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A SIMPLIFIED PROCEDURE FOR APPROXIMATE DETERMINATION OF ELECTRO-GEOMETRICAL PARAMETERS OF TWO-LAYER SOIL

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Original scientific paper

Determination of electro-geometrical parameters of two-layered soil represents a demanding task, which has been addressed by various techniques over the history. Development of personal computers (PCs) and the reduction of their price, provide application of numerical techniques and procedures for determining those parameters instead of the earlier graphic techniques. One such numerical procedure is described in detail in the International Standard IEEE STD 81-1983. According to the numerical procedure described in the mentioned standard, the parameters of two-layered soil can be determined with high accuracy. However, in some cases when high accuracy in determination of the parameters of two-layered soil is not required, simple algorithms and procedures which are well suited for use in mathematical and engineering simulation software packages of general purpose like Mathematica, MathCAD or MATLAB can be very acceptable, too. In this article one such numerical algorithm is described in detail. The presented numerical procedure is very simple, appropriate for use in general-purpose mathematical and engineering simulation software and is easily applicable in estimating electro-geometrical parameters of two-layered soil.

Keywords: *electro-geometrical soil parameters, numerical procedure, two-layer soil, Wenner electrode arrangement*

Pojednostavnjen postupak približnog određivanja elektro-geometrijskih parametara dvoslojnog tla

Izvorni znanstveni članak

Određivanje elektrogeometrijskih parametara dvoslojnog tla predstavlja zahtjevnu zadaću koja se povijesno gledano rješavala raznim tehnikama. Razvoj osobnih računala (PC) te pad njihove cijene, omogućavaju primjenu numeričkih tehnika i postupaka za određivanje tih parametara umjesto dotadašnjih grafičkih tehnika. Jedan takav numerički postupak pobliže je opisan u međunarodnoj normi IEEE Std. 81-1983. Sukladno navedenom numeričkom postupku opisanom u navedenoj normi, parametri dvoslojnog tla mogu biti određeni s visokom točnošću. Međutim, u nekim slučajevima kada se ne zahtijeva visoka točnost u određivanju parametara dvoslojnog tla, mogu jednako tako biti veoma prihvatljivi i jednostavni algoritmi i postupci, koji su jako prikladni za korištenje u matematičkim i inženjerskim simulacijskim programskim paketima opće namjene poput Mathematica, MathCAD ili MATLAB. Jedan takav numerički postupak pobliže je opisan u ovom članku. Prikazan numerički postupak je vrlo jednostavan, prikladan za korištenje u matematičkim i inženjerskim simulacijskim programima opće namjene te ga je lako koristi za procjenu elektro-geometrijskih parametara dvoslojnog tla.

Ključne riječi: *dvoslojno tlo, elektro-geometrijski parametri tla, numerički postupak, Wennerov raspored elektroda*

1 Introduction

Soil resistivity measurement is an unavoidable procedure in collecting data related to the design and construction of a grounding system [1, 2, 3] or in the graphic diagnostics of soil structure and soil composition [4, 5, 6]. Accuracy in determination of the soil composition and soil structure on the basis of measured data is very important, since it has a direct impact on the accuracy of the grounding parameters calculation and the cost of grounding. Often, a mapping of the soil composition and soil structure is required in some geographical areas. The data thus collected serve as guidelines in the assessment of investment costs related to the creation of grounding systems in these areas. In addition, such data can be useful to the geologists and the farmers, since a change of soil resistivity is in relation to the changes in the chemical soil composition and soil humidity [7, 8]. Thus for purposes of preliminary research, there is no need for high accuracy in determination of the soil electro-geometrical parameters in a particular area. Far more adequate for such purposes are faster measurement techniques, simpler algorithms and mathematical instruments for processing collected data. An algorithm appropriate for such a purpose must be simple, understandable to a wider range of engineers, appropriate for use in very widespread and affordable software packages for mathematics, and general-purpose engineering simulator programs like: Mathematica, MathCAD or the very popular MATLAB [9÷11]. One such numerical procedure is described in detail in this article. The procedure is simple and applicable in the mentioned programs for estimating electro-geometrical parameters of two-layered soil. For a better understanding of this procedure, mathematical expressions of all the relevant values are presented in detail, and its application is presented using the numerical example in which obtained results are presented analytically and graphically, and discussed.

