Prof. dr. Ivan Lončar
Fakultet organizacije i informatike
V a r a ž d i n

UDK: 51
Znanstveni rad

CONTINUITY OF THE PROPERTIES TrIndX≤α AND TrindX≤α

Let X be a limit of an inverse system $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$. We say that a topological property is continuous is X has when the spaces $X_\alpha, \alpha \in A$, have .

In the present paper we give some sufficient conditions for the continuity of the properties TrIndX≤α and TrindX≤α.

0. INTRODUCTION

0.1. A space X has a strong (weak) [20:161] inducive dimension -1, TrIndX = -1 (TrindX = -1) iff X = ∅; Let α be a transfinite ordinal number. If for every disjoint closed sets F and G (for any neighborhood U of any pont $x \in X$) of X there exists an open set V such that $F \subseteq V \subseteq X-G$ ($x \in V \subseteq U$), $TrIndFr(V) < \alpha$ ($TrindFr(V) < \alpha$), then X has a strong (weak) transfinite inductive dimension $TrIndX \leq \alpha$ ($TrindX \leq \alpha$).

0.2. A space X is (strongly) countable-dimensional [20:161] if $X = \cup\{X_i : i \in N\}$, where X_i, $i \in N$, are finite-dimensional (closed) subspaces of X.

0.3. [8: Theorem 3.]. Let $X = \cup\{X_i : i \in N\}$ be a hereditarily normal compact space. If $TrindX_i$, $i \in N$, are definied, then TrindX is definied.

0.4. [8: Theorem 6.]. If TrIndX is definied for compact space X, then X is weakly infinite-dimensional.

Mathematics subject classifications (1980); Primary 46M40; Secondary 55M10.
Key words and phrases: Continuity, inverse system, transfinite dimension.

0.5. [8: Theorem 1.]. Let X be a compact space. if TrindX is definied, then TrIndX is definied. If X is a hereditarily normal compact space, then TrIndX $\leq$ TrindX. ω_o.

0.6. If for normal (regular) space X TrIndX (TrindX) is definied and WX $< \aleph_\alpha$ (wX $\leq \aleph_\alpha$), then TrIndX $<\omega_{\alpha+1}$(TrindX $\omega_{\alpha+1}$), where WX is a big weight of X [2:495]. If X is a compact (metric) space such that wX$\leq \aleph_\alpha$, then TrIndX$\leq\omega_{\alpha+1}$ (TrIndX $\leq \omega_1$).

0.7. [2:498]. A) Let X be complete metric space. If X is countable-dimensional, then TrindX is definied. The converse holds if X is a complete separable metric space.

B) If X is a strongly metrizable space and TrindX is definied, then X is countable-dimensional.

C) If X is a compact metric space, then the following conditions are equivalent: (1) TrindX is definied, (2) TrIndX is definied, (3) X is countable dimensional.

0.8. [23]. Let $f:X \to Y$ be a closed surjection between metric spaces and $k \in N$. Then: (1) If $f^{-1}(y) \leq k$ for each $y \in Y$ and TrIndX is definied, then TrIndY is definied; (2) If Ind$f^{-1}(y)<k$ for each $y \in Y$ and TrIndY is definied, then TrIndX is definied.

1. TRANSFINITE DIMENSIONS, MAPPINGS AND INVERSE SYSTEMS

We say that a mapping $f:X \to Y$ is **dissipated** [22] of for each $x \in X$ and each open set $U \ni x$ there exists an open set $V \ni f(x)$ such that $f^{-1}(V) = U' \cup U''$, where U' and U'' are disjoint open set in $f^{-1}(V)$ (i.e. in X) and $x \in U' \subseteq U$.

1.1. LEMMA. [22]. If $f:X \to Y$ is a dissipated mapping then so is the mapping $f_A:Y \to Y$ for every $A \subseteq X$.

1.2. LEMMA. [22]. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system with dissipated mappings $f_{\alpha\beta}$. Then a projections f_α: $\lim \underline{X} \to X_\alpha$, $\alpha \in A$, are the dissipated mappings.

1.3. LEMMA. Let $f: X \to Y$ be a dissipated mapping. If $\mathrm{Trind}Y \leq \alpha$, then $\mathrm{Trind}X \leq \alpha$.

