CODEN STJSAO ISSN 0562-1887
ZX470/1534 UDK 621.833.01:539.419

# The Influence of Gear Parameters on the Surface Durability of Gear Flanks

***Zvonko SAJFERT[1], Mileta RISTIVOJEVIĆ[2], Božidar ROSIĆ[2], Dragan RADOVIĆ[3]* and *Živoslav ADAMOVIĆ[1]***

1) Univerzitet u Novom Sadu, Tehnički fakultet "Mihajlo Pupin" u Zrenjaninu (University of Novi Sad, Technical Faculty "Mihajlo Pupin" in Zrenjanin), Đure Đakovića bb, 23000 Zrenjanin, **Republic of Serbia**

2) Univerzitet u Beogradu, Mašinski fakultet (University of Belgrade, Faculty of Mechanical Engineering), Kraljice Marije 16, 11120 Beograd, **Republic of Serbia**

3) Alfa University Belgrade, Faculty of Management Novi Sad, **Republic of Serbia**

mristivojevic@mas.bg.ac.rs

**Keywords**

*Addendum modification*
*Contact stress*
*Gear*
*Teeth geometry*

**Ključne riječi**

*Geometrija zuba*
*Koeficijent pomicanja profila*
*Kontaktni naon*
*Zupčanik*

**Received (primljeno)**: 2010-12-01
**Accepted (prihvaćeno)**: 2011-08-09

Original scientific paper

Load gear pairs is limited to the surface and volume strength of teeth. Shape profile pair of teeth has a great influence on the surface durability of gear flanks. Contact stess and specific sliding on the flanks of the function extreme shapes of the pair of teeth. At the same time, the contact stress and specific sliding are the main criteria for evaluation of carrying capacity of tooth flanks. In this paper we analyzed the simultaneous influence of the inclination angle profile, the addendum modification coefficient and the number of teeth on the tooth flank load capacity of cylindrical gear pairs. Based on these results allow us to define the optimum parameters of cylindrical gear pairs in terms of specific sliding and the contact stresses on the flanks.

**Utjecaj parametara zubčanika na površinsku nosivost bokova zubaca**

Izvornoznanstveni članak

Opterećenje zupčastih parova ograničeno je površinskom i volumenskom čvrstoćom zubaca. Oblik profila zubaca ima veliki utjecaj na površinsku nosivost bokova zubaca. Kontaktni napon i specifično klizanje na bokovima zubaca su izrazita funkcija oblika profila zubaca. Istovremeno, kontaktni napon i specifično klizanje su glavni kriteriji za ocjenu nosivosti bokova zubaca. U ovom rada analiziran je istovrijemeni utjecaj kuta nagiba profila, koeficijenta pomicanja profila i broja zubaca na nosivost bokova zubaca cilindričnih zupčastih parova.
Na bazi ovih rezultata mogu se definirati optimalni parametri cilindičnih zupčastih parova u razvijenim matematičkim modelima za kontaktni napon i specifično klizanje na bokovima zubaca.

## 1. Introduction

Load carrying capacity of gears is limited by volume and surface damage of teeth. Active surface flanks of meshing gear pairs are exposed to different kinds of surface damage over the contact period . At present due to the complexity of the phenomenon of the surface damage exist.

A tooth surface can be damaged or destroyed in a variety ways, wear being one of the most frequent. The continuous wear of gear flanks, which depends on the value of contact pressure, sliding velocity and the thinkness of oil film, is the consequence of the presence of friction force [1-9]. Although there is no unified option in the literature, with regard to the exact mechanism of the surface damage due to fatigue, the contact pressure, the number of cycles, the hardness, the degree of lubrication, the temperature, rolling and sliding of contact surfaces all influence the pitting of active surfaces of the gear flanks. Pitting causes the increase in load concetration along undamaged parts of the gear flanks. Since reliable method for determing of damage of the surfaces of gear flanks is not available, then the intensity of contact stress provides a good way to determine the surface strenght of gear flanks. [ 3,6,9].

According to [10] registered a large number of destructive forms of tooth flanks. Failure that usually occur on the flanks of the wear, pitting and scoring.

