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MODEL OF AN OPEN MICROECONOMIC SYSTEM TAKING INTO ACCOUNT ITS OBJECTIVE FUNCTION

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ABSTRACT

An economic system is considered. The system consists of an active subsystem (a firm). There exists exchange between the firm and two economic reservoirs (markets) which are in environment of the system. Two cases are investigated; they are stationary and nonstationary regimes of the system functioning. The parameters of the market are constant in the stationary regime. Parameters of the market in the nonstationary regime are random; their distributions are known. Expressions for maximal profit and maximal profitability at given profit are obtained. Additional problem is considered for nonstationary regime. It is minimization of risks problem. Optimality conditions for this problem are obtained.

KEY WORDS

microeconomic system, market, stationary and nonstationary regimes, profit

CLASSIFICATION

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INTRODUCTION

In neoclassical microeconomics it is assumed that the only objective function of a firm as an economic system is its profit [1]. But really there are a lot of other objective functions of the firm. They account regime of the firm activity, its financial conditions, etc. One of the common objective functions is profitability. It is determined as a ratio of profit and production costs. It characterizes economic efficiency of the production process. Profitability, like a profit, depends on intensities of fluxes in the economic system. That is why it can be used in stationary regime only. If parameters of the economic system and its environment are not constants (they can be determined or random functions of time), then there is a risk of decrement of either profit or profitability. This risk should be accounted during the investigation of regimes of the firm activity too. So, in the nonstationary case amount of objective functions increases: one should calculate parameters of distributions of both profit and profitability, and parameters of the risk. Another increment of the number of the objective functions appears if the economic system includes several firms. Each firm can account objective functions of competitors to plan its competitive activity.

It is possible to find the optimal solution of the problem of optimization of the firm regime in a class of Pareto sets. The point of the set can be chosen arbitrary according to weight of each objective function.

MODEL DESCRIPTION

Let us consider a firm as an open economic system [2]. The firm exchangers a resource (a good) and a base resource (money) with its environment. The environment is two economic reservoirs (markets) with constant values of the resource v_1, v_2 ($v_2 > v_1$). The firm has got economic power, so it can control prices of the good p_1, p_2 while trading with the markets. Intensity g_i of trading depends on a function of the price p_i and the value v_i. Let that function be linear and let us introduce specifically

$$g_1 = \alpha_1(p_1 - v_1), \qquad g_2 = \alpha_2(p_2 - v_2), \tag{1}$$

where g_1 is a suply function, and g_2 a demand function.

Money fluxes are oppositely direct to fluxes of the good. Their intensities are equal to $p_1 g_1$ and $p_2 g_2$. We assume that production function of the firm is $g_2 = g_1$. In this case we can consider input production factor and yield product as the same good. Then $p_1 g_1$ are production costs of the firm and $p_2 g_2$ are receipts.

Equations of material and financial balances can be written as follows:

$$g_1 = g_2 = g\,, \Rightarrow \alpha_1(p_1 - v_1) = \alpha_2(p_2 - v_2), \tag{2}$$

$$\pi = p_2 g_2 - p_1 g_1, \Rightarrow \pi = \alpha_1(p_2 - p_1)(p_1 - v_1). \tag{3}$$

Here π is the firm's profit.

STATIONARY REGIME

Stationary regime corresponds to constant parameters of the system and its environment. Particularly, v_1 and v_2 are constants. Let as consider two problems. In the first problem the profit is the only objective function of the firm. The second problem is to find maximal profitability subject to the given profit.

CONDITIONS OF MAXIMUM OF THE FIRM'S PROFIT

Let us consider a problem of prices determination to maximize the profit of the firm:

$$\pi(p_1, p_2) \to \max_{p_1, p_2} . \tag{4}$$

Restriction (2) should be accounted for in solving the problem (4). This restriction means that there is no warehouse in the system. In our assumption of linear demand and supply functions it is convenient to express one of the prices through another and reduce the problem to unconditional optimization problem:

$$p_2 = v_2 + \frac{\alpha_1}{\alpha_2}(v_1 - p_1) \Rightarrow \pi(p_1) = \frac{\alpha_1}{\alpha_2}\left[\alpha_1 v_1 + \alpha_2 p_2 - (\alpha_1 + \alpha_2)p_1\right](p_1 - v_1) \to \max_{p_1} . \tag{5}$$

Let us denote

$$\beta = \frac{\alpha_2}{\alpha_1(\alpha_1 + \alpha_2)}, \; p_0 = \frac{\alpha_1 v_1 + \alpha_2 v_2}{\alpha_1 + \alpha_2} . \tag{6}$$