Proof. If α is a natural nimber, then see [22]. Suppose that α is a transfinite ordinal number and that Theorem is proved for every $\beta < \alpha$. Let $U \ni x$ be any open set about $x \in X$. There is an open set $V \ni f(x)$ such that $f^{-1}(V) = V' \cup V''$, where V' and V'' are disjoint open set in $f^{-1}(V)$ and $x \in V' \subset U$. Since $\mathrm{Trind}Y \leq \alpha$, we can assume that $\mathrm{Trind}\,\mathrm{Fr}V < \alpha$. This means that $\mathrm{Trind}f^{-1}(\mathrm{Fr}V) < \alpha$ since $f': f^{-1}(\mathrm{Fr}V) \to \mathrm{Fr}V$ is dissipated (1.1. Lemma). By virtue of $\mathrm{Fr}V' \subset f^{-1}(\mathrm{Fr}V)$ we have $\mathrm{Trind}\,\mathrm{Fr}V' < \alpha$ and $\mathrm{Trind}X \leq \alpha$. The proof is completed.

1.4. LEMMA. If X is a perfectly normal Lindelöf space (strongly paracompact strongly hereditarily normal space, σ-totally paracompact, order totally paracompact metrizable space) and $f: X \to Y$ is dissipated, then from $\mathrm{Ind}Y \leq n$ holds $\mathrm{Ind}X \leq n$ for every natural number n.

Proof. From [2:411] and [7:199,205] it follows that $\mathrm{ind}Y = \mathrm{Ind}Y$. Now we have $\mathrm{ind}Y \leq n$. By virtue of Lemma 1.3. it follows that $\mathrm{ind}X \leq n$. This means that $\mathrm{Ind}X \leq n$. Q.E.D.

1.5. LEMMA. Let X be a compact space and $f: X \to Y$ a dissipated mapping. If TrindY is definied, then TrindX is definied. If X is a hereditarily normal compact space, then $\mathrm{TrInd}X \leq \mathrm{TrInd}Y.\omega_o$.

Proof. If TrIndY is definied, then $\mathrm{Trind}Y \leq \mathrm{TrInd}Y$. From 1.3. it follows that TrindX is definied and from 0.5. that TrIndX is definied. Naow we have $\mathrm{TrInd}X \leq \mathrm{Trind}X.\omega_o \leq \mathrm{Trind}Y.\omega_o \leq \mathrm{TrInd}Y.\omega_o$. The proof is completed.

1.6. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system with dissipated mappings $\bar{f}_{\alpha\beta}$. If $\mathrm{Trind}X_\alpha \leq \lambda$, $\alpha \in A$, then $\mathrm{Trind}(\lim \underline{X}) \leq \lambda$.

Proof. Apply Lemmas 1.2. and 1.3.

1.7. COROLLARY. Let $\underline{X}$ be an inverse system as in Theorem 1.6. If X_α, $\alpha \in A$, are (strongly) ind-countable-dimensional, then $X = \lim \underline{X}$ so is.

Proof. If $X = \bigcup\{X_{\alpha,i} : i \in N\}$, then $X = \bigcup\{f_\alpha^{-1}(X_{\alpha,i}) : i \in N\}$. Apply Theorem 1.6.

1.8. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be a well-ordered inverse such that $cf(A) \neq \omega_1$. If a space X_α are perfectly normal Lindelöf spaces with $\mathrm{TrInd}X_\alpha \leq \lambda$ and a mappings $f_{\alpha\beta}$ are dissipated, then $X = \lim \underline{X}$ is a perfectly normal Lindelöf space such that $\mathrm{TrInd}X \leq \lambda$.

Proof. From [15] or [26] it follows that X is hereditarily Lindelöf. This means that X is perfectly normal [6:249]. In order to complete the proof it suffices to apply Theorem 1.6. and Theorem 8. from [2:411].

1.9. LEMMA. A) [22:143]. If $f: X \to Y$ is ind-zero-dimensional (i.e. $\mathrm{ind} f^{-1}(y) = 0, y \in Y$) closed mapping and if X is normal then f is dissipated,

B) [22:143]. If X is compact (locally compact), then each ind-zero-dimensional $f: X \to Y$ is dissipated,

C) Every open mapping $f: X \to Y$, such that $f^{-1}(y)$ $k \in N$ for each $y \in Y$, is dissipated,

D) [22:145]. If $f: X \to Y$ is a dissipated mapping of a compact (a Lindelöf, strongly paracompact) space X onto a metric space Y, then indX=IndX=dimX.

Proof. The statement C) follows from the fact that f is a closed homeomorphism [4:357].

