GLASNIK MATEMATIČKI
Vol. 33(53) (1998), 109–114

EXTENDING OPEN FAMILIES IN NONMETRIC SPACES AND AN APPLICATION TO OVERLAYS

Tadeusz Dobrowolski, Pittsburg, USA, **Rolando Jiménez,** Morelia, Mexico and **Witold Marciszewski,** Warsaw, Poland

Abstract. We show that a family $\mathcal{U}$ of open subsets of a subset X of a hereditarily normal paracompact space P which is locally finite in P admits a locally finite open extension $\mathcal{V}$ in P which is similar to $\mathcal{U}$. We apply this fact to prove an extension theorem for overlays over hereditarily paracompact spaces.

1. Introduction

Let P be a space and X be a subset of P. We are concerned with the problem of extending an open family $\mathcal{U}$ in X to an open family $\mathcal{V}$ in P so that whenever a finite subfamily of $\mathcal{V}$ has a nonempty intersection then the corresponding finite subfamily of $\mathcal{U}$ also has a nonempty intersection. If the last happens then we say that $\mathcal{U}$ and $\mathcal{V}$ are similar, see the beginning of Section 2 for more precise description. In case (P, d) is a metric space, Kuratowski's formula [6] $V_U = \{p \in P \mid d(p, U) < d(p, X \setminus U)\}$, $U \in \mathcal{U}$, yields a family $\mathcal{V} = \{V_U\}_{U \in \mathcal{U}}$ which is similar to $\mathcal{U}$. In this paper we consider the case of a hereditarily normal paracompact space P. Extending an old result due to Čech, we show that every open family $\mathcal{U}$ of subsets of an arbitrary set $X \subset P$ which is locally finite in P admits a locally finite open extension $\mathcal{V}$ which is similar to $\mathcal{U}$. (Previously Fox claimed a similar fact, however, his proof of [3, Lemma 5.5] contained a gap; see [5].)

Following an idea of Fox [3], we apply our result to prove an extension theorem for overlays. Let us recall that the concept of an overlay $e : \tilde{X} \to X$ is a shape-theory counterpart of a covering projection (introduced by Fox in [3]). Our extension theorem for overlays states that if X is an arbitrary subset of a hereditarily paracompact space P then, for an overlay $e : \tilde{X} \to X$, there exists an open neighborhood U of X in P and an overlay $\bar{e} : \tilde{U} \to U$ which extends e.

2. Extension of Covers

Let P be a space and X be a subset of P. Let $\mathcal{U} = \{U_\alpha \mid \alpha \in A\}$ and $\mathcal{V} = \{V_\alpha \mid \alpha \in A\}$ be families of subsets of X and P, respectively. The family $\mathcal{V}$ is

Mathematics subject classification (1991): 54D15, 54D20, 54C56.

Key words and phrases: hereditarily normal space, hereditarily paracompact space, similar families, overlay.

called an extension of the family $\mathscr{U}$ if, for every $\alpha \in A$, we have that $V_\alpha \cap X = U_\alpha$. The families $\mathscr{U}$ and $\mathscr{V}$ are said to be similar provided, for every finite set of indices $\{\alpha_1, \alpha_2, \dots, \alpha_n\} \subset A$, we have $U_{\alpha_1} \cap U_{\alpha_2} \cap \dots \cap U_{\alpha_n} = \emptyset$ if and only if $V_{\alpha_1} \cap V_{\alpha_2} \cap \dots \cap V_{\alpha_n} = \emptyset$.

THEOREM 2.1. *Let P be a hereditarily normal paracompact space and X be a subset of P. Then, for every family $\mathscr{U} = \{U_\alpha \mid \alpha \in A\}$ of open subsets of X which is locally finite in P, there exists an open locally finite family $\mathscr{V} = \{V_\alpha \mid \alpha \in A\}$ in P such that*

(1) *$\mathscr{V}$ is an extension of $\mathscr{U}$; and*
(2) *$\mathscr{U}$ and $\mathscr{V}$ are similar.*

The case when $\mathscr{U}$ is finite is a result of Čech [1]; for completeness we enclose a proof of it (which slightly differs from the original one).

LEMMA 2.2. *Let X be a subset of a hereditarily normal space P. Every finite open family $\mathscr{U}$ of subsets of X admits an open extension $\mathscr{V}$ in P which is similar to $\mathscr{U}$.*

Proof. Denote by $r = r(\mathscr{U})$ the greatest cardinality of a subfamily $\mathscr{W} \subset \mathscr{U}$ with a nonempty intersection. We will use induction on r.