Value p_0 is a price corresponding to a case when the economic reservoirs with the same demand and supply functions exchange the good in the absence of the firm. The same price corresponds the profitless regime of the firm activity if $p_1 = p_2 = p_0$. Generally $p_1 < p_0 < p_2$ [2]. Using notation (6) we can rewrite problem (5):

$$\pi = \frac{1}{\beta}(p_0 - p_1)(p_1 - v_1) \to \max_{p_1}, \tag{7}$$

and the necessary condition of optimality $\mathrm{d}\pi/\mathrm{d}p_1 = 0$ leads to the following solution:

$$\begin{cases} p_1^* = \dfrac{v_1 + p_0}{2}, \quad p_2^* = \dfrac{v_2 + p_0}{2}, \\ \pi^* = \dfrac{1}{\beta}\dfrac{(p_0 - v_1)^2}{4} = \dfrac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\dfrac{(v_2 - v_1)^2}{4} . \end{cases} \tag{8}$$

This result is known [3]. Profitability corresponding to the maximal profit is determined by the following expressions

$$\rho^* = \frac{\pi^*}{p_1^* g(p_1^*)} = \frac{p_2^* - p_1^*}{p_1^*}\frac{(v_2 - v_1)}{p_0 + v_1} = \frac{\alpha_1 + \alpha_2}{\alpha_2}\frac{p_0 - v_1}{p_0 + v_1} = \frac{v_2 - v_1}{\dfrac{\alpha_2(v_2 - v_1)}{\alpha_1 + \alpha_2} + 2v_1} . \tag{9}$$

It is a monotonously increasing function of v_1 and difference $v_2 - v_1$, such that:

$$\lim_{(v_2 - v_1) \to \infty} \rho^* = \frac{\alpha_1 + \alpha_2}{\alpha_2} > 1 . \tag{10}$$

DEPENDENCE OF THE PROFITABILITY ON THE PROFIT

Let us consider the dependency of the maximal value of profitability on the given value of profit $\pi^0 \in [0, \pi^*]$. To find this dependency we should solve the problem

$$\rho(p_1, p_2) \to \max_{p_1, p_2} \left\{ \begin{matrix} \pi(p_1, p_2) = \pi^0, \\ g_1(p_1) = g_2(p_2) \end{matrix} \right\} . \tag{11}$$

Here the number of controls equals to number of restrictions. It means that the degree of freedom for problem (11) equals zero: we can obtain a countable set of solution suspicious to be optimal using a set of restrictions only. Really, financial balance of the firm can be written taking into account equation (2):

$$(p_0 - p_1)(p_1 - v_1) - \beta\pi^0 = 0. \tag{12}$$

Using this equation we can express $p_1(\pi^0)$:

$$p_1(\pi^0) = p_1^* - \sqrt{p_1^2 - (p_0 v_1 + \beta\pi^0)}. \tag{13}$$

Here p_1^* is the solution (8). Minus sign in equation (13) corresponds to inequality $p_1 < p_1^*$. This solution, out of two solutions of the quadratic equation, leads to the profitability maximum. Thus we got two possible solutions, with one of them being the optimal one. Let us express π^0 from (12) and substitute it into the expression of profitability $\rho = \pi^0/(p_1 g)$to obtain

$$\rho = \frac{(p_0 - p_1)(p_1 - v_1)}{\beta\alpha_1 p_1 (p_1 - v_1)} = \frac{\alpha_1 + \alpha_2}{\alpha_2}\left(\frac{p_0}{p_1} - 1\right). \tag{14}$$

Finally, substitution of (13) into (14) brings about the required dependency

$$\rho(\pi^0) = \frac{\alpha_1 + \alpha_2}{\alpha_2}\left[\frac{p_0}{p_1^* - \sqrt{p_1^{*2} - (p_0 v_1 + \beta\pi^0)}} - 1\right]. \tag{15}$$

Taking into account equations (8), the(15) is rewritten as

$$\rho(\pi^0) = \frac{\alpha_1 + \alpha_2}{\alpha_2}\left[\frac{p_0}{p_1^* - \sqrt{\beta(\pi^* - \pi^0)}} - 1\right].$$

It is evident that if $\pi^0 = \pi^*$ the dependency $\rho(\pi^0)$ coincides with (9), while for $\pi^0 = 0$ (i.e. $p_1 = v_1$) one has

$$\rho(0) = \frac{\alpha_1 + \alpha_2}{\alpha_2}\left(\frac{p_0}{v_1} - 1\right) > \rho(\pi^*). \tag{16}$$

The obtained dependency $\rho(\pi^0)$ is a monotonously decreasing function of π^0 if $\pi^0 \in [0, \pi^*\rangle$. This dependency in form (15) determines Pareto set of solutions for the following problem:

$$\rho(p_1, p_2) \to \max_{p_1, p_2}, \qquad \pi(p_1, p_2) \to \max_{p_1, p_2}.$$

subjected to (2).

