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COMPLEMENTARITY OF TRAFFIC FLOW INTERSECTING METHOD WITH INTERSECTION CAPACITY ANALYSIS

ABSTRACT

The paper studies the complementarity of the methods from the field of traffic flow theory: methods of traffic flow intersecting intensity and the method for the at-grade intersection capacity analysis. Apart from checking mutual implications of these methods, the proportionality of mutual influences is assessed. Harmonized application of these methods acts efficiently on the entire traffic network, and not only on the intersections that are usually incorrectly represented as the only network operators. Theoretical considerations are illustrated by a practical example.

KEY WORDS

traffic flow theory, theory of organization and direction of traffic flows, intersecting intensity of traffic flows, intersection capacity, optimization

1. INTRODUCTION

The theory of organizing and directing traffic flows analyzes the organization of traffic flows on a network with the aim of their reorganization in order to use uniform load on the traffic network to positively affect the quality and safety of traffic. Its application tends to reduce the intensity of traffic flows intersecting, i.e. achieve a situation of their ideal (minimal) number of intersections. This theory can be applied to the majority of traffic processes. The idea of this theory follows all the postulates of good and safe road traffic operation.

The at-grade intersection capacity method deals with the analysis of traffic organization at an intersection in order to achieve greater throughput and safety of traffic. The at-grade intersection capacity is the dominant method for the analysis and optimization of urban traffic networks. The road network is, namely, usually described by the application of the theory of graphs, with intersections (graph nodes) representing places that determine the network parameters (capacity, travel speed/time, etc.) The roads between the intersections (graph links) play a secondary role, with usually predetermined static values of particular indicators, which change only with the change in intersection (graph node) parameters. The traffic engineering practice has also assigned a dominant role to the intersections, neglecting the integrity of the traffic network, i.e. forcing the intersections to assume the role of the only variable in the traffic models.

The task of this paper is to study the mutual implications and to try to establish a relation between these two methods, in order to make an integral consideration of the traffic process in the entire traffic network structure.

2. TRAFFIC FLOW INTERSECTING INTENSITY

Three methods are mentioned for the measuring of the intersecting intensity of the traffic flows ([3], p. 145):

1. method of minimal flow at the point of conflict:

$$I_{UI} = \sum_{i=1}^{n_I} \min(p_i, q_i) \tag{1.1}$$

2. method of the sum of traffic flows at the point of conflict:

$$I_{UI} = p + q, \forall p, q > 0 \tag{1.2}$$

3. method of the root of the conflicting area:

$$I_{UI} = \sum_{i=1}^{n_I} \sqrt{p_i \cdot q_i} \tag{1.3}$$

Experts usually use the third method as the best indicator.

An example of unnecessary intersecting is usually presented as in Figure 1. Unnecessary intersecting of two heavy traffic flows U and W occurs in two points (Figure 1a). Figure 1b shows how reorganization of the traffic flows can result in avoiding their unnecessary intersecting.

Considering the situation in Figure 1a through the theory of traffic flows and, without reducing the generalization, assuming two-way traffic, the traffic network situation is obtained as presented in Figure 2.

Traffic flows q_1 and q_2 from Figure 1a consist of the following flows:

$$U = \{q_{R3,4}, q_{R4,4}, q_{R1,3}\}$$
$$W = \{q_{R1,2}, q_{R2,2}, q_{R3,1}\} \tag{2.1}$$

The intersecting intensity of flows U and W according to (1.3) for the case in Figure 1a is:

$$I_{UI} = \sqrt{q_{R1,2} \cdot q_{R1,3}} + \sqrt{q_{R3,1} \cdot q_{R3,4}} \tag{2.2}$$

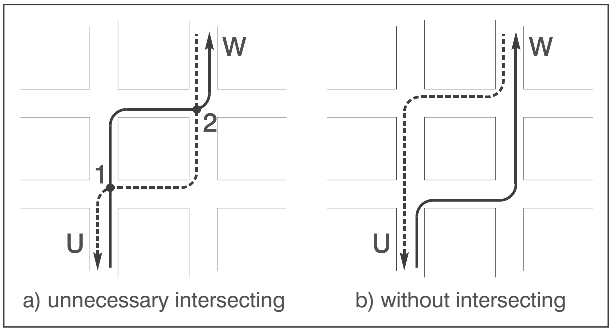

Figure 1 – Solution of unnecessary intersecting

Source: [3], p. 146

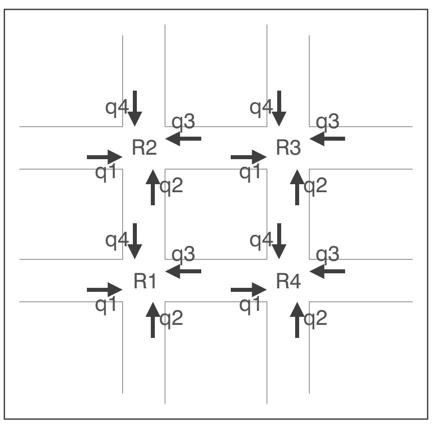

Figure 2 - Traffic network

3. AT-GRADE INTERSECTION CAPACITY

The traffic flow theory from the capacity aspect divides the at-grade intersections into two basic groups: (1) signalized intersections and (2) non-signalized intersections, so that the capacity indicators have to be found for both groups of intersections.