Suppose $r(\mathscr{U}) = 1$ for some cover $\mathscr{U}$; this means that $\mathscr{U}$ is pairwise disjoint. One can easily see that it is enough to consider the case when $\mathscr{U}$ has two elements. However, it is well-known that two disjoint open subsets of a subset X of a hereditarily normal space P can be extended to disjoint open subsets of P (see [2, Thm. 2.1.7]). This settles the case of $r = 1$.

Suppose that the assertion of our lemma holds for all families $\mathscr{U}$ with $1 \leqslant r(\mathscr{U}) < k$. Let $\mathscr{U} = \{U_0, U_2, \dots, U_{n-1}\}$ be a family of open subsets of X with $r(\mathscr{U}) = k$. If $n = k$ then $U_0 \cap U_1 \cap \dots \cap U_{n-1} \neq \emptyset$. In this case an arbitrary open extension $\mathscr{V}$ of $\mathscr{U}$ works. So, we can assume that $n > k$. Let $[n]^k$ denote the family of all subsets of the set $n = \{0, \dots, n-1\}$ of the cardinality k. Consider the family $\mathscr{W}$ consisting of all sets of the form $W_I = \bigcap\{U_i \mid i \in I\}$ where $I \in [n]^k$. For each such W_I and index $i \notin I$ we have that $W_I \cap U_i = \emptyset$. For each $I \in [n]^k$ and $i \in n \setminus I$ we choose open sets $W(I,i)$ and $U(I,i)$ in P that extend W_I and U_i, respectively, and such that $W(I,i) \cap U(I,i) = \emptyset$. We let $W_I^* = \bigcap\{W(I,i) \mid i \in n \setminus I\}$ and $U_i^* = \bigcap\{U(I,i) \mid I \in [n]^k,\ i \notin I\}$. We can additionally assume that $W_I^* \subset U_i^*$ provided $i \in I$. Let $W_0 = \bigcup \mathscr{W}$. Define $X' = X \setminus W_0 \subset P \setminus W_0 = P'$. Write $\mathscr{U}' = \{X' \cap U_0, X' \cap U_1, \dots, X' \cap U_{n-1}\}$ and notice that $\mathscr{U}'$ is an open family in X' with $r(\mathscr{U}') < k$. By the inductive assumption, there exists an open family $\mathscr{W}'$ in P' which extends $\mathscr{U}'$ and is similar to $\mathscr{U}'$. We can additionally assume that each element $W_i' \in \mathscr{W}'$ extending $X' \cap U_i$ is contained in U_i^*. Finally, for every $U_i \in \mathscr{U}$, define $V_i = W_i' \cup \bigcup\{W_I^* \mid I \in [n]^k \text{ and } i \in I\}$. The family $\{V_0, V_1, \dots, V_{n-1}\}$ is as required. $\square$

Proof of Theorem 2.1. Since $\mathscr{U}$ is locally finite in P, there exists an open cover $\mathscr{W}$ of P such that each element $W \in \mathscr{W}$ intersects only finitely many of elements $\mathscr{U}$. It follows that $\mathrm{cl}(W)$, the closure of W in P has the same property. By the paracompactness of P we can additionally assume that $\mathscr{W}$ is locally finite (in P). Clearly, the family $\{\mathrm{cl}(W) \mid W \in \mathscr{W}\}$ is also locally finite.

Fix $W \in \mathscr{W}$. Consider all U_α whose intersection with $\mathrm{cl}(W)$ is nonempty. There are only finitely many of such U_α, say $\{U_{\alpha_1}, U_{\alpha_2}, \dots, U_{\alpha_n}\}$. Apply Lemma 2.2 to the space P, the subset $X \cap \mathrm{cl}(W)$, and the family $\mathscr{U}_W = \{U_{\alpha_1} \cap \mathrm{cl}(W), U_{\alpha_2} \cap \mathrm{cl}(W), \dots, U_{\alpha_n} \cap \mathrm{cl}(W)\}$ to find a family $\mathscr{O}_W = \{O_{\alpha_1}, O_{\alpha_2}, \dots, O_{\alpha_n}\}$ of open subsets in P such that, for each α_i $(i = 1, 2 \dots, n)$, $O_{\alpha_i} \cap \mathrm{cl}(W) = U_{\alpha_i} \cap \mathrm{cl}(W)$, and the family $\mathscr{O}_W$ is similar to $\mathscr{U}_W$. For each element $U = U_{\alpha_i}$, let U_W denote the corresponding O_{α_i}. We let $U_W = \emptyset$ if $U_\alpha \cap \mathrm{cl}(W) = \emptyset$.

