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Cutting Force Analysis in Face Milling Using Rotatable Central Composite Design of Experiments and Taguchi Method

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This paper presents a study of the influence of cutting conditions on the cutting force components during face milling of steel 42CrMo4. Two experimental plans, rotatable central composite design and the Taguchi method with orthogonal arrays and signal-to-noise ratio, have been designed and performed on controlled machining with corresponding cutting conditions. Equations for the cutting force components, as a functions of cutting parameters, have been obtained by means of regression analysis. The Taguchi method has been used to analyse impact of cutting parameters on the cutting force components and to find optimal level of the cutting parameters. The comparison of results obtained by means of the rotatable central composite design and the Taguchi method was performed.

Analiza sila rezanja kod čeonog glodanja uporabom rotacijskog, centralno kompozicijskog plana pokusa i Taguchijeve metode

Izvornoznanstveni članak

U radu je istražen utjecaj parametara obrade na komponente sile rezanja pri čeonom glodanju čelika 42CrMo4. Eksperimenti su provedeni prema odabranim planovima pokusa, a to su rotacijski, centralno kompozicijski plan pokusa i Taguchijeva metoda s ortogonalnim nizovima te omjerom signala i šuma. Regresijskom analizom dobivene su jednadžbe komponenata sile rezanja u zavisnosti od parametara obrade. Analiza utjecaja parametara obrade na komponente sile rezanja te određivanje optimalnih parametara obrade za minimalne komponente sile rezanja provedeni su uporabom Taguchijeve metode. Na kraju su uspoređeni rezultati primjenom rotacijskog, centralno kompozicijskog plana pokusa i Taguchijeve metode.

1. Introduction

Contemporary manufacturing and technological process require implementation of sophisticated mathematical and other methods for the purpose of efficient control. Thus, research is needed to obtain mathematical approximations of machining processes and appearing phenomena as good as possible. Engineers are faced with two main practical problems in manufacturing. The first is to determine the values of the process parameters that will provide the desired product quality and the second is to maximize manufacturing system performance with available resources. Cutting force is one of the important physical variables that comprise relevant process information in machining. Such information can be used to assist in understanding machining features such as machinability, tool wear fracture, machine tool chatter, machining accuracy and surface finish [1].

The subject of this study is to analyse dependence of the cutting force components on three cutting parameters in face milling (cutting speed v_c, feed per tooth f_t and depth of cut a_p). In this work the response surface method (RSM) based on the rotatable central composite design (RCCD) has been used with an analysis of variance (ANOVA) and regression analysis (RA). A comparison of results obtained by means of the Taguchi method with the regression model was carried out.

2. Experimental settings

For the present work, all experiments have been performed at the Tool machine laboratory at the Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, Split. A vertical machining center VC560 Spinner, equipped with a 12000 rpm electrospindle and the SK 40 tool holder, has been used for the milling experiments. The face milling experiments were

Symbols/Oznake

N	- number of experimental points - broj pokusa	n	- number of replication of experiment - broj ponavljanja pokusa
k	- number of input parameters - broj ulaznih parametara	v_c	- cutting speed, m/min - brzina rezanja
b_0, b_i, b_{ii}, b_{ij}	- regression coefficients - regresijski koeficijenti	f_z	- feed per tooth, mm/tooth - posmak po zubu
X_i, X_j	- coded values of input parameters - kodirane vrijednosti ulaznih parametara	a_p	- depth of cut, mm - dubina rezanja
n_0	- design number of the average level - broj pokusa na srednjoj razini		**Greek letters/Grčka slova**
n_α	- design number on the central axes - broj pokusa na centralnoj osi	α	- point on the central axes - točka na centralnoj osi
y_i	- measured value of output variable - izmjerena vrijednost izlazne varijable		

performed by a tool CoroMill 390 with three TiN coated inserts, produced by Sandvik. Test samples were made of steel 42CrMo4, with dimensions 250x110x110 mm. All of the inserts in every experiment were new to eliminate the effect of tool wear. The cutting forces were measured by utilizing dynamometer Kistler type 9271A produced in Winterhur Switzerland. The dynamometers signals were then processed via charge amplifiers and analog/digital converter to a computer. Cutting parameters as the cutting speed, the feed per tooth and the depth of cut were taken within their region of interest of 120 to 140 m/min, 0.10 to 0.20 mm/tooth and 1.0 to 1.5 mm, respectively.