3.1 Signalized intersection capacity

Operative analysis of a signalized intersection regarding capacity is done in accordance with the Highway Capacity Manual (HCM) method. The basic condition regarding capacity at signalized intersections is:

$$\lambda_i = \frac{g_i}{C} \geq \frac{q_i}{S_i} = y_i \tag{3}$$

where:

g_i – effective green for *i-th* group of lanes [s],

C – cycle length [s],

q_i – volume [veh/h, veh/s],

S_i – saturation flow [veh/h, veh/s];

The analysis with the congested flow value as an invariant value usually leads to error in the range of $\pm(4$ to $6)\%$. In urban networks that are mostly close to the at-capacity status, this error affects substantially the final solution. Since the majority of the theories neglect this influence, in this paper the congested flow will also be considered as a constant value.

Whereas the capacity is a quantitative indicator of the status, the degree of saturation is a qualitative (relative) indicator, which also represents an evidence of lack of uniformity (3):

$$x_i = \frac{q_i}{Q_i} = \frac{q_i}{\frac{g_i}{C} \cdot S_i} = \frac{y_i}{\lambda_i} \tag{4}$$

Average delay is taken as a measure of quality (level of service). In practical solutions of urban networks in case of short cycles under the "*at capacity*" or better conditions, the level of saturation and the average delay represent complementary values, although generally they are not correlated.

The analysis of the saturation degrees of all the traffic flows at signalized intersections is not necessary. The critical degree of saturation of a signalized intersection X_C represents the value which describes the capacity of the entire intersection and is defined by:

$$X_C = \sum_{i=1}^{n} y_i \cdot \left(\frac{C}{C-L}\right) = Y \cdot \frac{C}{C-L} \tag{5}$$

where:

Y – sum of the critical flow ratios,

L – total lost time in the cycle [s].

Each phase is characterized by the critical flow ratio (traffic flow with the highest value of y_i) and if this flow, according to (3), receives sufficient green phase, all other flows in this phase will have sufficient capacity. A set of critical movements (I_C) is a set of movements with the maximum flow ratio. The ratio of the critical degree of saturation of a signalized intersection with the critical flow ratios is defined by the following relation (for proof see [10]):

$$X_C \leq 1 \Leftrightarrow (\forall y_i)(\exists \lambda_i,):(y_i \leq \lambda_i), \forall i \in I_C \tag{6}$$

The critical degree of saturation of a signalized intersection is smaller than one if and only if the signal timing plan is such that no critical flow is saturated. Expression (6) shows that maintaining of value $X_C \leq 1.00$

ensures capacity for all the traffic flows at a signalized intersection, i.e. that minimization of value X_C ensures the largest reserve of the intersection capacity.

3.2 Capacity of the non-signalized intersection

The method of determining the capacity of a non--signalized intersection is based on determining the relations of the priority and secondary flows – *ranks of movement.* Right-of-way movements are ranked as 1, and all the rest are of lower ranks depending on the right of way (Figure 3). Models that have been scientifically based and confirmed in practice are based on the time intervals which describe the process at the intersection. The equation for the calculation of potential capacity according to model *HBS* and *HCM 1994* for non-signalized intersection is:

$$Q_{p,i} = \frac{3600}{t_{f,i}} \cdot e^{-\frac{q_{c,i}}{3600} \cdot \left(t_{g,i} - \frac{t_{f,i}}{2}\right)} \tag{7}$$

where:

$Q_{p,\ i}$ – potential capacity of a vehicle in rank 2 or lower in the i-th group of lanes [veh/h],

$q_{c,\ i}$ – conflicting traffic flow for movement i [veh/h],

$t_{g,\ i}$ – critical gap; average interval in the main flow that allows entry into the intersection for i-th flow [s],

$t_{f,\ i}$ – average follow-up time for minor movement in i-th flow [s].

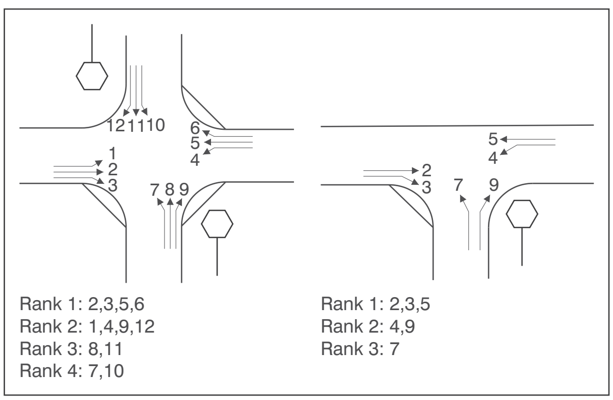

Figure 3 – Traffic flows at non-signalized intersection

HCM 2000 changed the model of potential capacity in relation to HCM 1994 and HBS. In small conflicting flows there is no difference, and in large conflicting flows the difference is about 6%.

In order to calculate the practical capacity from the potential capacity one has to see whether there are queues in priority movements that would reduce the potential capacity of the secondary flow. For rank 2 movements there is equality between the potential and practical capacity (Qm). For all other movements in rank 3 and lower, the occurrence of the queue in conflicting flows of higher rank reduces the capacity. The probability that there is a queue in a particular flow is:

$$p_{0,i} = 1 - \frac{q_i}{Q_{m,i}} \tag{8.1}$$

where:

$p_{0,\ i}$ – probability of queue in the considered flow,

q_i – volume,

$Q_{m,\ i}$ – practical capacity.

The calculated probabilities are used to determine the factor of impedance f_i which affects the potential capacity:

$$f_i = \prod_j p_{o,j} = \prod_j \left(1 - \frac{q_j}{Q_{m,j}}\right) \tag{8.2}$$

so that the practical capacity for the third rank movements is calculated as follows:

$$Q_{m,i} = f_i \cdot Q_{p,i}, \quad i \in \{7,8,10,11\} \tag{9}$$

This is a very simplified presentation of the method of calculating the capacity at a non-signalized intersection. It is illustrative enough for the needs of this paper since it shows the main factors of the capacity of the non-signalized intersection:

- volume and structure of the conflicting traffic flow,
- traffic flow structure which affects the intervals.