Now, fix $U = U_\alpha$. Look at the family $\{F(U, W) \mid W \in \mathscr{W}\}$, where $F(U, W) = \mathrm{cl}(W) \setminus U_W$. Since $\{F(U, W) \mid W \in \mathscr{W}\}$ is inscribed in locally finite family $\{\mathrm{cl}(W) \mid W \in \mathscr{W}\}$, it is locally finite as well. Consequently, the set $F_U = \bigcup\{F(U, W) \mid W \in \mathscr{W}\}$ is a closed set in P. We set V_α to be $P \setminus F_U$. We claim that the family $\mathscr{V} = \{V_\alpha \mid \alpha \in A\}$ is as required.

It follows from the definition that each V_α is an open subset of P. We will show that if a finite subfamily $\{V_{\alpha_1}, V_{\alpha_2}, \dots, V_{\alpha_n}\}$ of $\mathscr{V}$ has a nonempty intersection then the corresponding subfamily $\{U_{\alpha_1}, U_{\alpha_2}, \dots, U_{\alpha_n}\}$ of $\mathscr{U}$ has a nonempty intersection. Fix $x \in P$. Let $x \in V_{\alpha_1} \cap V_{\alpha_2} \cap \dots \cap V_{\alpha_n}$ (the intersection may reduce to a single set). Select $W \in \mathscr{W}$ such that $x \in W$. Consequently, we have $x \in V_{\alpha_1} \cap V_{\alpha_2} \cap \dots \cap V_{\alpha_n} \cap W$. We now conclude that, for each $U = U_{\alpha_i}$ $(i = 1, 2, \dots, n)$, $x \notin F(U, W)$. It follows that $x \in U_W$ for each such U. Hence, we have that $x \in \bigcap\{U_W \mid U = U_{\alpha_i}\} \neq \emptyset$. Since the last family is similar to the family $\{U_{\alpha_1} \cap \mathrm{cl}(W), U_{\alpha_2} \cap \mathrm{cl}(W), \dots, U_{\alpha_n} \cap \mathrm{cl}(W)$, we conclude that $U_{\alpha_1} \cap U_{\alpha_2} \cap \dots \cap U_{\alpha_n} \cap \mathrm{cl}(W) \neq \emptyset$. In particular, we have that $U_{\alpha_1} \cap U_{\alpha_2} \cap \dots \cap U_{\alpha_n} \neq \emptyset$. In a similar way we obtain that, for every $\alpha \in A$, $V_\alpha \cap X \subseteq U_\alpha$. On the other hand, for every $U = U_\alpha$, V_α contains U because every $F(U, W)$ is disjoint from U. This shows that, for every $\alpha \in A$, $V_\alpha \cap X = U_\alpha$.

Moreover, the family $\mathscr{V}$ is locally finite in P. It is clear that each $W \in \mathscr{W}$ intersects only finitely many of V_α (those for which U_α intersects $\mathrm{cl}(W)$). □

COROLLARY 2.3. *Let X be a closed subset of a hereditarily normal paracompact space P. Then, every open locally finite family $\mathscr{U}$ in X admits an open locally finite extension $\mathscr{V}$ in P so that $\mathscr{V}$ is similar to $\mathscr{U}$.*

Proof. Notice that $\mathscr{U}$ is locally finite in P, and apply Theorem 1. □

COROLLARY 2.4. *Let X be a subset of a hereditarily paracompact space P. Then, every open locally finite family $\mathscr{U} = \{U_\alpha \mid \alpha \in A\}$ in X admits an open extension $\mathscr{V}$ in P so that $\mathscr{V}$ is locally finite in $\bigcup \mathscr{V}$ and is similar to $\mathscr{U}$.*

Proof. There exists an open neighborhood V_0 of X in P so that the family $\mathscr{U}$ is locally finite in V_0. Now, we can apply Theorem 1 to X regarded as a subset of a hereditarily normal paracompact space V_0. □

3. Extending overlays

Let us recall that a covering projection $e : \tilde{X} \to X$ is a map such that every point $x \in X$ has an open neighborhood U such that $e^{-1}(U)$ is a disjoint union of open subsets $\tilde{U}^\alpha$ and, for each α, $e|\tilde{U}^\alpha$ is a homeomorphism of $\tilde{U}^\alpha$ onto U. We do not assume here that X is locally connected. The following modification of this notion is due to Fox [3].