All experiments were carried out without cooling and lubrication agents.

3. Application of rotatable central composite design (RCCD)

Design of experiment (DOE) has been achieved by the rotatable central composite design (RCCD). The RCCD models the response using the empirical second-order polynomial:

$$y = b_0 + \sum_{i=0}^{k} b_i \cdot X_i + \sum_{i=0}^{k} b_{ij} \cdot X_i \cdot X_j + \sum_{i=0}^{k} b_{ii} \cdot X_i^2. \tag{1}$$

In order to determine the required number of experimental points for RCCD the following expression is used:

$$N = 2^k + 2k + n_0 = n_k + n_\alpha + n_{0,} \tag{2}$$

Rotatability provides equal precision of estimation in all directions and the central composite design is made rotatable by adding the points $\alpha = \pm\ 1.682$ to the central axes. The coordinate α was calculated using expression: $\alpha = (2^k)^{1/4}$. The 3-factorial RCCD of experiment requires 8 experiments (3 factors on two levels, 2^3), 6 experiments on the average level and 6 experiments on the central axes, makes a total of 20 experiments.

In order to collect data for RA, software Design-Expert 6.0 has been used to generate experimental points. Considering material of workpiece and tool producer recommendations for cutting parameters, Table 1 has been created for a 3-factor design of experiments. Measurements of component cutting forces have been presented in Table 2.

By applying the regression analysis the coefficients of regression, multi-regression factors, standard false evaluation and the value of t-test have been assessed [2]. After omitting insignificant factors the mathematical models for components of cutting force Fx, Fy, Fz are obtained as follows:

$$Fx = 1526.97 - 15.69 \cdot v_c + 882.34 \cdot f_t - \\ - 654.16 \cdot a_p + 7743.04 \cdot f_t^2 + 162.64 \cdot a_p^2, \tag{3}$$

$$Fy = 796.95 - 9.38 \cdot v_c + 357.74 \cdot f_t - 144.94 \cdot a_p + \\ + 0.042 \cdot v_c^2 + 86.93 \cdot a_p^2 + 660 \cdot f_t \cdot a_p, \tag{4}$$

$$Fz = -63.81 + 379.57 \cdot f_t - 3.25 \cdot v_c \cdot f_t - 0.75 \cdot v_c \cdot a_p. \tag{5}$$

Table 1. Physical and coded values of input factors for second order design of experiments

Tablica 1. Fizičke i kodirane vrijednosti ulaznih faktora za plan pokusa drugog reda

	Input factors/Ulazni faktori					
Cod.val./ Kodirane vrijedn.	Levels/ Razine	-1.682	-1	0	1	1.682
Physical values/ Fizičke vrijednosti	$X_1=v_c$ m/min	113.18	120	130	140	146.82
	$X_2=f_z$ mm/tooth	0.07	0.10	0.15	0.20	0.23
	$X_3=a_p$ mm	0.83	1.00	1.25	1.50	1.67

Table 2. Results of components of cutting forces according to multifactor second order design

Tablica 2. Rezultati mjerenja komponenti sila rezanja prema višefaktorskom planu pokusa

Exp. No.	Input factors/ Ulazni faktori			*F*x, N	*F*y, N	*F*z, N
	X_1	X_2	X_3			
1.	-1	-1	-1	196	135	36
2.	1	-1	-1	157	132	40
3.	-1	1	-1	290	150	48
4.	1	1	-1	235	145	51
5.	-1	-1	1	192	135	36
6.	1	-1	1	198	131	38
7.	-1	1	1	316	192	56
8.	1	1	1	261	168	46
9.	-1.682	0	0	205	165	45
10.	1.682	0	0	185	142	39
11.	0	-1.682	0	160	103	33
12.	0	1.682	0	308	175	54
13.	0	0	-1.682	166	134	40
14.	0	0	1.682	250	180	45
15.	0	0	0	190	140	41
16.	0	0	0	188	142	42
17.	0	0	0	190	141	42
18.	0	0	0	192	139	43
19.	0	0	0	189	141	42
20.	0	0	0	187	140	40

3.1. Analysis of results

The analysis of variance and the regression analysis for *F*x indicate:

- variables which are significant for the mathematical model are: cutting speed v_c, feed per tooth f_t, depth of cut a_p, square of feed per tooth f_t^2 and square of depth of cut a_p^2,
- feed per tooth f_t has the most significant influence,
- coefficient of determination is R^2=0.9468, which means that the model is representative, because it clarifies 94.68 % of deviations, which are the result of the variable's influence.