The capacity of the non-signalized intersection consists of the sum of the traffic flows in the main flow (rank 1) and capacity of other movements in rank 2 and lower.

For the method of calculating the capacity of the roundabouts formally equation (7) can be taken where potential capacity of the approaches is equal to the practical capacity. The capacity of the roundabout is equal to the sum of approach capacities.

4. OPTIMIZATION FROM THE ASPECT OF TRAFFIC FLOW CHANGES

The traffic flows are organized optimally when the number of their intersecting is minimal in relation to the ideal model of traffic flows in the network, i.e. when intersections, diverging and merging of flows are reduced to the actual minimum ([3], p. 147). Traffic network *TN* which contains *r* intersections is optimally solved ($TN_{opt}(I_{UI})$) if the traffic flows are organized through r_{opt} intersections (whose number can be equal, larger or smaller in relation to the existing number) so that the total intensity of intersecting is minimal:

$$\begin{gathered} TN_{opt}(I_{UI}) = \min \sum_{i=1}^{r_{opt}} I_{UI}(R_i),\ R_i \in TN_{opt} \\ \sum_{i=1}^{r_{opt}} I_{UI}(R_i) \leq \sum_{k=1}^{r} I_{UI}(R_k),\ R_i \in TN_{opt},\ R_k \in TN \end{gathered} \tag{10}$$

From the aspect of capacity, the optimal solution of the traffic network is in achieving maximal capacity of the intersection:

$$TN_{opt}(Q) = \max \sum_{j=1}^{r_{opt}} Q(R_j),\ R_j \in R_{opt}$$
$$\sum_{j=1}^{r_{opt}} Q(R_j) \geq \sum_{k=1}^{r} Q(R_k),\ R_j \in TN_{opt}, R_k \in TN \tag{11}$$

4.1 Optimization of traffic flow intersecting intensity

Apart from the trivial solution (the existing network is optimally solved), minimizing of the intensity of traffic flow intersections changes the structure of the traffic network and the very traffic process: volumes of some traffic flows are changed, some flows are eliminated and/or new are created:

$$\Delta_{ij} = \begin{cases} > 0 & - \text{flow at intersection } R_i \text{ for } j\text{-th phase or movement is increased} \\ 0 & - \text{traffic flow does not change} \\ \langle -q_{ij}, 0 \rangle & - \text{flow at intersection } R_i \text{ for } j\text{-th phase or movement is reduced} \\ -q_{ij} & - j\text{-th flow (phase / movement) at intersection } R_i \text{ is eliminated}, \\ +q_{ij} & - j\text{-th flow (phase / movement) at intersection } R_i \text{ is added}, \end{cases} \tag{12}$$

Applying (1.3), the general solution has the following form:

$$\min \sum_{i=1}^{r} \sqrt[n_i]{\prod_{j=1} q_{ij}} = \sum_{i=1}^{r_{opt}} \sqrt[n_{i_{opt}}]{\prod_{j=1} (q_{ij} + \Delta_{ij})} \tag{13}$$

and the non-equation is valid:

$$\sum_{i=1}^{r_{opt}} \sqrt[n_{i_{opt}}]{\prod_{j=1} (q_{ij} + \Delta_{ij})} \leq \sum_{i=1}^{r} \sqrt[n_i]{\prod_{j=1} q_{ij}} \tag{14}$$

which yields:

$$\sum_{i=1}^{r_{opt}} \sum_{j=1}^{n_{i_{opt}}} \Delta_{ij} \leq 0 \tag{15}$$

i.e. such plan of intersection in the network is found and the distribution of traffic flows that, as a total, have minimal intensity of intersections.

Optimizing (1.1) and (1.2) the general solution has the form:

$$\min \sum_{i=1}^{r} \min\{q_{ij}; j = 1, \ldots, n_i\} = $$
$$= \sum_{i=1}^{r_{opt}} \{q_{ij} + \Delta_{ij}; j = 1, \ldots, n_i\} \tag{16.1}$$

$$\min \sum_{i=1}^{r} \sum_{j=1}^{n_i} q_{ij} = \sum_{i=1}^{r_{opt}} \sum_{j=1}^{n_i} (q_{ij} + \Delta_{ij}) \tag{16.2}$$

In both cases (15) is valid.

4.2 Optimization of signalized intersection capacity

For signalized intersections equation (5) represents a function of two variables:
- degree of critical load of the intersection: *Y*,
- signal timing plan described by values: *C, L*.

Regarding the reduction of intersecting intensity, the model of traffic flow change is interesting, where the model of optimizing the i-th signalized intersection has the following form:

$$\min X_{C_i}(Y) = \sum_{j=1}^{n_i} \left(\frac{q_{ij} + \Delta_{ij}}{S_{ij}} \right) \cdot \frac{C_i}{C_i - L_i},\ j \in I_{C_i} \tag{17}$$

under condition (15).

4.3 Optimization of non-signalized intersection capacity

Equation (7) for the calculation of potential capacity represents a function of three variables:
- conflicting traffic flow: $q_{c,j}$,
- intervals in the main flow: $t_{g,j}$,
- interval of the secondary flow vehicle entry: $t_{f,j}$.