Because these types of destruction of the greatest number of theoretical and experimental research. In [3,5,6,7,8,9], based on experimental and theoretical studies established the dependence between] the position a pitch point in the contact period and location of damage on the flanks.

A mathematical model for analyzing the degree of wear of tooth flanks of cylindrical gear pairs to change the contact stress was developed in [8].

In the majority of cases, gear with pressure angle of the basic rack of 20° are being used [1-9]. However, more stringent requirements in terms of specific mass, load carrying and dimensions of the gear require gears with a pressure angle of the tool other then 20°. In addition, above mentioned requirements are influenced by addendum modification coefficient. Therefore, it is necessary to take into account the individual as well as the combined effect of those two parameters (the pressure angle of the tool and the addendum modification coefficient) on the surface of gear flanks.

To this end, this paper analyzes the influence of the shape profile of the tooth contact stresses and specific sliding tooth flank of cylindrical gear pairs. Contact stress and the maximum specific sliding affect the carrying capacity of tooth flanks. The shape profile of teeth is altered by changing the angle of inclination profile tools, profiles and displacement coefficients of teeth meshed gears. Based on these results we can define the optimal shape profile of the pair of teeth in terms of capacity tooth flank.

## 2. Tool – the rack geometry

Gears are commonly made by milling, while casting and cold forming are seldom used. Machining of the gears are usually done two days either throught form cutting or generating cutting. The former process is being superseded by the latter one. This procedure is based on the basic law of conjugation: the tooth profile presents the envelope of all succesive positions of the profile of the tool.

Teeth of the cylindrical gears are usually prosuced with a racked shaped tool Figure 1. The sides of an involute rack toot are straight, while the teeth profiles of the corresponding teeth pair are involute curves, Figure 1.

During the simultaneous translation of basic rack and rotation of meshing gear, the instantaneous axis – forms konematic surfaces. A cylindrical surface is created by gear, and a plane by the rack. At the time, point C forms konematic lines: pitch circle and pitch straight line, Figure 1.

During manufacturing the basic rack position in relation to the gear is determined by the relative position of the axis symmetry of the basic rack to the pitch line of the gear. This position is defined by the distance between the pitch line and the axis of addendum modification coefficient (x), Figure 1.

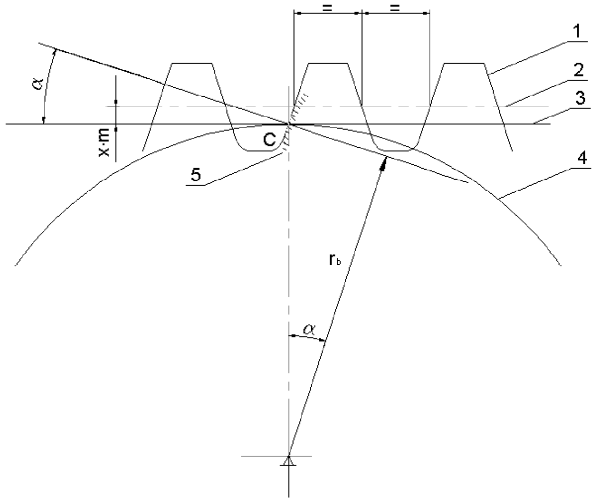


**Figure 1.**The basic rack position in relation to the gear; 1 profile of the rack, 2 axsis of symmmetry of the basic rack, 3 pitch straight line, 4 pitch circle, 5 profile of the gear

**Slika 1.** Medjusobni položaj osnovne zupčaste letve i zupčanika: 1 profil osnove zupčaste letve, 2 srednja linija osnovne zupčaste letve, 3 kinematska linija, 4 kinematski krug, 5 profil zupca

Depending on the relative position between the pitch circle and the axis of symmetry of the basic rack, the addendum modification coefficient could be positive, negative or equal to zero.

When the pitch circle intersect the axis of symetry of the basic rack, addendum modification is nagative ($x<0$), when the pitch circle and the axis of symetry of the basic rack have no common point Figure 1.

The pressure angle of the tool defines the degree of the curvature of the involute profile. The greater angle, the higher the degree of curvature.