Definition. Let $\tilde{X}$ and X be two Hausdorff spaces, $\mathscr{M} = \{M_\lambda\}_{\lambda\in\Lambda}$ be a collection of subsets of X, and $e : \tilde{X} \to X$ be a map. A collection $\tilde{\mathscr{M}} = \{\tilde{M}_\lambda^\alpha\}_{\lambda\in\Lambda}^{\alpha\in A_\lambda}$ of subsets of $\tilde{X}$ is said to **lie evenly** over the collection $\mathscr{M}$ if

(1) $e^{-1}(M_\lambda) = \bigcup\limits_{\alpha\in A_\lambda} \tilde{M}_\lambda^\alpha$ for each index $\lambda \in \Lambda$;

(2) for each $\alpha \in A_\lambda$, the set $\tilde{M}_\lambda^\alpha$ is open in $e^{-1}(M_\lambda)$;

(3) for each $\alpha \in A_\lambda$, the set $\tilde{M}_\lambda^\alpha$ is mapped by $e_\lambda^\alpha = e|\tilde{M}_\lambda^\alpha$ homeomorphically onto M_λ; and

(4) if $M_\lambda \cap M_{\lambda'} \neq \emptyset$ then, for each $\alpha \in A_\lambda$, the set $\tilde{M}_\lambda^\alpha$ meets exactly one of the sets $\tilde{M}_{\lambda'}^\beta$, $\beta \in A_{\lambda'}$, (in particular, we have that $\tilde{M}_\lambda^\alpha \cap \tilde{M}_\lambda^\beta = \emptyset$ whenever $\alpha \neq \beta$).

The map $e: \tilde{X} \longrightarrow X$ will be called an **overlay** if $\tilde{X}$ has an open cover $\tilde{\mathscr{M}}$ that lies evenly over some open cover $\mathscr{M}$ of X.

Notice that from the fact that $\tilde{\mathscr{M}}$ is an open cover and condition (3) it follows that e is continuous.

Let η_λ denote the cardinality of the family $\{\tilde{M}_\lambda^\alpha \mid \alpha \in A_\lambda\}$. Thus A_λ can be identified with the set of ordinals $\eta_\lambda = \{\alpha \mid 0 \leqslant \alpha < \eta_\lambda\}$. In general η_λ need not be the same as $\eta_{\lambda'}$, but of course they must be the same whenever M_λ and $M_{\lambda'}$ intersect. In such a case $\eta_\lambda = \eta_{\lambda'}$ and a permutation $\omega_{\lambda\lambda'}$ of η_λ is determined as follows:

$$\omega_{\lambda\lambda'}(\alpha) = \beta,$$

where $\tilde{M}_\lambda^\alpha \cap \tilde{M}_{\lambda'}^\beta \neq \emptyset$. Note that $\omega_{\lambda\lambda}$ is the identity permutation. Moreover, we have that $\omega_{\lambda'\lambda} = \omega_{\lambda\lambda'}^{-1}$, and $\omega_{\lambda\lambda'}\omega_{\lambda'\lambda''} = \omega_{\lambda\lambda''}$ whenever $M_\lambda \cap M_{\lambda'} \cap M_{\lambda''} \neq \emptyset$.

The following fact presumably belongs to mathematical folklore; we present a proof of it because we could not find one in the literature.