From the aspect of traffic flow change it is interesting to consider the changes of the conflicting traffic flow volume, whereas the intervals retain their constant values. The error in calculating the capacity, if the intervals are considered as invariant values, can be as much as ±11%. Since the theory of organization and direction of traffic flows indirectly positively affects the structure of flow, the positive influence of change of interval will be neglected. The optimization of the potential capacity from the aspect of change in traffic flows is represented by the following model:

$$\max \sum_{j=1}^{n_i} Q_{(p,j),i} = \sum_{j=1}^{n_{i_{opt}}} \left[\frac{3600}{t_{f,j}} \cdot e^{-\frac{(q_{(c,j),i} + \Delta_{ij})}{3600} \cdot \left(t_{g,j} - \frac{t_{f,j}}{2} \right)} \right] \tag{18}$$

under condition (15).

5. IMPLICATION OF THE METHODS

Since the theory of organization and direction of traffic flows best shows its efficiency in the existing traffic network, the mutual influence of the methods in conditions when they do not change will be analyzed:

- intersections and their construction and geometrical characteristics – the same traffic network structure is retained,
- cycle lengths at signalized intersections,
- priorities of movements at non-signalized intersections.

The change in network topology would require a dominant role of other scientific and expert disciplines (mainly civil engineering and architectonic). Since this paper considers the problem from the aspect of traffic and transport science, the proposed limitations are logical and justified.

5.1 Influence of capacity on the intersecting intensity

Obviously, the following implications **do not hold** for the traffic network *TN* which has *r* intersections, out of which r_s are signalized and r_u non-signalized:

$$\min \sum_{i=1}^{r_s} X_{C_i} \Rightarrow \min \sum_{i=1}^{r_s} \sqrt[n_i]{\prod_{j=1} q_{ij}} \tag{19.1}$$

$$\max \sum_{i=1}^{r_u} Q_{(p,j),i} \Rightarrow \min \sum_{i=1}^{r_u} \sqrt[n_i]{\prod_{j=1} q_{ij}} \tag{19.2}$$

The proofs are trivial. In the group of signalized intersections the optimization of the signal change plan is performed. In this way the capacity is improved (minimizing the critical degree of saturation of a signalized intersection X_C) and there is no influence on the traffic flows. This intervention may even increase the intersecting since improved traffic conditions can attract new vehicles. At non-signalized intersections it is possible to change the right of way in order to adapt to the actual traffic demand, thus increasing the potential and practical capacity of intersections without making any formal changes in the physical organization of the traffic flows. The intersection reconstruction will reduce the intensity of intersecting of the traffic flows, but this does not mean achieving the minimum at the level of the network.

This proves the following statement:

Statement 1. *The increase in the capacity at the at-grade intersection group does not affect the intensity of traffic flows intersecting.*

5.2 Influence of the change in intersecting intensity on the capacity

Using the optimization model (13) it should be checked whether and how this change affects the capacity of the intersection. Traffic network *TN* has a total of *r* intersections out of which r_s are signalized and r_u are non-signalized. Not changing the structure of the network the following solution is obtained:

$$\min I_{UI}(TN) = \sum_{i=1}^{r_s} \sqrt[n_i]{\prod_{j=1} (q_{ij} + \Delta_{ij})} + \sum_{i=1}^{r_u} \sqrt[n_i]{\prod_{j=1} (q_{ij} + \Delta_{ij})} \tag{20}$$

and here (15) holds.

It is analyzed what influence does (20) have on the intersection capacity.

5.2.1 Signalized intersections

Not reducing the general characteristics of model (17) only those flows are considered that represent critical movements. A set of critical movements (I_C) and a set of critical flow ratios (Y) at *i-th* intersection are defined as:

$$I_{C_i} = \left\{ q_{ij} : \frac{q_{ij}}{S_{ij}} = y_{ij} \right\} \tag{21.1}$$

$$\frac{q_{it}}{S_{it}} \leq \frac{q_{ij}}{S_{ij}}, \quad \forall q_{it} \notin I_{C_i} \tag{21.2}$$

$$Y_i = \{ y_{i1}, y_{i2}, \ldots, y_{in_i} \} \tag{22}$$

The optimization of the traffic flow intersecting intensity will reduce the sum of critical flow ratios. Two solutions are possible:

a) the set of critical movements is not changed $I_{C_i} = I_{C_i}(I_{UI})$ and there are r_{s1} *such intersections*;

$$\left(\frac{q_{ij} + \Delta_{ij}}{S_{ij}} \geq \frac{q_{ik}}{S_{ik}}, \forall q_{ij} \in I_{C_I}, \forall q_{ik} \notin I_{C_I} \right) \Rightarrow$$
$$\Rightarrow Y_i = Y_i(I_{UI}), \quad i = 1, \ldots, r_{s1} \tag{23.1}$$

b) the set of critical movements at r_{s2} intersections has changed. For a critical movement *l*, y_l has changed so that there is movement *p* for which: $y_p > y_l$. There are r_{s2} such intersections:

$$\left(\exists q_{il} : \frac{q_{il} + \Delta_{il}}{S_{il}} < \frac{q_{ip} + \Delta_{ip}}{S_{ip}}, q_{il} \in I_{C_i}, q_{ip} \notin I_{C_i} \right) \Rightarrow$$
$$\Rightarrow Y_i(I_{UI}) = Y_i - \{y_{il}\} + \{y_{ip}\}, \quad i = 1, \ldots, r_{s2} \tag{23.2}$$

In the former case the value of the sum of critical degrees of saturation is:

$$\sum_{i=1}^{r_{s1}} Y_i(I_{UI}) = \sum_{i=1}^{r_{s1}} \sum_{j=1}^{n_i} \frac{q_{ij} + \Delta_{ij}}{S_{ij}} = \sum_{i=1}^{r_{s1}} \left(Y_i + \sum_{j=1}^{n_i} \frac{\Delta_{ij}}{S_{ij}} \right) \tag{24.1}$$