## 3. Contact stresses on the flanks

Flank contact teeth carrying the load in the contact period are exposed to both rolling and sliding. As a result, their active surface are exposed to different types of damage [10 ]. Completely reliable estimate of resistance against the tooth flank surface damage does not exist, because they can not accurately take into account all factors influencing the process of surface destruction. According to the conventional procedure of calculation of the maximum size of the surface pressure in the contact zone of the pair of teeth is an essential feature for assessing the surface hardness of tooth flanks.

Active surface of the teeth flanks in the loaded condition, contacting along the current lines of contact that are common generating involute surface. Stress analysis in the active tooth flank surfaces, contact-involute pair of teeth surfaces are approximated with a mechanical model that matches the contact of two cylinders of parallel axes.

Based on this mechanical model, according to the theory of Hertz, the voltage (surface pressure) on the flanks can be written as:

$$\sigma_H^2 = z_E^2 \frac{F_n}{\rho b}, \tag{1}$$

where

$F_n$ - is normal tooth load

$b$ - is tooth face width

$\rho$ - is reduced radius of curvature

$z_E$ - is elasticity factor.

Expression (1), stress (surface pressure) on the flanks, can be written as:

$$\sigma_H^2 = z_E^2 k,$$

where

$k$ - is reduced contact surface pressure.

Reduced contact surface pressure depends only on the geometric size of teeth in contact and the load that is transferred. That is why he is suitable for stress analysis in the flanks.

## 4. Influence of gear parameters on the contact stress

The addendum modification of coefficient and pressure angle are two independent parameters influencing the geometry of active surface flanks of conjugated teeth, as well as the contact pressure.

To analyse the effect of the pressure angle of the tool and of the addendum modification coefficient on the load carrying capacity, the contact pressure on active surface of gear flanks has been considered in [6].

In order to address the influence of the addendum modification coefficient on the value of contact pressure of conjugated teeth, the following relation has been formed:

$$\frac{k}{k_0} = \frac{tg20^\circ}{tg\alpha_w},$$

where

$$tg\alpha_w - \alpha_w = inv20^\circ + \frac{\sum x}{\sum z} tg20^\circ,$$

$\sum x = x_1 + x_2$ - the sum of the addendum modification coefficients of meshing gears

$\sum z = z_1 + z_2$ - the tooth sum of meshing gears

$k_0$ - modified contact pressure at the gear flanks when $\alpha = 20^\circ$ and $\sum x = 0$

$k$ - modified contact pressure at the gear flanks when $\alpha = 20^\circ$ and $\sum x \neq 0$

$\alpha_w$ - pressure angle.

The effect of the addendum modification coefficient and the tooth sum of sum meshing gears on the contact prssure is shown in Figure 2.

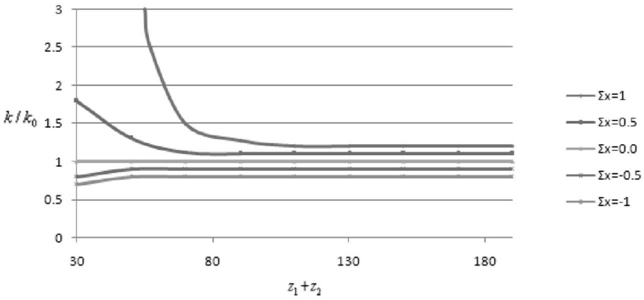


**Figure 2.** Dependence of contact pressure on the addendum modification coefficient and the teeth of meshing gears, $\alpha = 20^\circ$

**Slika 2.** Ovisnost kontaktnog napona od koeficijenata pomicanja profila i broja zubaca, za $\alpha = 20^\circ$

Contact pressure is affected by the addendum modification coefficient, to a greater exent, for a small tooth sum of meshing gears and for negative values of adendum modification coefficient. Positive values of the addendum modification coefficients have a weak effect on the value of contact pressure. For gears having large tooth sum $\sum z > 120$ the influence of the addendum modification coefficient on contact pressure is negligible.