2 Soil resistivity measurement

A number of measurement techniques for soil resistivity have been described in detail in the ANSI/the IEEE STD 81 -1983 [12]. This standard provides suitable methods for determination of the upper soil layer resistivity and lower soil layer resistivity for the two-layer soil model, as well as the thickness of the upper layer. Several electricity techniques prevail in the soil resistivity measurements [4, 5, 6, 12, 13]: Wenner, Schlumberger, Lee, Dipole-Dipole. These techniques differ from each other in the arrangement of measuring electrodes (Fig. 1). Each of these measurement techniques has its advantages and disadvantages. These mainly concern the speed at which the measurement is performed, i.e. the necessary movement of the electrodes, the required sensitivity of the voltmeter, sensitivity to external interference, etc. [14÷17]. For example during the soil resistivity measurement, the Schlumberger measurement technique requires movement of the outer (current) electrodes only, and that is an advantage in comparison with the Wenner measurement technique, where all four measurement electrodes must be moved.

Thus, Schlumberger's measurement technique represents faster measurement technique. However, due to

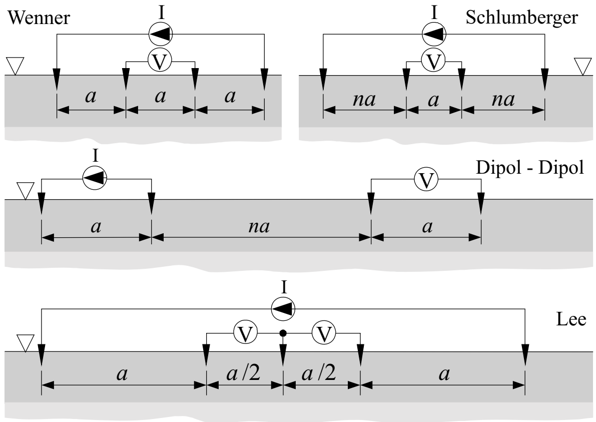

Figure 1 The Wenner-, Schlumberger-, Dipol-dipol. and Lee- electrode arrangements for the soil resistivity measurement

the larger distance between the current and voltage electrodes this technique requires a voltmeter of greater sensitivity, and higher measurement currents. Nevertheless, Wenner method is the most recommended and the most frequently used technique for measurement of the soil resistivity [12, 13]. It involves a symmetric configuration of four equally spaced electrodes (Fig. 2). A simple interpretation of measurement results is the reason for wide-spread utilization of this technique.

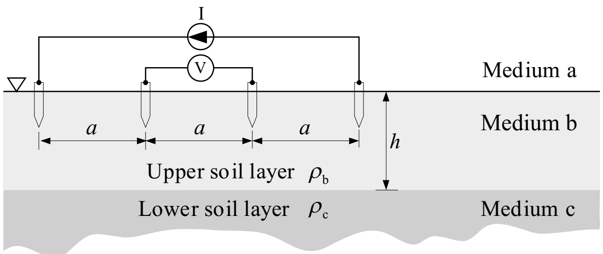

Figure 2 Model of the double layer soil and the Wenner electrode arrangement

The specified measurement technique is very appropriate for determination of the parameters for two-layered soil. The measurement procedure for this purpose is carried out with a special battery-powered measuring instrument. The measuring instrument consists of an alternating current source (ACS) with a frequency which must be different from system frequency, i.e. 50/60 Hz. Moreover, this frequency does not coincide with the frequency of possible system harmonic components. It also includes: a filter which separates the alternating measured voltage with the same frequency as the ACS frequency and attenuates other frequencies, high impedance voltmeter and ammeter. Although the influence of the electricity disturbances during soil resistivity measurement is often negligible, in certain circumstances they can completely prevent the measurement and thus the followed interpretation of measurement data [17], too.

