x^2}{2R_2} - \frac{x^4(R_1 - R_2)}{6L_{\mathrm{E}}^2 \cdot R_1 \cdot R_2}. \tag{27}$$

The shift of the circular curve according to Eq. (22) can be expressed as

$$\Delta R = \frac{L_{\mathrm{E}}^2(R_1 - R_2)}{48R_1R_2}. \tag{28}$$

While calculating the ordinates according to the circular curve

$$y_{\mathrm{I}} = y_{\mathrm{II}} = \frac{x^4(R_1 - R_2)}{6L_{\mathrm{E}}^2 \cdot R_1 \cdot R_2} \tag{29}$$

can be derived [3, 7, 10, 6].

Table 4 The coordinates obtained for Parabola I (By predicating the tangent at starting point (O))

Parabola I		
Distance, m	X/m	Y/m
0	0	0
50	49,985	1,046
100	99,879	4,241
149	148,579	9,617
150	149,570	9,751
151	150,560	9,887
152	151,551	10,023
153	152,542	10,161

Table 5 The coordinates obtained for Parabola II (By predicating the tangent at end point (E))

Parabola II		
Distance, m	X/m	Y/m
0	0	0
50	49,968	1,557
100	99,747	6,165
147	146,226	13,111
148	147,211	13,285
149	148,195	13,460
150	149,180	13,635
151	150,164	13,812

In Fig. 5, the coordinate values of the fourth degree parabolas, used as an egg curve, are calculated for $R_1 = 1200$ m, $R_2 = 800$ m and $L_{\mathrm{E}} = 300$ m values by means of a program in Tab. 4 and Tab. 5.

The coordinate values of Parabola I and Parabola II are obtained as in Tab. 6 by using Helmet transformation. Apart from this, the basic component values and the coordinates of the fourth degree parabola are given in Tab. 6.

By giving the radiuses of circular curve 1 and circular curve 2 with the length of the transition curve L_{E}, R value is obtained by placing Eq. (28) in the proper place. The

Table 6 The coordinates of the fourth degree parabola used as an egg curve

Distance, m	The Fourth Degree Parabola X/m	Y/m
0	0,000	0,000
50	49,985	1,046
100	99,879	4,241
150	149,570	9,751
200	198,905	17,840
250	247,690	28,760
300	295,716	42,640
τ_{E}	19,8944 gon	
ΔR	0,78125 m	
X_{M}	149,560 m	
M_1M_2	399,219 m	

coordinates of the final point (E) of the transition curve are calculated with the following equations:

$$\tau_1 = \frac{200\,L_1}{\pi R_1} \tag{30}$$

$$\tau_2 = \frac{200\,L_2}{\pi R_2} \tag{31}$$

$$M_1M_2 = d = R_1 - (R_2 + \Delta R) \tag{32}$$

$$\gamma = 100 - (\tau_1 + \tau_2) \tag{33}$$

$$b = d\sin\tau_1,\ c = R_2\cos\gamma \tag{34}$$

$$a = d\cos\tau_1,\ e = R_2\sin\gamma \tag{35}$$

$$A = (R_1 - \frac{\Delta R}{2})\cos\tau_1 \tag{36}$$

$$B = (R_1 - \frac{\Delta R}{2})\sin\tau_1 \tag{37}$$

$$Y_{\mathrm{M}} = R_1 - A \tag{38}$$

$$X_{\mathrm{M}} = B \tag{39}$$

$$Y_{\mathrm{E}} = R_1 - a - e \tag{40}$$

$$X_{\mathrm{E}} = b + c. \tag{41}$$

In Fig. 6, the coordinates of the interval point (P) at L_{P} distance from E point at the egg curve are calculated by the following equations:

$$\tau_{\mathrm{P}} = \frac{200L_{\mathrm{P}}}{\pi R_2} \tag{42}$$

$$\alpha = 100 - (\tau_1 + \tau_2 - \tau_{\mathrm{P}}) \tag{43}$$

$$b = d\sin\tau_1 \tag{44}$$

$$S = \frac{b}{\cos\alpha},\ t = S\cdot\sin\alpha \tag{45}$$

$$y_{\mathrm{P}} = \frac{L_{\mathrm{P}}^4(R_1 - R_2)}{6L_{\mathrm{E}}^2 \cdot R_1 \cdot R_2} \tag{46}$$

$$a = d\cos\tau_1 = (R_1 - (R_2 + \Delta R))\cos\tau_1 \tag{47}$$

$$K = (R_2 + S + y_\mathrm{P})\cos(\tau_1 + \tau_2 - \tau_\mathrm{P}) \tag{48}$$

$$Y_\mathrm{P} = R_1 + t - a - K \tag{49}$$

$$X_\mathrm{P} = (R_2 + S + y_\mathrm{P})\sin(\tau_1 + \tau_2 + \tau_\mathrm{P}). \tag{50}$$

2.4 The fourth degree parabola between two circle arches at the reverse way (as an S curve)

The transition curve, as shown in Fig. 7, is applied between a circular curve with R_1 radius and a circular curve with R_2 radius.