Taking into account both the pressure angle of tool and addendum modification coefficient on the contact pressure the following equation was formed:

$$\frac{k}{k_0} = \frac{tg\alpha'_w \cos^2 20^\circ}{tg\alpha_w \cos^2 \alpha}, \tag{2}$$

where:

$$tg\alpha_w - \alpha_w = inv20^\circ + 2\frac{\sum x}{\sum z} tg20^\circ, \tag{3}$$

$$tg\alpha'_w - \alpha'_w = inv20^\circ + 2\frac{\sum x}{\sum z} tg20^\circ. \tag{4}$$

$\alpha_w'$ - the pressure angle when $\alpha = 20^\circ$

$\alpha_w$ - the pressure angle when $\alpha \neq 20^\circ$.

By following equations (3) and (4) with the Newton – Raphson method, graphical represantations of expresion (2) for values of pressure angle of 16º and 25º respectively, are shown in Figure 3. and Figure 4. Based on these figures one can conclude the following:

1. For positive values of the of the addendum modification coefficient $\sum x > 0$ the influence of the tooth sum of the meshing gears on contact pressure in negligible.
2. Negative values of the sum of the addendum modification coefficient $\sum x < 0$, have a pronounced effect on the stress state of gear flanks, provided the tooth sum of meshing gears is less than 100.
3. The effect of pressure angle on contact of gear flanks depends on the values of the tooth sum of meshing gears and the sum of the addendum modification coefficient. The lower the sum of the addendum modification coefficient the more pronounced is the effect of pressure angle of contact pressure. For negative values of the sum of the addendum modification coefficients and the small tooth sum of meshing gears ($\sum z < 100$), the pressure angle has strong effect on the stress state of gear flanks.

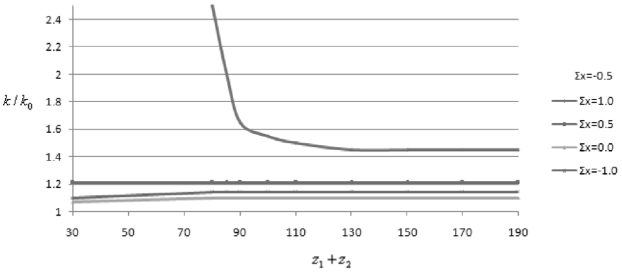


**Figure 3.** Dependence of contact pressure on the addendum modification coefficient and the teeth of meshing gears, α =16º

**Slika 3.** Ovisnost kontaktnog napona od koeficijenata pomicanja profila i broja zubaca, za α = 16º

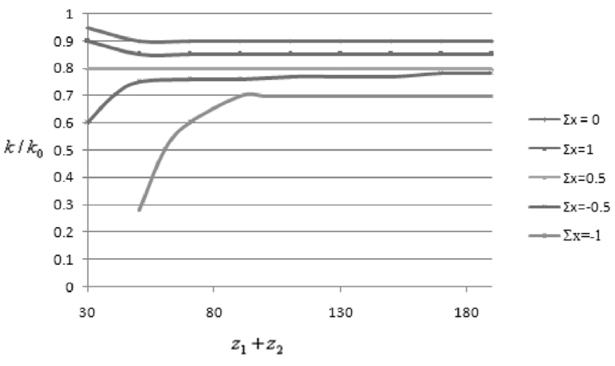


**Figure 4.** Dependence of contact pressure on the addendum modification coefficient and the teeth of meshing gears, α =25º

**Slika 4.** Ovisnost kontaktnog napona od koeficijenata pomicanja profila zubac, za α = 25º

## 5. Influence of gear parameters on the specifying sliding

The manner in which both the pressure angle of the tool and the addendum modification coefficient influence wear of gear flanks is shown in terms of specific sliding [5].

Assuming uniform wear of the meshing gear flanks at the ends of the active line of action, point E2A1 and E1A2 at the Figure 5, the following equation is formed:

$$\rho_{E2}\rho_{A2} = u^2 \rho_{E1}\rho_{A1}, \tag{5}$$

where:

$u$ - kinematic ratio of tooth pair

$\rho_{A1}$, $\rho_{A2}$ - radius of curvature of involute profiles of the tooth of the addendum circle of mating gears

$\rho_{E1}$, $\rho_{E2}$ - radius of curvature of involute profiles at the points E1 and E2 of the root diameter of driving and driven gear.