1.10. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of locally compact spaces X_α with $\mathrm{Trind}X_\alpha \leq \lambda$ and perfect ind-zero-dimensional mappings $f_{\alpha\beta}$. Then $\mathrm{Trind}(\lim \underline{X}) \leq \lambda$.

Proof. The projections $f_\alpha : \lim \underline{X} \to X_\alpha$, $\alpha \in A$, are perfect ind-zero-dimensional mappings. This means that $\lim \underline{X}$ is a locally compact space. By virtue of B) (Lemma 1.9.) it follows that f_α are dissipated. Lemma 1.3. completes the proof.

1.11. REMARK. If $f_{\alpha\beta}$ are only ind-zero-dimensional, then $\lim \underline{X}$ is not locally compact but f_α remain a dissipated mappings. This means that is $\mathrm{Trind}(\lim \underline{X}) \leq \lambda$.

1.12. THEOREM. Let $\underline{X} = \{X_n, f_{nm}, N\}$ be an inverse sequence of a perfectly normal (Lindelöf, normal countably compact) spaces X_n and closed ind-zero-dimensional mappings f_{nm}. If $\mathrm{Trind}X_n \leq \lambda$, then $\mathrm{Trind}(\lim \underline{X}) \leq \lambda$.

Proof. The projections $f_n : \lim \underline{X} \to X_n$, $n \in N$, are closed [16] and ind-zero-dimensional. The space $\lim \underline{X}$ is perfectly normal (Lindelöf [15], normal countably compact [16]). From A) Lemma 1.9. it follows that f_n are dissipated mappings. Applying Lemma 1.3. we complete the proof.

For σ-directed inverse systems we have

1.13. THEOREM. Let a metric space X be a limit of a σ-directed inverse system $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ of compact metrizable X_α, $\alpha \in A$. If $\mathrm{Trind}X_\alpha$ or $\mathrm{TrInd}X_\alpha$, $\alpha \in A$, are definied, then TrindX and TrIndX are definied.

Proof. From 0.7.c) it follows that it suffices to prove that X is countable-dimensional. This follows from the fact that X_α, $\alpha \in A$, are countable-dimensional and the next theorem.

1.14. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be a σ-directed inverse system of a compact metric countable-dimensional spaces X_α. If $X = \lim \underline{X}$ is a metrizable space, the X is countable-dimensional.

Proof. Let $\{U_i : i \in N\}$ and $\{F_i : i \in N\}$ be a family of open and closed subsets of X such that $F_i \subseteq U_i$, $i \in N$. For every pair (F_i, U_i) there exists $\alpha_i \in A$ and an open set $U_{\alpha_i} \subset X_{\alpha_i}$ such that $f^{-1}_{\alpha_i\beta}(U_{\alpha_i}) \supseteq f_\beta(F_i)$ for each $\beta \geq \alpha_i$. From the σ-directedness of A it follows that there exists $\beta > \alpha_i$, $i \in N$. Since X_β is countable-dimensional metric space, there exist open sets $W_i \subseteq X_\beta$, $i \in N$, such that $f_\beta(F_i) \subseteq W_i \subseteq f^{-1}_{\alpha_i\beta}(U_{\alpha_i})$ and $\mathrm{ord}\{Fr(W_i) : i \in N\} < \aleph_0$ [20:167 Clearly, $F_i \subseteq f^{-1}_\beta(W_i) \subseteq U_i$ for each $i \in N$. By virtue of [20:167] it follows that X is a countable-dimensional space since ord $\{Fr(f^{-1}(W_i)) : i \in N\}$ is finite. The proof is completed.

1.15. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be a well-ordered inverse system with $cf(A) > \omega_1$. If X_α, $\alpha \in A$, are compact metric spaces and $\mathrm{Trind}X_\alpha$ or $\mathrm{TrInd}X_\alpha$ are definied, then TrindX and TrindX, $X = \lim\underline{X}$, are definied.

Proof. By [26] it follows that $w(X) \leq \aleph_0$. Apply Theorem 1.14.

1.16. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of a compact spaces X_α and dissipated mappings $f_{\alpha\beta}$. If $\mathrm{TrInd}X_\alpha$. are definied, then $\mathrm{TrInd}(\lim\underline{X})$ is definied.

Proof. A projections $f_\alpha : \lim\underline{X} \to X_\alpha$, $\alpha \in A$, are dissipated (Lemma 1.2.). By virtue of Lemma 1.3. it follows that $\mathrm{Trin}(\lim X)$ is definied. From 0.5. it follows that the proof is completed.