The analysis of variance and the regression analysis for *F*y indicate:

- variables which are significant for the mathematical model are: cutting speed v_c, feed per tooth f_t, depth of cut a_p, square of cutting speed v_c^2 and square of depth of cut a_p^2 and interaction of feed per tooth and depth of cut $f_t \cdot a_p$,
- feed per tooth f_t and depth of cut a_p have the most significant influence,
- coefficient of determination is R^2=0.9607, which means that the model is representative, because it clarifies 96.07 % of deviations, which are the result of the variable's influence.

The analysis of variance and the regression analysis for *F*z indicate:

- variables which are significant for the mathematical model are: feed per tooth f_t, the interactions of cutting speed and feed per tooth $v_c \cdot f_t$, cutting speed and depth of cut $v_c \cdot a_p$,
- feed per tooth f_t has the most significant influence,
- coefficient of determination is R^2=0.9402, which means that the model is representative, because it clarifies 94.02 % of deviations, which are the result of the variable's influence.

The criterion for deciding about "most significant influence" is the value of Prob>F. The real meaning of Prob>F is in testing of H_0 hypothesis (there is no factor effect) against alternative H_1 hypothesis (there is a factor effect). Namely, this is the probability of getting the F Value if the term did not have an effect on the response. The F Value for a term is the test for comparing the variance associated with that term with the residual variance. It is the mean square for the term divided by the mean square for the residual. In general, a term that has a probability value less than 0.05 would be considered to have a significant effect. A probability value greater than 0.10 is generally regarded as not significant.

In this analysis of variance, the values of Prob>F for feed per tooth (for all cutting force components) and depth of cut (only for *F*y) have the smallest value, less than 0.001, and therefore feed per tooth and dept of cut have the most significant influence on the corresponding responses.

4. Application of Taguchi method

The Taguchi's design of experiments uses orthogonal arrays, the basic characteristics of which is balance, i.e.

both balance of elements of columns and balance between the columns. This means that every factor appears on the same number of levels and that every factor on any level will be in all combinations with other factors [4].

The number of input factors, as in the RCCD, is three, but the levels of input factors have been changed. In the design of experiments based on othogonal array L9 (3^4), three levels have been used. The necessary number of test runs is nine, which represents a big advantage since the number of tests is reduced in relation to RCCD. Robust design and experimental results, together with result transformations into signal-to-noise ratio are given in Table 3.

The core criterion for analysis of experimental data is signal-to-noise ratio, i.e. ratio S/N [5-6]. For the minimal components of cutting forces, the best solution is “smaller is better”, ratio *S/N* is determined:

$$S / N = -10 \cdot \log\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{2}\right). \tag{6}$$

4.1. Analysis of results

Influence of control factor on S/N ratio has been presented in Figures 1, 2 and 3. The response graphics of components of cutting forces have been shown for all three control factors. The smallest components of cutting forces are achieved using the cutting parameters where the S/N ratio is maximal.

Parameter influence on the response could be presented by means of response graphs, which show the change of the S/N ratio in the moment of the level change of control parameter from the first to the third for applied design of experiment L9 (3^4). Influence of terms on the process response is expressed graphically by the slope of the line connecting different levels of parameters. The higher slope means greater parameter influence on the response. The greatest influence of feed per tooth on all components of cutting force can be seen from presented graphs.