NONSTATIONARY REGIME

Let us consider a nonstationary regime. It occurs when some parameters of the system are not constant. We choose value v_1 to be either determined or random function of time with distribution given by time-independent density $f(v_1)$. Note that mathematical estimation $\bar{v}_1$ and variation σ_1^2 are known because $f(v_1)$ is given.

Three problems are formulated in the nonstationary regime:

- to determine the maximal value of the averaged (the mathematical expectation of) profit of the firm, $\bar{\pi}$,
- to determine the maximal value of the averaged (the mathematical expectation of) profitability, $\bar{\rho}$, as a function of the averaged profit,
- to determine the minimal value of a risk decreasing of the profit below the given value (e.g. zero level) at each moment of time: $P(\pi(t) < 0) \to \min$.

Controls in all three problems are prices of the good. Prices should be constant value, not functions of time. These problems are common for real economic activity of the firm, because forecast of the markets parameters cannot be exact especially in the case when markets are distributed. In a case of distributed markets the last problem corresponds to a problem of determination of the minimal part of territory where profit is less than a given (planned) value.

CONDITIONS OF THE MAXIMUM OF THE AVERAGED PROFIT

Since conditions (2) and (3) should be fulfilled for any value of v_1, then the expression

$$\pi(v_1, p_1) = \frac{1}{\beta}\left[p_0(v_1) - p_1\right](p_1 - v_1) \tag{17}$$

is dependency of the profit of the firm on parameters v_1 and p_1. Here v_1 is an independent parameter, and p_1 is a control. So, the problem on maximization of the averaged profit can be written as

$$\bar{\pi}(p_1) = \frac{1}{\beta}\left[-p_1^2 + p_1\overline{(p_0(v_1) + v_1)} - \overline{p_0(v_1)v_1}\right] \to \max_{p_1}. \tag{18}$$

In (18) the bar means averaging over set V of possible values of v_1. For example,

$$\bar{\pi}(p_1) = \int_V \pi(v_1, p_1) f(v_1) \mathrm{d}v_1 \to \max_{p_1}. \tag{19}$$

The necessary condition of optimality $\mathrm{d}\bar{\pi}/\mathrm{d}p_1 = 0$ leads to equation

$$p_1^* = \frac{1}{2}\overline{(p_0(v_1) + v_1)}.$$

Note that $p_0(v_1)$ is a linear dependence. That is why solution of the problem (18) is

$$p_1^* = \frac{p_0(\bar{v}_1) + \bar{v}_1}{2} \Rightarrow p_2^* = \frac{p_0(\bar{v}_1) + \bar{v}_2}{2}. \tag{20}$$

But in this case

$$\bar{\pi}^* \neq \frac{1}{\beta}\frac{\left[p_0(\bar{v}_1) - \bar{v}_1\right]^2}{4}. \tag{21}$$

Indeed, if we write expression for the averaged profit and rearrange it:

$$\bar{\pi}^* = \frac{1}{\beta}\left[-\left(\frac{p_0(\bar{v}_1) + \bar{v}_1}{2}\right)^2 + \frac{p_0(\bar{v}_1) + \bar{v}_1}{2}\left(p_0(\bar{v}_1) + \bar{v}_1\right) - \overline{p_0(v_1)v_1}\right],$$

$$\bar{\pi}^* = \frac{1}{\beta}\left[\left(\frac{p_0(\bar{v}_1) + \bar{v}_1}{2}\right)^2 - \overline{p_0(v_1)v_1}\right]. \tag{22}$$

Then we obtain the following expression for the averaged profit:

$$\overline{\pi}^* = \frac{1}{\beta}\left[\frac{\left(p_0(\overline{v_1}) + \overline{v_1}\right)^2}{4} - \frac{\alpha_1}{\alpha_1 + \alpha_2}\sigma_1^2\right]. \tag{23}$$

Here $\sigma_1^2 = \overline{v_1^2} - \overline{v_1}^2$ is a variation of the value v_1. The variation is nonnegative always. That is why the inequality (21) is true.