LEMMA 3.1. *Let $e: \tilde{X} \longrightarrow X$ be a covering projection. If X is a hereditarily paracompact space, then $\tilde{X}$ is a hereditary paracompact space.*

Proof. Let $\tilde{U}$ be an open subspace of $\tilde{X}$. We must show that $\tilde{U}$ is paracompact. Since $e: \tilde{X} \to X$ is a covering projection the set $e(\tilde{U}) = U$ is an open subset of X, and there is an open cover $\{M_\lambda\}$ of U so that $e^{-1}(M_\lambda)$ is the disjoint union of an open family $\{\tilde{M}_\lambda^\alpha\}$ of $\tilde{X}$ and $e|\tilde{M}_\lambda^\alpha : \tilde{M}_\lambda^\alpha \to M_\lambda$ is a homeomorphism. Since X is a hereditarily paracompact space we can find a closed locally finite cover $\mathscr{V}$ of U inscribed in the cover $\{M_\lambda\}$. For a given $V \in \mathscr{V}$, being a disjoint union of paracompact spaces, the space $U_V = e^{-1}(V) \cap \tilde{U}$ is paracompact. Now, since $\tilde{U}$ is covered by $\{U_V \mid V \in \mathscr{V}\}$, a locally finite closed family of paracompact spaces, it is paracompact (see [2, Thm. 5.1.34]). $\square$

If $\bar{e}: \tilde{U} \longrightarrow U$ is an overlay and $X \subset U$, then letting $\tilde{X} = \bar{e}^{-1}(X)$, the map $e = \bar{e}|\tilde{X}: \tilde{X} \longrightarrow X$ is also an overlay. We will say that e is a restriction of $\bar{e}$ and that $\bar{e}$ is an extension of e.

THEOREM 3.2. *If X is a subset of a hereditarily paracompact space P and $e: \tilde{X} \longrightarrow X$ is an overlay, then for some open neighborhood U of X in P there exists an overlay $\bar{e}: \tilde{U} \longrightarrow U$ that is an extension of e. Moreover, $\tilde{U}$ is hereditarily paracompact.*

Proof. We will follow the lines of the proof of [3, Theorem 5.2].

Since $e: \tilde{X} \longrightarrow X$ is an overlay there is an open covering $\{M_\lambda\}$ of X over which lies evenly an open covering $\{\tilde{M}_\lambda^\alpha\}$ of $\tilde{X}$. Since X is a paracompact space it may be assumed that $\{M_\lambda\}$ is locally finite in X. By Corollary 2.4, there exists an open family $\{V_\lambda\}$ in P which is similar to the cover $\{M_\lambda\}$ and such that $\{V_\lambda\}$ is locally finite in the union of $\{V_\lambda\}$.

Let $U = \bigcup\limits_\lambda V_\lambda$; this is an open neighborhood of X in P. For each $\lambda \in \Lambda$ and $\alpha \in \eta_\lambda$, let V_λ^α be a topological space homeomorphic to V_λ , and let r_λ^α be a homeomorphism of V_λ^α onto V_λ. In the discrete union V of all the spaces V_λ^α, $\alpha \in \eta_\lambda, \lambda \in \Lambda$, let us introduce the following equivalence relation:

(1) if $p \in V_\lambda^\alpha$ and $p' \in V_{\lambda'}^\beta$, then $p \approx p'$ if and only if $r_\lambda^\alpha(p) = r_{\lambda'}^\beta(p')$ and $\omega_{\lambda\lambda'}(\alpha) = \beta$.

The relation $\approx$ is symmetric and reflexive. The fact that, for any triple $\lambda, \lambda', \lambda''$, if $M_\lambda \cap M_{\lambda'} \cap M_{\lambda''} = \emptyset$ then $V_\lambda \cap V_{\lambda'} \cap V_{\lambda''} = \emptyset$ yields the transitivity of $\approx$. Hence the quotient space $\tilde{U} = V/\approx$ is well-defined. Let us denote by q the quotient map of V onto $\tilde{U}$ and by q_λ^α the restriction $q|V_\lambda^\alpha$ of q to V_λ^α. Clearly, q_λ^α maps V_λ^α homeomorphically onto a subset $\tilde{V}_\lambda^\alpha$ of $\tilde{U}$. Since $q^{-1}(\tilde{V}_\lambda^\alpha) = \bigcup\limits_{\lambda,\beta}(r_\lambda^\beta)^{-1}(V_\lambda)$, the set $\tilde{V}_\lambda^\alpha$ is open in $\tilde{U}$. We infer that $\tilde{U} = \bigcup\limits_{\lambda,\alpha} \tilde{V}_\lambda^\alpha$.