Substituting the expression for the radii of curvature at the caracteristic contact point on the pinion – and gear tooth involute profiles into equation (5), the following expression is derived:

$$\begin{aligned}&[a\sin\alpha_w - (r_{a1}^2 - r_{b1}^2)^{1/2}](r_{a2}^2 - r_{b2}^2)^{1/2} = \\ &= u^2[a\sin\alpha_w - (r_{a2}^2 - r_{b2}^2)^{1/2}](r_{a1}^2 - r_{b1}^2)^{1/2}\end{aligned}, \tag{6}$$

where:

$a$ - center distance

$r_{a1}$, $r_{a2}$ - tip diameter of pinion, wheel

$r_{b1}$, $r_{b2}$ - base diameter of pinion, wheel.

Based upon equation (6) the course of change of specific sliding of meshing teeth profiles over the contact period, for α=20º and α=25º are depicted in Figure 5 and Figure 6 respectively.

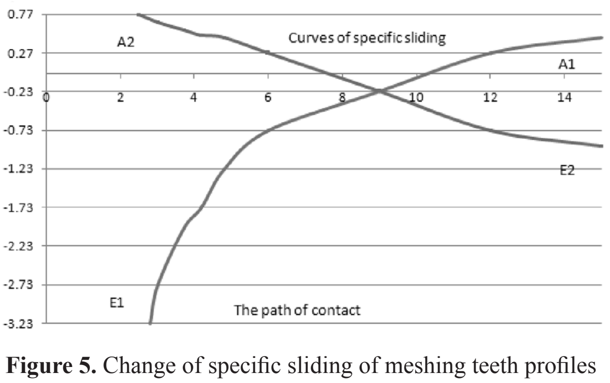


**Figure 5.** Change of specific sliding of meshing teeth profiles over the contact period, for α=20º

**Slika 5.** Promjena specifičnog klizanja na profilima spregnutih zubaca u tijeku dodirnog perioda, za α = 20º

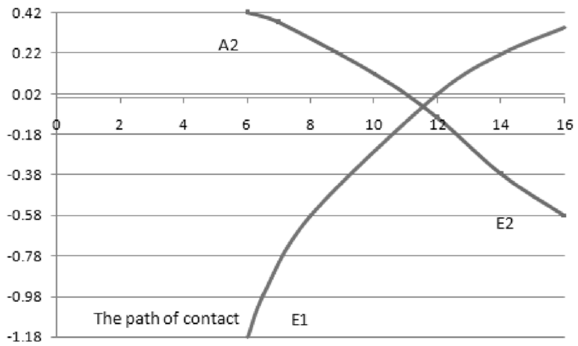


**Figure 6.** Change of specific sliding of meshing teeth profiles over the contact period, for α=25º

**Slika 6.** Promjena specifičnog klizanja na profilima spregnutih zubaca u tijeku dodirnog perioda, za α=25º

## 6. Conclusion

Based upon the perfomed it follows that the pressure angle of the tool and the addendum modification coefficient have significant influence on the load carrying of gear flanks, both to fatigue and to the unifomity of wear of active gear flanks. The magnitude of this effect depends on the value of the pressure angle of the tool, the intensity and the sign of the sum of meshing gears.

The influence of the addendum modification coefficient on contact stress of gear flanks is more pronounced for a small tooth sum and negative values of the addendum modification coefficient, while positive values of the addendum modification coefficients have no influence on the load carrying capacity of gear flanks.

The contact stress of gear flanks affected, as well, for small sums of the addendum modification coefficient and decreasing number of teeth of mating gears, the influence of the pressure angle of the tool and contact stress is becoming marked. Over the contact period and along the contact path, the absolute value of specific sliding pinion – and gear – tooth profiles decreases with the increase of the pressure angle of the tool. Using this analysis, during the design stage of gears, optimal assembly satisfying the most stringent tehnical specifications can be achived.

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