DEPENDENCY OF THE AVERAGED PROFITABILITY ON THE AVERAGED PROFIT

The problem on maximization of the averaged profitability subject to the given averaged profit has zero degree of freedom as in the stationary case. Equations (2) and (3) can be used to obtain dependency $\overline{\rho}(\overline{\pi}^0)$ (here $\overline{\pi}^0$ is the given averaged profit of the firm) accounting characteristics of averaging operation. Solution of the equation

$$p_1^2 - p_1(p_0(\bar{v}_1) + \bar{v}_1) + (\overline{p_0(v_1)v_1} + \bar{\pi}^0\beta) = 0,$$

allows us to obtain the dependency of $p_1(\bar{\pi}^0)$:

$$p_1(\bar{\pi}^0) = p_1^* - \sqrt{p_1^{*2} - (\overline{p_0 v_1} + \beta\bar{\pi}^0)}, \tag{24}$$

where p_1^* is determined by (20). Value of p_1 is determined by value of the averaged profit; the profitability $\rho(v_1, \bar{\pi}^0)$,

$$\rho(v_1, \bar{\pi}^0) = \frac{\pi(v_1, p_1(\bar{\pi}^0))}{p_1(\bar{\pi}^0)g(p_1(\bar{\pi}^0))} = \frac{(p_0(v_1) - p_1(\bar{\pi}^0))(p_1(\bar{\pi}^0) - v_1)}{\beta\alpha_1 p_1(\bar{\pi}^0)(p_1(\bar{\pi}^0) - v_1)} =$$

$$= \frac{\alpha_1 + \alpha_2}{\alpha_2}\left(\frac{p_0(v_1)}{p_1(\bar{\pi}^0)} - 1\right), \tag{25}$$

linearly depends on v1. That is why the averaged profitability equals

$$\rho(\bar{\pi}^0) = \frac{\alpha_1 + \alpha_2}{\alpha_2}\left(\frac{p_0(\bar{v}_1)}{p_1(\bar{\pi}^0)} - 1\right) = \frac{\alpha_1 + \alpha_2}{\alpha_2}\left(\frac{p_0(\bar{v}_1)}{p_1^* - \sqrt{p_1^{*2} - (\overline{p_0(v_1)v_1} + \beta\bar{\pi}^0)}} - 1\right). \tag{26}$$

Taking into account equation (22) this dependency can be rewritten in the form analogues to the stationary case:

$$\rho(\bar{\pi}^0) = \frac{\alpha_1 + \alpha_2}{\alpha_2}\left(\frac{p_0(\bar{v}_1)}{p_1^* - \sqrt{\beta(\pi^* - \bar{\pi}^0)}} - 1\right). \tag{27}$$

The following resume is appropriate here:

- the averaged profitability of the firm corresponding to the maximal averaged profit is determined by the expression (9) in both stationary and nonstationary cases; but the maximal averaged profit in nonstationary case is less than the maximal stationary profit,
- if the averaged profit of the firm tends to zero then the more the variation is the less the averaged profitability is.

These two statements can be represented as follows:

$$\min\overline{\rho}(\bar{v}_1, \sigma_1) = \min\rho(\bar{v}_1), \quad \max\overline{\rho}(\bar{v}_1, \sigma_1) < \max\rho(\bar{v}_1).$$

THE CONDITION OF THE MINIMAL RISKS

The next problem is to minimize the risk of negative profit:

$$P(\pi(v_1, p_1) < 0) \to \min_{p_1}, \tag{28}$$

where $P(X)$ is probability of a random event X. Here, random event is parameterized by value v_1. Since density of distribution $f(v_1)$ is given, we can find the probability $P(\pi(v_1, p_1) < 0)$ as follows

$$P(\pi(v_1, p_1) < 0) = 1 - \int_{V_+} f(v_1)\mathrm{d}v_1,, \tag{29}$$

where $V_+(p_1)$ is set of values v_1 corresponding to the condition $\pi(v_1, p_1) \geq 0$. To determine this set we will use the equation of the financial balance of the firm:

$$\beta\pi = (p_0(v_1) - p_1)(p_1 - v_1)., \tag{30}$$

That dependency is a parabola. That is why the set V_+ is a segment with boundaries $\breve{v}_1$, $\widehat{v}_2$ corresponding to zero profit:

$$\breve{v}_1 = \frac{\alpha_1 + \alpha_2}{\alpha_1} p_1 - \frac{\alpha_2}{\alpha_1} v_2 < \widehat{v}_1 = p_1. \tag{31}$$

Necessary condition of optimality for the problem (28) leads to the following inequality

$$\left[f(p_1) - \frac{\alpha_1 + \alpha_2}{\alpha_1} f\left(\frac{\alpha_1 + \alpha_2}{\alpha_1} p_1 - \frac{\alpha_2}{\alpha_1} v_2 \right) \right] \delta p_1 \leq 0, \tag{32}$$

where δp_1 is a possible sweep of the price p_1. Solution of this inequality depends on a law of distribution of v_1.