The apparent resistance of soil $\rho(a)$ expressed by measured voltage U_V (V), measured current I (A) and the geometric coefficient K (m) is by definition [12]:

$$\rho(a) = \frac{U_V}{I}\frac{1}{K}. \qquad \rho(a) = \rho_{soil} = \rho_b \tag{1}$$

When the soil is single-layered, the apparent resistivity $\rho(a)$ does not depend on the variable a, and is equal to soil resistivity, i.e. can be written as: $\rho(a) = \rho_{soil} = \rho_b$. Based on a theoretical model and the objective assumption that low-frequency current field during this measurement is identical in form and amount to the current field that would be obtained if the alternate source (current electrode) were replaced by the DC source, the expression for the geometric factor of the Wenner electrode configuration can be obtained, as follows:

$$\frac{1}{K} = 2\pi a. \tag{2}$$

The geometric coefficient (2) of this arrangement enables one to determine the apparent soil resistivity when the soil is layered. By inserting the term (2) in the expression (1) the expression for the soil resistivity is obtained using the Wenner electrode configuration [12]:

$$\rho(a) = 2\pi a \frac{U_V}{I}. \tag{3}$$

3 Determination of apparent soil resistivity

Analysis of data obtained by measuring of the analyzed soil area is based on their comparison with a theoretical model of two-layer soil.

Methods for interpretation of measurement results can be divided into the following groups:

- Approximate methods
- Direct methods
- Iterative methods, [12]
- Combined direct-iterative methods, [14]
- Method of artificial intelligence, [18].

Until now the most widely used numerical procedure for determining parameters of two-layer soil is described in [12]. According to the IEEE Std. 81-1983, the apparent soil resistivity is described by the term:

$$\rho(a) = \rho_b \left[1 + 4\sum_{n=1}^{N} \left(\frac{\beta^n}{\sqrt{1+\left(\frac{2nh}{a}\right)^2}} - \frac{\beta^n}{\sqrt{4+\left(\frac{2nh}{a}\right)^2}} \right) \right], \tag{4}$$

in which the reflection factor β is expressed by:

$$\beta = \frac{\rho_c - \rho_b}{\rho_c + \rho_b}. \tag{5}$$

Where:
ρ_c – upper soil layer resistivity, Ω·m
ρ_b – lower soil layer resistivity, Ω·m
β – reflection factor, –
h – thickness of the upper soil layer, m
N – number of images in the respective multiple reflections.

Readers can find in [16] a much more detailed procedure for derivation of the expression (4) than that described in the International Standard [12].

According to the International Standard IEEE Std. 81-1983 [12], let $\psi(\rho_b, \beta, h)$ be an error function given by expression (6). This function is defined as the relative deviation between the apparent soil resistivity value ρ_m^0 as measured by the Wenner method and the calculated soil resistivity value ρ_m assuming that soil is a two-layer configuration. A ρ_m is given by (4) and (5). Both ρ_m^0 and ρ_m are functions of the electrode spacing a.

$$\psi(\rho_b, \beta, h) = \sum_{m=1}^{M} \left(\frac{\rho_m^0 - \rho_m}{\rho_m^0} \right)^2 . \qquad (6)$$

Where
M – total number of measured soil resistivity values with probe spacing a, as the parameter.

In order to obtain the best fit $\psi(\rho_b, \beta, h)$ must be minimal. Thus, the task is to determine such values of ρ_b, β and h to minimize the error.

The procedure of minimizing the error function (6) has been described in detail in [12]. For its implementation it is necessary to be familiar with numerical mathematics and programming in some of the newer programming languages. In this paper is presented a **simplified equivalent procedure,** instead of numerical procedure defined in [12]. Due to its simplicity, this procedure is suitable for the broader engineering population. The methodology which is the base of this simplified numerical procedure for determining the parameters of two-layer soil is described below. Since this is the first time that this procedure is presented, all math derivations i.e. mathematical statements including all derivation steps and used assumptions will be presented in detail as follows.

4 Interpretation of measurement data

The expression (4), which describes the theoretical curve of the apparent soil resistivity gives a smooth curve for the known electro-geometrical parameters of the soil: ρ_b, ρ_c and h. Although the expression (4) is derived under the assumption that the measuring electrodes are "sufficiently" spaced out [16], and that they can be replaced with adequate spherical electrodes, research in [16] has shown that in most applications the expression (4) provides a very good description of two-layer soil. The curves of apparent soil resistivity obtained by equation (4) have a characteristic form, as shown in Figs. 3a, 3b, 4a and 4b.