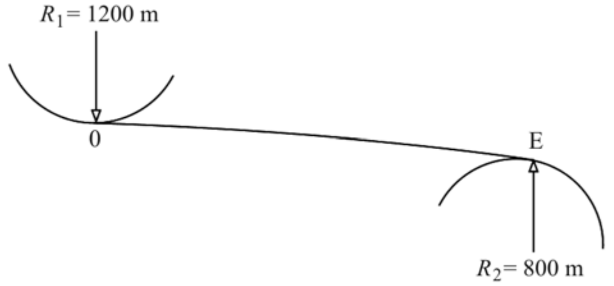

Figure 7 The application of the fourth degree parabola as S curve between two circle curves at the reverse way

The curvature and the general equations of Parabola I and Parabola II are obtained through Equations (24) and (25), and Equations (26) and (27), respectively. The shift of the circular curve according to Eq. (22) is obtained.

$$\Delta R = \frac{L_\mathrm{E}^2(R_1 - R_2)}{48\cdot R_1\cdot R_2}. \tag{51}$$

While calculating the ordinates according to the circular curve, the following equations are used

$$y_\mathrm{I} = y_\mathrm{II} = \frac{x^4(R_1 - R_2)}{6L_\mathrm{E}^2\cdot R_1\cdot R_2}. \tag{52}$$

Table 7 The coordinates obtained for Parabola I (By predicating the tangent at starting point (O))

Parabola I		
Distance / m	X / m	Y / m
0	0	0
50	49,986	1,017
100	99,908	–3,779
148	147,781	–7,271
149	148,778	–7,344
150	149,776	–7,417
151	150,773	–7,490
152	151,770	–7,562

In Tab. 7, the fourth degree parabolas are used as a reverse curve and coordinate values in Tab. 7 and Tab. 8 are calculated for R_1 = –1200 m, R_2 = 800 m and L_E = 300 m values by means of the program. The coordinate values of Parabola I and Parabola II are obtained as in Tab. 9 by using Helmet transformation. Apart from this, the basic component values and the coordinates of the fourth degree parabola are given in Tab. 9.

Table 8 The coordinates obtained for Parabola II (By predicating the tangent at end point (E))

Parabola II		
Distance / m	X / m	Y / m
0	0	0
50	49,969	1,538
100	99,777	5,858
148	147,404	11,871
149	148,395	11,952
150	149,386	12,087
151	150,377	12,222
152	151,367	12,357

Table 9 The coordinates of the fourth degree parabola used as an S curve

Distance, m	The Fourth Degree Parabola X /m	Y /m
0	0,000	0,000
50	49,986	–1,017
100	99,908	–3,779
150	149,776	–7,417
200	199,600	–11,583
250	249,224	–17,658
300	298,430	–26,488
τ_E	3,9789 gon	
ΔR	3,910 m	
X_M	149,852 m	
M_1M_2	2003,910 m	

3 Conclusion

Taking into account the geometric and dynamic criteria in terms of the road design, it is obligatory to use a transition curve. A transition curve is an obligatory component in terms of the security and aesthetics in road design. Similarly, in artificial river and canal constructions, equipment of the route with a transition curve is increasingly gaining importance. The fourth degree parabola that is explained in this article is used in various countries especially in designing the railway lines required for a comfortable and secure service of high speed trains. The problems that appear at high speeds because of the clothoid can be eliminated by using the fourth degree parabola as demonstrated in this study.

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Authors' addresses

Atınç Pirti, Associated Prof. Dr.
Yildiz Technical University
Faculty of Civil Engineering
Department of Surveying Engineering
Davutpaşa/Esenler, İstanbul, Turkey
e-mail: atinc@yildiz.edu.tr

M. Ali Yücel, Assistant Prof. Dr.
Canakkale Onsekiz Mart University
Faculty of Engineering and Architecture
Department of Geomatic Engineering
Canakkale-Turkey
e-mail: aliyucel@comu.edu.tr