In the remaining part of this Section we study the relations between TrindX and TrindY if $f: X \to Y$ is an open mapping.

1.17. LEMMA. If $f: X \to Y$ is a local homeomorphism, then TrindX (TrindX $\leq \lambda$) is definied iff TrindY is definied (TrindY $\leq \lambda$).

For a mapping $f: X \to Y$ we denote by HL_f the set of all points $x \in X$ in which f is a local homeomorphism.

1.18. LEMMA. Let $f: X \to Y$ be a mapping such that HL_f is dense in X. If X is a hereditarily paracompact, then f is a local homeomorphism.

Proof. For every $h \in HL_f$ we have an open set U_h h such that $f/U_h : U_h \to f(U_h)$ is a homeomorphism onto an open set $f(U_h) \subset Y$. By regularity of X it follows that there is an open set V_h, $h \in HL_f$, such that $h \in V_h \subset \bar{V}_h \subset U_h$. Since $\cup\{V_h : h \in HL_f\}$ is paracompact, there is an open locally finite cover $\{W_\alpha : \alpha \in A\}$ which refines $\{V_h : h \in HL_f\}$. Now we have $X = \overline{HL_f} \subset \overline{\cup\{V_h : h \in HL_f\}} \subseteq \overline{\cup\{W_\alpha : \alpha \in A\}} = \cup\{\bar{W}_\alpha : \alpha \in A\} \subset \cup\{U_h : h \in HL_f\}$. This means that $X = \cup\{U_h : h \in HL_f\}$. Q.E.D.

If $f: X \to Y$ is an open mapping such that $|f^{-1}(y)| = k$ for all $y \in Y$ and X is T_2 space, then f is a closed local homeomorphism (Lemma 1.9.C). By 1.17. we have

1.19. LEMMA. Let $f: X \to Y$ be an open surjection between regular spaces such that $|f^{-1}(y)| = k$ for each $y \in Y$. TrindX $\leq \lambda$ iff TrindY $\leq \lambda$.

1.20. THEOREM. Let $\underline{X} = \{X_\alpha, f_{\alpha\beta}, A\}$ be an inverse system of regular spaces X_α and open mappings such that $|f_{\alpha\beta}^{-1}(x_\alpha)| \leq k \in N$. If for each $x_\alpha \in X_\alpha$ there exists $\beta > \alpha$ such that $f_{\alpha\beta}^{-1}(x_\alpha) = k$, then Trind$X_\alpha \leq \lambda$ iff Trind(lim$\underline{X}$) $\leq \lambda$.

Proof. The projections $f_\alpha : \lim \underline{X} \to X_\alpha$, $\alpha \varepsilon A$, are open such that $f_\alpha^{-1}(x_\alpha) = k$. Lemma 1.19. completes the proof.

1.21. THEOREM. Let $f: X \to Y$ be an open surjection such that $|f^{-1}(y)| \leq \aleph_0$ for all $y \varepsilon Y$. If X is locally compact (or Čech-complete) hereditarily paracompact space, then $\mathrm{Trind}X \leq \lambda$ iff $\mathrm{Trind}Y \leq \lambda$.

Proof. From [2: 457, Theorem 4.] it follows that HL_f is dense in X. Lemma 1.18. completes the proof.

1.22. THEOREM. Let $\underline{X} = \{X_n, f_{nm}, N\}$ be an inverse sequence of a complete separable metric spaces X_n and open mappings f_{nm} such that $f_{nm}^{-1}(x_n) \leq \alpha k$. $\mathrm{Trind}(\lim \underline{X}) \leq \alpha$ if $\mathrm{Trind}X_n \leq \alpha$ for every $n \varepsilon N$.

Proof. Apply 1.17. and 1.21.

1.23. REMARK: By virtue of [2:500] it follows that $X = \lim \underline{X}$ in Theorem 1.22. is countable-dimensional iff $X_n, n \varepsilon N$, are countabl[e-] dimensional. Furthemore, on can prove that if $f: X \to Y$ is an ope[n] mapping between locally compact (or compelte) metric separable spaces such that $|f^{-1}(y)| \leq \aleph_0$ for all $y \varepsilon Y$, then X is (strongly countable-dimensional iff Y is (strongly) countable-dimensional.

1.24. LEMMA. Let $f: X \to Y$ be an open surjection between complet[e] separable metric spaces such that $f^{-1}(y), y \varepsilon Y$, has an isolated point. If X has a dimension TrindX then Y has a dimension TrindY.