Figs. 3a and 3b present families of curves of apparent soil resistivity for different thicknesses of the upper layer in two-layer soil model, while the upper and lower soil layer resistivity are constant, and hence the reflection factor (5) is constant, too. Fig. 3a refers to the case when the upper soil layer resistivity is greater than the lower soil layer resistivity ($\rho_b > \rho_c$). Fig. 3b refers to the case when the upper soil layer resistivity is less than the lower soil layer resistivity ($\rho_b < \rho_c$). In Figs. 3a and 3b, the curves of the apparent soil resistivity h_1 and h_3 refer to the cases when the thickness of the upper soil layers is minimum and maximum, respectively. Figs. 3a and 3b indicate that the family of curves of apparent soil resistivity have common horizontal asymptotes, whose values are determined by the upper and lower soil layer values.

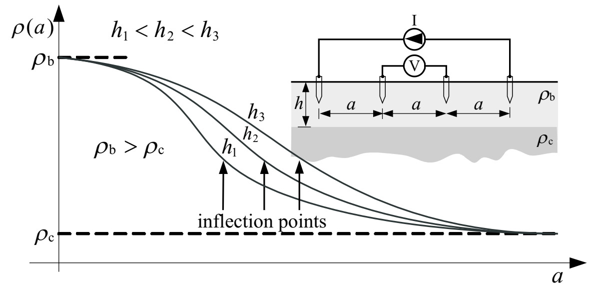

Figure 3a The curves of soil apparent resistivity at $\rho_b > \rho_c$ and the inflection point shift due to changes in the thickness of the upper soil layer

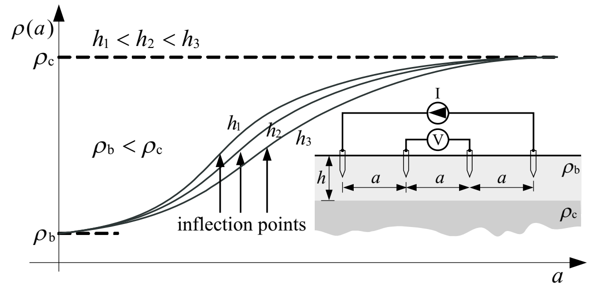

Figure 3b The curves of soil apparent resistivity at $\rho_b < \rho_c$ and the inflection point shift due to changes in the thickness of the upper soil layer

As for their form, the curves are sigmoid increasing or decreasing functions (logistic function) [19], which in our case, i.e. with constant electrical parameters of soil, have a point of inflection that depends on the geometrical parameters of the soil, i.e. the thickness of the upper soil layer. This indicates that the inflection point is a unique distinguishing feature of the presented family of curves. Also, a shift of the inflection point on the curves of apparent soil resistivity can be observed at a constant thickness of the upper soil layer, under a change of the electrical parameters of soil, i.e. the upper and lower soil layer resistivity, which results in a variable reflection factor, as shown in Figs. 4a and 4b.

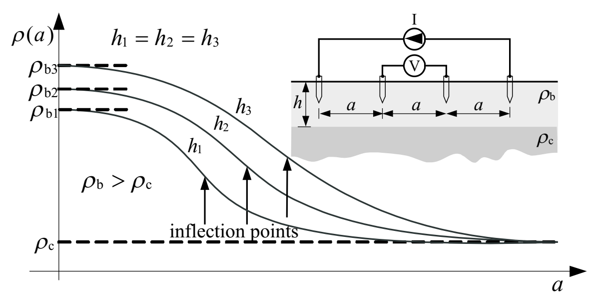

Figure 4a Curves of apparent resistivity of soil at $\rho_b > \rho_c$ and and the inflection point shift due to changes of the reflection factor

Fig. 4a refers to the case where the upper soil layer resistivity is greater than the lower soil layer resistivity i.e. $\rho_b > \rho_c$. For easier analysis and without loss of generality it is taken that the lower soil layer resistivity is constant (ρ_c = const.) and the upper soil layer resistivity is changeable.