In the latter case the sum of values Y has the value:

$$\sum_{i=1}^{r_{s2}} Y_i(I_{UI}) = \sum_{i=1}^{r_{s2}} \left(\sum_{j=1}^{n_i - l_i} \frac{q_{ij} + \Delta_{ij}}{S_{ij}} + \sum_{p=1}^{p_i} \frac{q_{ip} + \Delta_{ip}}{S_{ip}} \right) =$$
$$= \sum_{i=1}^{r_{s2}} \left(\sum_{j=1}^{n_i - l_i} y_{ij} + \sum_{p=1}^{p_i} y_{ip} \right) + \sum_{i=1}^{r_{s2}} \left(\sum_{j=1}^{n_i - l_i} \frac{\Delta_{ij}}{S_{ij}} + \sum_{p=1}^{p_i} \frac{\Delta_{ip}}{S_{ip}} \right) \tag{24.2}$$

where:

l_i – number of movements that exit from set Y_i,

p_i – new movements that enter set Y_i

If for all the signalized intersections, and there are r_s of them, (15) holds, then due to (21.2) the following also holds:

$$\sum_{i=1}^{r_s}\sum_{j=1}^{n_i}\Delta_{ij} \leq 0 \Rightarrow \sum_{i=1}^{r_s} Y_i(I_{UI}) \leq \sum_{i=1}^{r_s} Y_i \tag{25}$$

Proof: According to (24.1) in the group of intersections r_{s1} the sum of critical degrees of load at intersections changes for the amount of

$$\sum_{i=1}^{r_{s1}}\sum_{j=1}^{n_i}\frac{\Delta_{ij}}{S_{ij}}.$$

In the group of intersections r_{s2} the sum

$$\sum_{j=1}^{n_i-l_i} y_{ij} + \sum_{p=1}^{p_i} y_{ip}$$

changes according to (24.2) for

$$\sum_{j=1}^{n_i-l_i}\frac{\Delta_{ij}}{S_{ij}} + \sum_{p=1}^{p_i}\frac{\Delta_{ip}}{S_{ip}}.$$

Because of (15)

$$\sum_{i=1}^{r_{s1}}\sum_{j=1}^{n_i}\frac{\Delta_{ij}}{S_{ij}} + \sum_{j=1}^{n_i-l_i}\frac{\Delta_{ij}}{S_{ij}} + \sum_{p=1}^{p_i}\frac{\Delta_{ip}}{S_{ip}} \leq 0,$$

i.e. sums of critical degrees of load are reduced. ■

If optimal volume of the intensity of traffic flows intersecting is achieved in the group of signalized intersections, the sum of critical flow ratios is reduced, thus, according to (5), reducing also the critical degree of saturation of a signalized intersection, i.e. the reserve of the capacity is increased – the capacity of the signalized intersections is increased.

The question is whether this influence is proportional, i.e. whether by reducing the intensity of traffic flow intersecting the capacity of the signalized intersection is also proportionally increased. Only exceptionally is there a real solution of the equation:

$$\frac{\min I_{UI}(TN_s)}{I_{UI}(TN_s)} = \frac{X_C(\min I_{UI})}{X_C} \tag{26}$$

At *i-th* intersection it is necessary to solve the equation which has $4\cdot n_i$ or $4\cdot(n_i - l_i + p_i)$ unknowns (traffic flow and its change, degree of critical saturation and its change), and for the traffic network the system of *r* equations whose number of unknowns depends on the number of traffic flows considered in the network has to be solved.

For the relation of the change in the intensity of traffic flow intersecting and the critical degree of saturation of a signalized intersection the following statement can be claimed:

> **Statement 2**. *If in a set of signalized intersections the intensity of traffic flows intersecting is reduced, their capacity will increase. The value of changes is not proportional.*

5.2.2 Non-signalized intersections

The analysis of non-signalized intersections is much more complex, since there is no unique indicator for the entire intersection. Therefore, the change in the intensity of traffic flow intersecting on movement ranks 2 and lower is analyzed, considering the intervals in the main and secondary flows as invariant values. For the sake of simplicity of recording the following substitutions into model (7) will be introduced:

$$T_{f,i} = \frac{3600}{t_{f,i}} > 0, T_{0,j} = -\frac{1}{3600}\cdot\left(t_{g,i} - \frac{t_{f,i}}{2}\right) < 0 \tag{27}$$

so that the expression for practical capacity (9) can be written as:

$$Q_{m,j} = T_{f,j}\cdot e^{q_{c,i}\cdot T_{0,j}}\cdot\prod_k\left(1-\frac{q_{c,k}}{Q_{m,k}}\right) \tag{28}$$

Optimization of the intensity of traffic flow intersecting will affect the capacity in the set of non-signalized intersections in the following way:

$$\sum_{i=1}^{r_u} Q_i(\min I_{UI}) = \sum_{i=1}^{r_u}\sum_{j=1}^{n_i} T_{f,j}\cdot e^{(q_{c,i}+\Delta_{ij})\cdot T_{0,j}}\cdot$$
$$\cdot\prod_k\left(1-\frac{q_{c,k}+\Delta c,k}{Q_{m,k}}\right) = \sum_{i=1}^{r_u}\sum_{j=1}^{n_i} e^{\Delta_{ij}\cdot T_{0,j}}\cdot Q_{p,j}\cdot$$
$$\cdot\prod_k\left(p_{0,k}-\frac{\Delta_{c,k}}{Q_{m,k}}\right) \tag{29}$$