For any point $\tilde{p}$ of $\tilde{U}$ there is at least one point p of V for which $q(p) = \tilde{p}$, and if p and p' are two such points then $p \approx p'$. Hence, if we define $\bar{e}(\tilde{p})$ to be the image under an appropriate r_λ^α of a point of $q^{-1}(\tilde{p})$ the definition is unambiguous. Obviously $\bar{e}q_\lambda^\alpha = r_\lambda^\alpha$ for every λ, α. Since, by the definition, $\bar{e} : \tilde{U} \to U$ maps each $\tilde{V}_\lambda^\alpha$ topologically onto V_λ, it is easily seen that $\bar{e}$ is an overlay. We will show that $\bar{e}$ is an extension of e up to some homeomorphism of $\tilde{X}$.

Consider a point $\tilde{x}$ of $\tilde{X}$. It belongs to at least one of the set $\tilde{M}_\lambda^\alpha$. Therefore the point $x = e(\tilde{x})$ must belong to M_λ and hence to V_λ. Let p_λ^α denote the point of V_λ^α that is mapped by r_λ^α onto x, and let $\tilde{p} = q(p_\lambda^\alpha)$. Let us define $f(\tilde{x})$ to be the point $\tilde{p}$. This is unambiguous definition for if $\tilde{x} \in \tilde{M}_\lambda^\alpha \cap \tilde{M}_{\lambda'}^\beta$, then $x \in M_\lambda \cap M_{\lambda'}$ and $p_\lambda^\alpha \approx p_{\lambda'}^\beta$. Since $f|\tilde{M}_\lambda^\alpha$ is obviously a homeomorphism of $\tilde{M}_\lambda^\alpha$ onto a subset of $\tilde{V}_\lambda^\alpha$, and $\{M_\lambda\}$ is locally finite, it is easy to show that f is a homeomorphism onto $f(\tilde{X})$. Then, finally we note that

$$\bar{e}(f(\tilde{x})) = \bar{e}(\tilde{p}) = \bar{e}(q_\lambda^\alpha(p_\lambda^\alpha)) = r_\lambda^\alpha(p_\lambda^\alpha) = x = e(\tilde{x}).$$

It can be easily observed that $f(\tilde{X}) = \bar{e}^{-1}(X)$.

Since P is hereditarily paracompact space, the space U is also hereditarily paracompact. By Lemma 3.1, the space $\tilde{U}$ is hereditarily paracompact. $\square$

Remark. Let X be a closed subset of a hereditarily normal paracompact space P. Then, every overlay $e : \tilde{X} \to X$ admits an extension to an overlay $\bar{e} : \tilde{U} \to U$, where U is some open neighborhood of X in P. (To obtain this, replace Corollary 2.4 by Corollary 2.3 in the proof of Theorem 3.2.) $\square$

Our original project was to use Theorem 3.2 for an extension of the classical lifting property of covering projections to overlays $e : \tilde{X} \to X$. This was accomplished by Fox [3,4] for the case of metrizable X. Actually we are able to carry out Fox's program for some hereditarily paracompact spaces X. However, after having learned that Mrozik [7] (using a different approach) has recently obtained the lifting property for overlays over an arbitrary space X, we decided not to include our partial results on that subject herein.

REFERENCES

[1] E. Čech, *Théorie générale de la'homologie dans un espace quelconque*, Fund. Math. **19** (1932), 149–183.
[2] R. Engelking, *General Topology*, PWN Warszawa, 1985.
[3] R. H. Fox, *On shape*, Fund. Math. **74** (1972), 47–71.
[4] R. H. Fox, *Shape theory and covering spaces*, Lectures Notes in Math., 375, Springer-Verlag, Berlin, 1974, pp. 71–90.
[5] R. H. Fox, *Errata to the paper "On shape"*, Fund. Math. **75** (1972), 85.
[6] K. Kuratowski, *Topology*, I, PWN Warszawa, 1985.
[7] P. Mrozik, *Images and exactness in the category of pro-groups and the lifting problem for covering projections*, Glasnik Mat. **28(48)** (1993), 209–226.

(Received July 15, 1997)

Department of Mathematics
Pittsburg State University
Pittsburg KS 66762
USA
e-mail: tdobrowo@mail.pittstate.edu

Instituto de Matemáticas
Unidad Morelia
Nicolás Romero # 150 Col Centro
Morelia, Mich. Mexico
e-mail: rolando@gauss.matem.unam.mx

Faculty of Mathematics and Computer Science
Vrije Universiteit
De Boelelaan 1081 a, 1081 HV Amsterdam
The Netherlands
and University of Warsaw
Poland
e-mail: wmarcisz@cs.vu.nl