Proof. Let A_i be the sets from the proof of 1.12.6. Theorem of [7]. Since $\mathrm{Trind}A_i, i \varepsilon N$, are definied and $Y = \bigcup\{f(A_i) : i \varepsilon N\}$, it follows that Y is countable-dimensional. This means that TrindY is definied [2:500].

Similarly, on can prove the following

1.25. THEOREM. Let $f:X \to Y$ be an open onto mapping between complete separable metric spaces such that for each $y \in Y$ a set $f^{-1}(y)$ is a discrete subspace of X. Then TrindX is definied iff TrindY is definied.

As the applications we prove the following theorems.

1.26. THEOREM. Let $\underline{X} = \{X_n, f_{nm}, N\}$ be an inverse sequence of a countable-dimensional metric spaces X_n. If f_{nm} are open surjection such that $f_{nm}^{-1}(x_n)$ is discrete, then $X = \lim \underline{X}$ is a countable-dimensional space.

Proof. Let $X_n = \cup\{X_{n,i} : i \in N\}$, where $X_{n,i}$ is finite-dimensional subspace for each $i \in N$. By virtue of [7:288]we have $\dim f_{nm}^{-1}(X_{n,i}) \leq \dim X_{n,i}$ for each $i \in N$. We obtain an inverse sequence $\underline{Y}_i = \{Y_{mi}, f_{nm}/Y_{m,i}\}$, $Y_{m,i} = f_{nm}^{-1}(X_{n,i})$, which satisfies Theorem 4.1.22. [7]. Thus, $\dim(\lim \underline{Y}_i) \leq \dim X_{n,i}$ for each $i \in N$. The proof is completed since $X = \cup\{\lim \underline{Y}_i : i \in N\}$.

If f_{nm} are open with $f_{nm}^{-1}(x_n) \leq k \in N$, then $f_n : X \to X_n$, $n \in H$, have the property $f_n^{-1}(x_n) \leq k$. Hence, by virtue of [3:9.1. Theorem] we have

1.27. THEOREM. Let $\underline{X} = \{X_n, f_{nm}, N\}$ be a sequence of a metric spaces X_n and open onto mappings f_{nm} such that $f_{nm}^{-1}(x_n) \leq k$. A limit $X = \lim \underline{X}$ is (strongly) countable-dimensional iff the spaces X_n, $n \in N$, are (strongly) countable-dimensional.

From the preceding theorems it follows

1.28. THEOREM. Let $\underline{X} = \{X_n, f_{nm}, N\}$ be a sequence of a complete separable metric spaces X_n and open mappings f_{nm} with discrete fibers $f^{-1}_{nm}(x_n)$. If $TrindX_n, n \in N$, are definied, then $Trind(\lim \underline{X})$ is definied.

Proof. From 0.7. it follows that X_n are countable-dimensional. By theorem 1.26. we infer that $\lim \underline{X}$ is countable-dimensional. Applying 0.7. we complete the proof.

Similarly, from 0.7. and Theorem 1.27. it follows

1.29. THEOREM. Let $\underline{X} = \{X_n, f_{nm}, N\}$ be an inverse sequence of a complete separable metric spaces X_n and open mappings with $f^{-1}_{nm}(x_n) \leq k$. $Trind(\lim \underline{X})$ is definied iff $TrindX_n, n \in N$, are definied.

1.30. REMARK. If in Theorem 1.29. X_n are compact metric spaces, then $TrInd(\lim \underline{X})$ is definied iff $TrIndX_n, n \in N$, are definied. The proof holds from 0.7.C).

We close this Section with the following theorems.

1.31. THEOREM. Let $\underline{X} = \{X_n, f_{nm}, N\}$ be a sequence of a metric spaces X_n with open-and-closed mappings f_{nm} such that $|f^{-1}_{nm}(x_n)| \leq \aleph_0$. If the spaces $X_n, n \in N$, are countable-dimensional, then $X = \lim \underline{X}$ is countable-dimensional.

Proof. In the proof 1.26. apply [7:288(c)] instead [7:288(a)].

1.32. THEOREM. Let $\underline{X} = \{X_n, f_{nm}, N\}$ be an inverse sequence of a complete separable metric spaces X_n with open-and-closed mappings f_{nm} such that $|f^{-1}_{nm}(x_n)| \leq \aleph_0$. Limit $X = \lim \underline{X}$ has a dimension $Trind(\lim \underline{X})$ if the spaces $X_n, n \in N$, have the dimension $TrindX_n$.