Table 3. Ortogonal array L9 (3^4) with experimental results and calculated S/N ratios

Tablica 3. Ortogonalni niz L9 (3^4) s izmjerenim veličinama i izračunatim S/N omjerom

Input factors/Ulazni faktori									
Levels/Razine				-1		0		1	
$X_1=v_c$, m/min				120		130		140	
$X_2=f_t$, mm/tooth				0.10		0.15		0.20	
$X_3=a_p$, mm				1.00		1.25		1.50	
Nr	X_1	X_2	X_3	*F*x, N	*S/N*	*F*y, N	*S/N*	*F*z, N	*S/N*
1.	-1	-1	-1	196	-45,85	135	-42,61	36	-31,13
2.*	-1	0	0	215	-46,65	140	-42,92	44	-32,87
3.	-1	1	1	316	-49,99	192	-45,67	56	-34,96
4.*	0	-1	0	160	-44,08	122	-41,73	38	-31,60
5.*	0	0	1	220	-46,85	156	-43,86	43	-32,67
6.*	0	1	-1	249	-47,92	144	-43,17	48	-33,62
7.	1	-1	1	198	-45,93	131	-42,35	38	-31,60
8.*	1	0	-1	170	-44,61	137	-42,73	43	-32,67
9.*	1	1	0	230	-47,23	159	-44,03	51	-34,15

* - additonally performed experiments

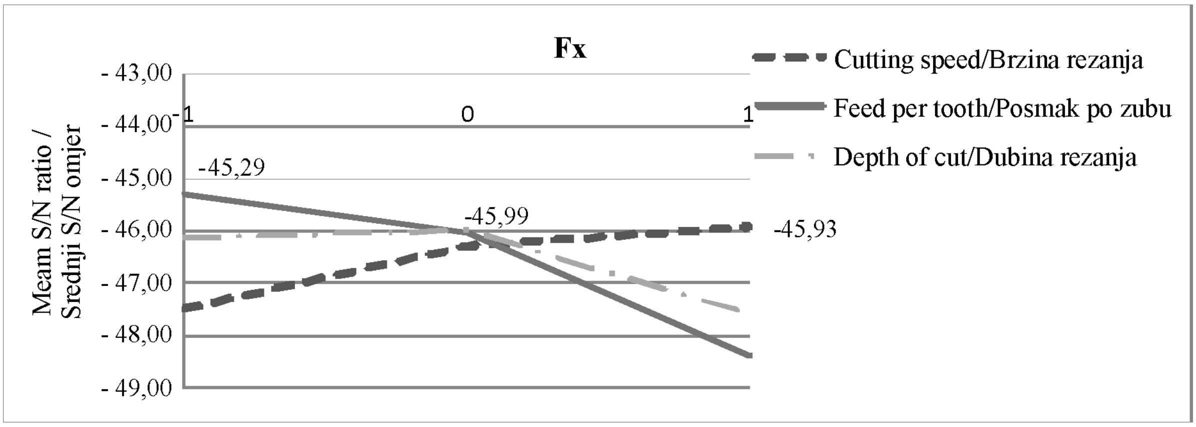

Figure 1. Influence of control factor on S/N ratio for *Fx*
Slika 1. Utjecaj kontrolnih faktora na S/N omjer za *Fx*