MODEL OF COMPETITIVE MARKET

Another possibility to revise the objective function of the firm is a problem on optimal regime of the firm at a competitive market.

Competition means a possibility to change conditions of operation for other firms trading with the same market. Monopoly where there is no other firm and the perfect competition where there is no possibility to influence on other firms activity are not real competitive markets.

Competition has a science between these extreme types of markets. Let us consider a market in midrange namely duopoly. Even in this case the firm can be indifferent to a competitor. Such an indifferent firm can be described by the mentioned objective functions: profit and profitability. To account the special features of the competitive markets we should introduce a new objective function including characteristics of the competitor.

Let us consider a market buying the good as nonhomogeneous economic agent. It is divided into two parts x_{I} and x_{II} . This distribution is a result of competition: sellers of the part x_{I} buy the good made by the firm I and sellers of the part x_{II} buy the good produced by the firm II. A new balance equation for the market described this distribution:

$$\dot{x}_{\mathrm{I}} = \kappa(p_{2,\mathrm{II}} x_{\mathrm{II}} - p_{2,\mathrm{I}} x_{\mathrm{I}}). \tag{33}$$

In the stationary case $p_{2,\mathrm{II}} x_{\mathrm{II}} = p_{2,\mathrm{I}} x_{\mathrm{I}}$. It corresponds to the following parts of the market trading with the firms:

$$x_{\mathrm{I}} = \frac{p_{2,\mathrm{II}}}{p_{2,\mathrm{I}} + p_{2,\mathrm{II}}}, \quad x_{\mathrm{II}} = \frac{p_{2,\mathrm{I}}}{p_{2,\mathrm{I}} + p_{2,\mathrm{II}}}. \tag{34}$$

The demand function depends also on x_{I} and x_{II}:

$$g_{2\nu} = \alpha_{2\nu} x_\nu (v_2 - p_{2\nu}), \quad \nu \in \{\mathrm{I}, \mathrm{II}\}.$$

The objective function of the firm should account both results of the firm (e.g. I) and its competitor (II):

$$f_{\mathrm{I}} = \pi_{\mathrm{I}}(p_{2,\mathrm{I}} p_{2,\mathrm{II}}) + k\pi_{\mathrm{II}}(p_{2,\mathrm{I}} p_{2,\mathrm{II}}) \to \min_{p_{2,\mathrm{I}}, p_{2,\mathrm{II}}}, \tag{35}$$

The necessary condition of optimality in additional assumptions $\alpha_{i,\nu}$ = const., production costs c_ν = const. (i $\in$ {1, 2}, $\nu \in$ {I, II}) leads to the following if optimality leads to the following expression for $p_{2,\mathrm{I}}$:

$$p_{2,\mathrm{I}} = p_{2,\mathrm{II}}\left[\sqrt{1+\frac{p_{2,\mathrm{II}}(v_2+c_\mathrm{I})+k(p_{2,\mathrm{II}}-c_\mathrm{II})(v_2-p_{2,\mathrm{II}})+c_\mathrm{I}v_2}{p_{2,\mathrm{II}}^2}}-1\right]. \qquad (36)$$

Various values of $k < 0$ correspond to various types of competition: the larger the $|k|$, the more active influence the firm should provide to the competitor.

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MODEL OTVORENOG MIKROEKONOMSKOG SUSTAVA KOJI UZIMA U OBZIR NJEGOVU FUNKCIJU CILJA

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SAŽETAK

Razmatran je ekonomski sustav. Sustav se sastoji od aktivnog podsustava, tvrtke. Postoji izmjena razmjena između tvrtke i dva ekonomska rezervoara, tržišta, koji su u okolini sustava. Dva su slučaja istražena; stacionarni, odnosno nestacionarni režim funkcioniranja sustava. Parametri tržišta su konstantni u stacionarnom režimu. Parametri tržišta su nasumični u nestacionarnom režimu, ali je njihova raspodjela poznata. Izvedeni su izrazi za maksimalnu dobit i maksimalnu isplativost pri danoj dobiti. Dodatno je za nestacionarni režim razmatran problem minimiziranja rizika. Dobiveni su uvjeti optimalnosti navedenog problema.

KLJUČNE RIJEČI

mikroekonomski sustav, tržište, stacionarni i nestacionarni režim, dobit