It should be checked whether $Q_{m,\ j}(\min I_{UI}) \geq Q_{m,\ j}$. This is correct, because, due to (15) and (27):

$$(\Delta_{c,j} \leq 0, T_{0,j} < 0) \Rightarrow e^{\Delta_{c,j}\cdot T_{0,j}} \geq 1$$

$$\Delta_{c,k} \leq 0 \Rightarrow \left(p_{0,k} - \frac{\Delta_{c,k}}{Q_{m,k}} \geq p_{0,k}\right)$$

By optimizing the traffic flow intersecting intensity at non-signalized intersections the following is obtained, due to (14):

$$\frac{\min I_{UI}(TN_u)}{I_{UI}(TN_u)} = \sum_{i=1}^{r_u}\sqrt{\frac{\prod_{j=1}^{n_i}(q_{ij}+\Delta_{ij})}{\prod_{j=1}^{n_i} q_{ij}}} \leq 1 \tag{30}$$

Regarding the relation of the change in the capacity the result is as follows:

$$\sum_{i=1}^{r_u}\frac{Q_i(\min I_{UI})}{Q_i} = \sum_{i=1}^{r_u}\sum_{j=1}^{n_i}\frac{e^{\Delta_{ij}\cdot T_{0,j}}\cdot\prod_k\left(p_{0,k}-\frac{\Delta_{c,k}}{Q_{m,k}}\right)}{\prod_k p_{0,k}} \geq 1 \tag{31}$$

As in (26), only exceptionally is there a solution in the set of real numbers for (30) and (31) so that for the relation of the change in the intensity of traffic flow intersecting and the capacity of the non-signalized intersection the following holds:

Statement 3. *If in a set of non-signalized intersections the intensity of traffic flows intersecting is reduced, their capacity will increase. The amount of changes is not proportional.*

5.2.3 Influence of intersecting intensity on the traffic network

Since signalized and non-signalized intersections have been analyzed separately, the question is whether the reduction in the intensity of traffic flows intersecting increases the capacity of the entire traffic network at at-grade intersections. The following statement has to be proven:

Statement 4. *Traffic network TN(R, L) consists of a set R which represents at-grade intersections and a set L which represents links (traffic flows) between the intersections. If the intensity of traffic flows intersecting is reduced in the traffic network, the capacity of the traffic network will be increased. The amount of changes is not proportional.*

$$\min \sum_{i=1}^{r} I_{UI}(R_i) \Rightarrow \sum_{i=1}^{r} Q_i(\min I_{UI}) \geq \sum_{i=1}^{r} Q_i$$

$$\sum_{i=1}^{r} \frac{I_{UI}(R_i)}{\min I_{UI}(R_i)} \neq \sum_{i=1}^{r} \frac{Q_i(\min I_{UI})}{Q_i}$$

Proof. If statements 2 and 3 are valid for the subsets of signalized and non-signalized intersections, then this statement has been proven. Otherwise, the following has to be proven:

$$\sum_{i=1}^{r_s} Q_i(\min I_{UI}) + \sum_{i=1}^{r_u} Q_i(\min I_{UI}) \geq \sum_{i=1}^{r_s} Q_i + \sum_{i=1}^{r_u} Q_i$$

$$\sum_{i=1}^{r_s} [(Q_i(\min I_{UI}) - Q_i)] + \sum_{i=1}^{r_u} [(Q_i(\min I_{UI}) - Q_i)] \geq 0 \tag{i}$$

Since the assumption of the statement is the reduction in the intensity of traffic flows, strict inequality holds

$$\sum_{i=1}^{r} \sum_{j=1}^{n_i} \Delta_{ij} < 0 \tag{ii}$$

It is assumed that the reorganization of the traffic flows has not resulted in the reduction of the intersecting intensity in the subset of signalized intersections, thus not having increased their capacity either:

$$\sum_{i=1}^{r_s} \sum_{j=1}^{n_i} \Delta_{ij} \geq 0 \Rightarrow \sum_{i=1}^{r_s} [Q_i(\min I_{UI}) - Q_i] \leq 0 \tag{iii}$$

In that case, due to (ii):

$$\sum_{i=1}^{r_u} \sum_{j=1}^{n_i} \Delta_{ij} < 0 \Rightarrow \sum_{i=1}^{r_u} [Q_i(\min I_{UI}) - Q] > 0 \tag{iv}$$

If according to (iii) the capacity of signalized intersections is not increased, due to (iv) the capacity of non-signalized intersections will certainly increase, i.e. the capacity of the network will generally increase and (i) is proven. The case for non-signalized intersections is proven in the analogue manner. ■

6. CASE STUDY

An example of unnecessary traffic flow intersecting can be found in the central part of the City of Zagreb (Figure 4). Part of the traffic network is studied, which consists of six intersections (Table 1).