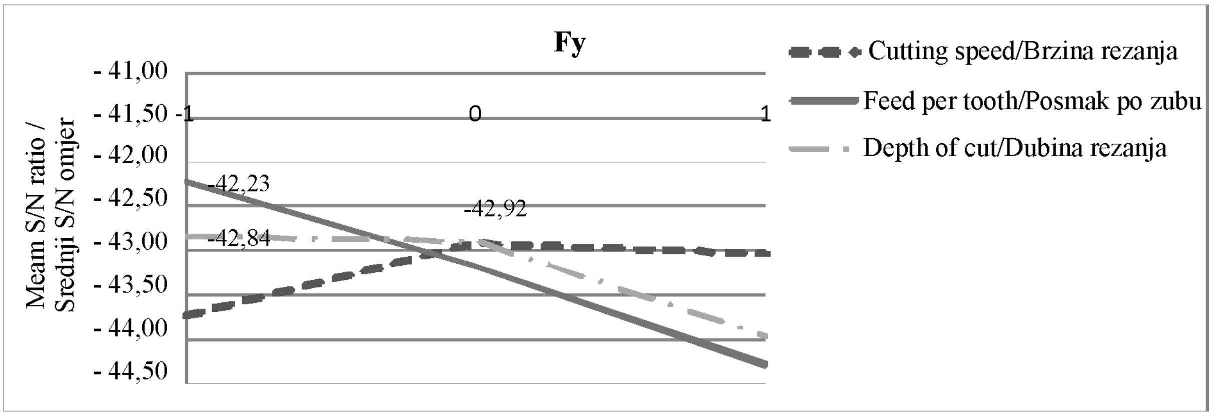

Figure 2. Influence of control factor on S/N ratio for *Fy*
Slika 2. Utjecaj kontrolnih faktora na S/N omjer za *Fy*

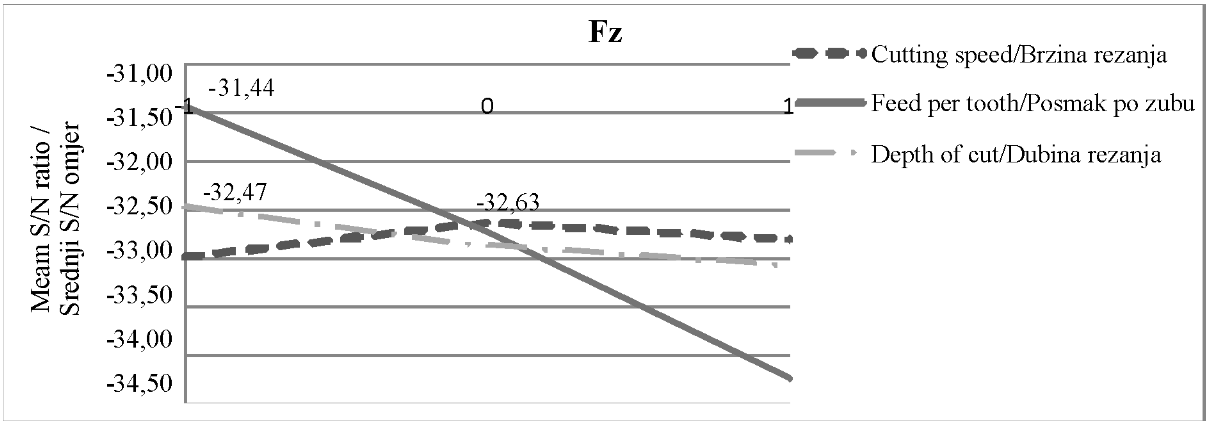

Figure 3. Influence of control factor on S/N ratio for *Fz*
Slika 3. Utjecaj kontrolnih faktora na S/N omjer za *Fz*

Cutting speed and depth of cut has certain influence on components *F*x and *F*y, and minor influence on cutting force component in axial direction, *F*z. The optimization of cutting parameters inside the provided factors levels, with regard to criterion "smaller is better", gives the combination of control factors:

For *F*x: X_1=1, X_2=-1, X_3=0

For *F*y: X_1=0, X_2=-1, X_3=-1

For *F*z: X_1=0, X_2=-1, X_3=-1

Namely, this combination of control factors, which is within the tested range, enables the smallest components of cutting forces.

5. Conclusions

Based on performed experiments and by comparing the test results acquired with RCCD method and the Taguchi method, it can be concluded:

The advantage of the classical experimental design method is the possibility to obtain a mathematical model, which exactly determines a response for certain cutting parameters values within the experimental domain, and it enables a high quality analysis of experiments range as well as achieving optimal exact values. On the other side, there is the Taguchi method by means of which values of optimal parameters are obtained among control factor levels. In addition, Taguchi method is better for parameters with discrete values in contrast to classical optimization technique and continuous values.

The main advantage of the Taguchi method, in relation to classical experimental design method, is the effectivness of robust design itself which can be seen through reliable results and reduced number of test runs. Taguchi method uses a special design of orthogonal arrays to study the entire parameter space with a small number of experiments. Furthermore, to obtain optimal value of process parameters the classical method needs the prediction model to be used for optimization procedure, which is not necessary for orthogonal arrays design. Also, the parameters value needs to be defined strictly numerical in RCCD, unlike the description of state as in the Taguchi method.

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