Proof. apply 0.7.c) and Theorem 1.31.

REFERENCES

[1] Aleksandrov P.S., O nekotoryh osnovny napravljenijah v obščej topologii, UMN 19(1964), 3-46.

[2] Aleksandrov P.S., Pasynkov B.A., Vvedenije v teoriju razmernosti, Nauka, Moskva, 1973.

[3] Arhangel'skij A.V., Otobraženija otkritye i bliskie k otkritym. Svjazi meždu prostranstvami, Trudy Mosk.mat.obšč.15 (1966), 181-223.

[4] Arhangel'skij A.V., Ponomarev V.I., Osnovy obščej topologii v zadačah i upražnenijah, Nauka, Moskva 1974.

[5] Baladze V.H., O funkcijah razmernostnogo tipa, Trudy Tbilis. mat. inst. 68 (1982), 5-41.

[6] Engelking R., General Topology, PWN, Warszawa 1977.

[7] ...., Dimension Theory, PWN, Warszawa 1978.

[8] Fedorčuk V.V., Beskonečnomernye bikompakti, Izv.AN SSSR. Ser.mat.42 (1978), 1162-1178.

[9] .... Metod razvertivaemyh spektrov i vpol'ne zamknutyh otobraženij, UMN 35 (1980), 112-121.

[10] Gordh G.R., Mardešić S., Characterizing local connectedness in inverse limits, Pacific J.Math.58(1975), 411-417.

[11] Hisao Kato, A note on infinite-dimension under refinable maps, Proc.Amer.Math.Soc.88(1983), 177-180.

[12] Jayne J.E., Rogers C.A., Functions fermeės en partie, C.r.Acad.Sci. 13(1980), 667-670.

[13] Keesling J.E., Open and closed mappings and compactification, Fund. Math. 65 (1969), 73-81.

[14] Koyama Akira, Refinable maps in dimension theory, Topol. and Appl. 17 (1984), 247-255.

[15] Lončar I., Lindelöfov broj i inverzni sistemi, Zbornik radova Fakulteta organizacije i informatike Varaždin 7 (1983), 115-123.

[16] ...., Inverse limits for spaces which generalize compact spaces, Glasnik mat. 17 (37) (1982), 155-173.

[17] Mardešić S., Lokalno povezani, uređeni i lančasti kontinuumi, Radovi JAZU, Zagreb, 1960, 147-166.

[18] Nagami K., Countable paracompactnes of inverse limits and products, Fund.Math. 73 (1972), 261-270.

[19] ...., Mappings of finite order and dimension theory, Japan J.Math. 30 (1950), 25-44.

[20] Nagata J., Modern dimension theory, Amsterdam, 1965.

[21] Pasynkov B.A., Faktorizacionnye teoremy v teoriji razmernosti, UMN 36 (1981), 147-175.

[22] ...., Častičnye topologičeskie proizvedenija, Trudy Mosk. mat. obšč. 13 (1965), 136-245.

[23] Polkovski L.T., Open and closed mappings and infinite dimension, Gen.Topol.and Relat.Mod.Anal.and Algebra. 5, Berlin, 1983, 561-564.

[24] Pol E., Limiting mappings and projections of inverse systems, Fund.Math. 80 (1973), 81-97.

[25] Ščepin E.V., Funktory i nesčetnye stepeni kompaktov, UMN 36 (1981), 3-62.

[26] Tkačenko M.G., Cepy i kardinaly, DAN SSSR 239 (1978), 546-549.

[27] Väisälä J., Local topological properties of countable mappings, Duke Math. J. 41 (1974), 541-546.

[28] Yajima Yukinobu, On the dimension of limits of inverse systems, Proc. Amer. Math. Soc. 91 (1984), 461-466.

Primljeno: 1986-07-08.

Lončar I. Neprekidnost svojstava TrIndX$\le\alpha$ i TrindX$\le\alpha$

S A Ž E T A K

U radu se izučava ponašanje svojstava TrindX ... i TrIndX<... pri prelasku na limes inverznog sistema. Osnovna pažnja posvećena je disipativnim preslikavanjima među prostorima inverznog sistema. Dokazano je da tada inverzni limes ima transfinitne induktivne dimenzije $\le\alpha$ *ako ih imaju prostori inverznog sistema.*