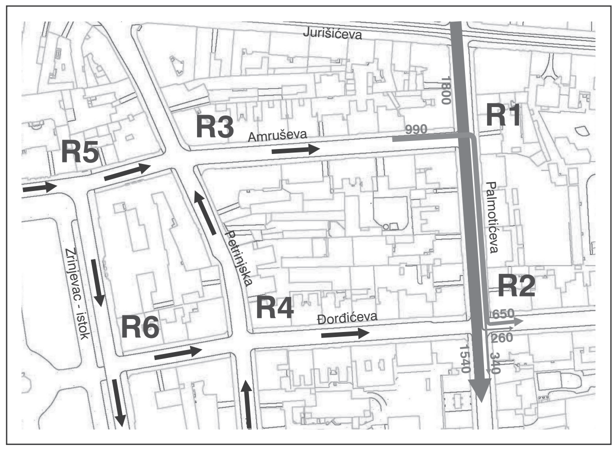

Figure 4 – Intersecting of traffic flows: Palmotićeva – Amruševa Streets

Signalized intersections are mutually connected and synchronized. The current organization of traffic leads to unnecessary intersecting of traffic flows at intersections R1 and R2. They unnecessarily merge traffic flows, whereas intersections R6 and R4 are very lightly loaded. Since this refers to heavy traffic flows, such a solution causes frequent congestions of the Palmotićeva Street on the section between Jurišićeva and Đorđićeva Streets and of the Amruševa Street between Petrinjska and Palmotićeva Streets. The R3 – R4 – R5 – R6 quadrant of intersections accommodates court institutions and institutions of the Ministry of the Interior which require special security measures so that reducing the congestion in this area is of interest to all these institutions.

There are two possible solutions of managing the traffic flow through Amruševa Street, presented in Figure 5. The first solution is proposed by this paper because:

– there are no changes in the organization of traffic in the zone of Zrinjevac and Petrinjska Street,

- there are least interventions in the traffic organization,
- it is possible to reach Zrinjevac through Petrinjska Street,
- there is more space for the increased queue in Đorđićeva Street: from R6 to R2.

Table 1 - Intersections in the zone of Palmotićeva and Amruševa Streets

Code	Intersection	Type	Total traffic at intersection [veh/h]
R1	Palmotićeva – Amruševa	signalized	2790
R2	Palmotićeva – Đorđićeva	signalized	2910
R3	Amruševa – Petrinjska	signalized	990
R4	Đorđićeva – Petrinjska	non-signalized	190
R5	Amruševa – Zrinjevac-east	non-signalized	1080
R6	Đorđićeva – Zrinjevac-east	non-signalized	160

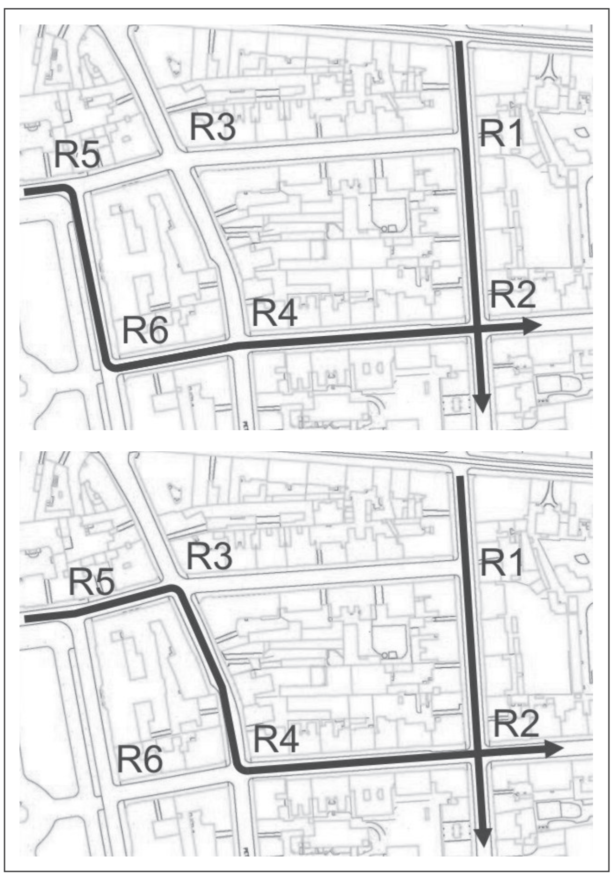

Figure 5 – Proposals for reducing intersecting intensity

In order to realize this solution it is necessary to install traffic lights at intersection R4, whereas traffic light can be eliminated at intersection R3, i.e. the solution keeps the same number of signalized intersections. Although the traffic light at intersection R1 can also be eliminated, the signalized pedestrian crossing is retained.

The simulation and analysis of the traffic process has been carried out by the Synchro software package. Figure 6 presents the simulation of the current condition, and Figure 7 the simulation of the proposed solution. The figures illustrate how the solution affects the increase in the capacity of the Palmotićeva and Amruševa Streets, not disturbing at the same time the capacity of the rest of the network – the rest of the network is "used" in a better way.

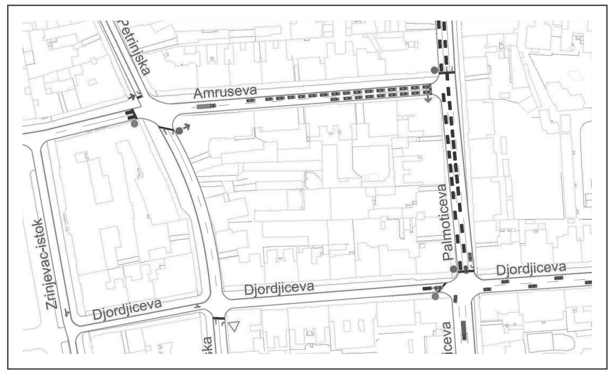

Figure 6 – Simulation of the current condition

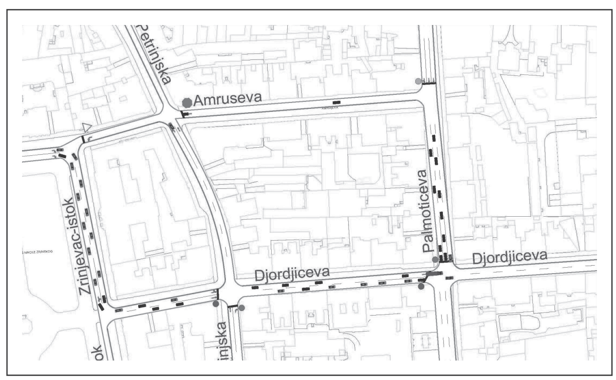

Figure 7 – Simulation of the proposed solution

What remains is to use the presented models to analyze the change in the traffic flow intersections intensity and the intersection capacity. Intersections R5 and R6 need not be analyzed, since they practically do not represent intersections but rather diverging points of one traffic flow.

The change in the traffic flow intersecting intensity according to the method of the root of conflicting area (model 1.3) is presented in Table 2. Since the proposed solution does not feature traffic flow conflict at intersection R1, intersections R1 and R2 are considered as a unique traffic whole.

Table 2 – Intersecting intensity according to conflicting area root model

Intersection	CURRENT			PROPOSAL			Difference	
	q1	q2	I_UI	q1	q2	I_UI	I_UI	%
R1	1800	990	1335	1800	0	0	-546	-29%
R2	2790	120	579	1700	1100	1367		
R3	920	70	254	100	110	105	-149	-59%
R4	70	120	92	1090	120	362	270	295%
Ponder. mean **			825			712	-113	-14%

** - Pondered arithmetic mean is used: ponder of value is share of traffic at intersection in total network traffic

Table 3 - Value Xc and serviceability level

Intersection	CURRENT		PROPOSAL		DIFFERENCE	
	X_c	RU	X_c	RU	X_c	%
R1	0.80	B	0.59	A	-0.21	-26%
R2	0.72	B	0.84	A	0.12	17%
R3	0.35	A				
R4		A	0.40	A	0.05	14%
Pond. mean	0.68		0.65		-0.03	-5%

Table 4 – Network indicators

Indicators at the network level	Current	Proposal	DIFFERENCE	%
Total lost time [h]	148	24	-124	-84%
Average speed [km/h]	8	23	15	188%
Total travel time [h]	178	66	-112	-63%
Total travelled distance [km]	1497	1531	34	2%
Total consumed fuel [l]	705	282	-423	-60%
Economy of consumption [km/l]	2.12	5.42	3.3	156%
Emission of carbon compounds (CO) [kg]	13.10	5.24	-7.86	-60%
Emission of nitrogen compounds (NO) [kg]	2.53	1.01	-1.52	-60%

The intersecting intensity on the critical section of intersection R1 – R2 decreased by 546 veh/h, i.e. 29%, whereas on the studied section of the network this decrease is on the average 14%.

The analysis of capacity indicators at signalized intersections is presented in Table 3. Since there is change in the type of intersection between intersection R3 and R4 (signalized - non-signalized), these two intersections are studied together. Also, the big change in the load of western approach to intersection R2 conditions re-programming (change of the green phase relation) within the current cycle length.

The values are expected; significant reduction of the degree of saturation at intersection R1 with increase at R2. Regarding R3 and R4, since the values of the degree of saturation are small, the difference is not relevant.

The proposed reorganization of the traffic flows will reduce the intersecting intensity in the network by 19% which will result in an increase in the level of service and the intersection capacity at the critical section of 5%. Such results can question the proposed change. However, considering the effects on the entire network shows the real results of this intervention. Table 4 confirms that the new traffic solution significantly improves the traffic, economic and environmental effects, i.e. shows that it is incorrect to analyze and optimize just the intersections as the only traffic network operators.

The total costs of introducing the new regulation amount to about 598,700 euro, and the annual loss due to the new regulation is 423,500 euro. Annual savings amount to 877,000 euro so that already during the second year savings are achieved, and during the following years the profits are double the losses.

7. CONCLUSION

The method of traffic flows intersecting intensity and the methods for the analysis of the capacity of at-grade intersections are complementary methods regarding improvement of the quality and safety of road traffic. This paper has studied their interdependence through the generally accepted and commonly used traffic models.

The change in the capacity of the at-grade intersection does not affect the traffic flows intersecting intensity. In the opposite case the following implication has been proven: the change in the traffic flow intersecting intensity affects the intersection capacity. The changes are not proportional.

The case study presents the application of the method of reducing the traffic flows intersecting intensity and its influence on the increase in the intersection capacity, mainly on the traffic network efficiency. The case study shows how the implementation of scientific and technical methods from the area of traffic sciences can achieve, at low costs, fast and significant results in the actual road traffic network.

The results of this paper confirm that, apart from applying compositions of different methods in their logical sequence, it is necessary to continuously study the possibilities of individual and combined methods and models in order to increase the efficiency of the entire traffic process. The entire structure of the traffic network should be studied, rather than only intersections as its only operators.

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SAŽETAK

KOMPLEMENTARNOST METODE PRESIJECANJA PROMETNIH TOKOVA S METODAMA PROPUSNE MOĆI RASKRIŽJA

U radu se ispituje komplementarnost metoda iz područja teorije prometnog toka: metode intenziteta presijecanja prometnih tokova i metoda za analizu propusne moći raskrižja u razini. Uz provjeru međusobnih implikacija ocjenjuje se i proporcionalnost međusobnih utjecaja. Usklađenom primjenom ovih metoda učinkovito se djeluje na cijelu prometnu mrežu, a ne samo na raskrižja koja se najčešće pogrešno predstavljaju kao jedini operatori mreže. Teoretska razmatranja ilustrirana su praktičnim primjerom.

KLJUČNE RIJEČI

teorija prometnog toka, teorija organiziranosti i usmjeravanja prometnih tokova, intenzitet presijecanja prometnih tokova, propusna moć raskrižja, optimizacija

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