Fig. 4b refers to the case where the upper soil layer resistivity ρ_b is lower than the upper soil layer resistivity ρ_c. In Fig. 4b, it was taken that the upper soil layer resistivity is constant ρ_b = const., while the lower soil layer resistivity is variable. The upper and lower horizontal asymptotes are determined by the values of the upper and lower soil layer resistivity. Shifts of the inflection points in Figures 4a and 4b depend on the value of the reflection factor, which is the highest when the difference between the upper and lower soil layer resistivity is the greatest. Therefore, in Figs. 4a and 4b the inflection point represents a unique distinguishing feature by which each curve is differentiated within a family of curves.

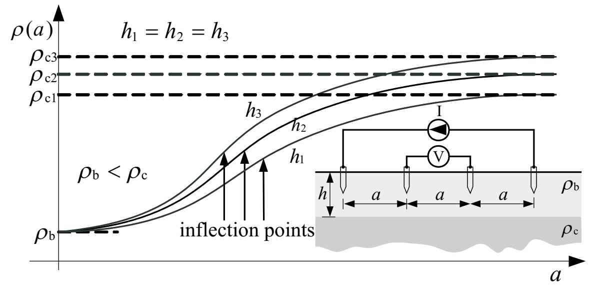

Figure 4b Curves of apparent resistivity of soil at $\rho_b < \rho_c$ and the inflection point shift due to changes of the reflection factor

Based on the presented graphic examples in Figs. 3a, 3b, 4a and 4b, which show the dependence of the inflection point position on the curve of apparent soil resistivity on electro-geometrical soil parameters, it could be concluded, intuitively, that if the functional relation were known, then could the parameters of two-layer soil be determined. Thus, the functional relationship between the position of the inflection point and the electro-geometrical soil parameters should be established. The inflection point on the curves of apparent soil resistivity is determined by:

$$\left.\frac{\mathrm{d}^2\rho(a)}{\mathrm{d}a^2}\right|_{a=a_i} = 0, \tag{7}$$

where a_i designates the position of the inflection point.

Unfortunately, the solution of the equation (7) can be obtained only numerically. The reason is in the mathematical form of the apparent soil resistivity $\rho(a)$, i.e. expression (4). The position of the inflection point could be easy to determine by means of expression (7), when the function of the apparent soil resistivity is some simpler function, similar in shape to the function $\rho(a)$, described in equation (4), and which has the same dependence on electro-geometrical parameters (ρ_b, ρ_c and h).

In other words, it is necessary to know the equivalent function, which equally describes two-layer soil. Such a function may, but need not have a perfectly uniform dependence on the variable a (distance between neighbouring electrodes), nor on the electro-geometrical soil parameters. To understand the properties which replaced function must have, it is necessary to explore the properties of the original function of the apparent soil resistivity given by (4). The properties of the function (4) are easiest to explore through the implementation of successive calculations for which known parameters are assumed. For that purpose, it is most appropriate to assume the reflection factor β as known value, and for the parameter to adopt the ratio of the inflection point distance and thickness of the upper soil layer (a_i/h). When the dependence of the ratio a_i/h on the reflection factor is known, a replacement function must have the same shape and functional dependence of these two values.

5 Dependence of the position of inflection point on electro-geometric soil parameters

By consecutively solving the equation (4) for different values of the reflection factor and thickness of the upper soil layer, the position of the inflection point in its dependence of the reflection factor is obtained by a numerical procedure (Tabs. 1 and 2). Since the reflection coefficient can be positive or negative, the dependency of the ratio of the inflection point and the upper layer thickness (a_i/h) on the positive or negative reflection factor are shown in separate tables.

Table 1 Dependence of the ratio of the inflection point and thickness of the upper soil layer on the amount of positive reflection factor

β	0,95	0,90	0,85	0,80	0,75	0,70	0,65	0,60	0,55	0,50
$\frac{a_i}{h}$	1,898	1,714	1,605	1,528	1,470	1,423	1,385	1,352	1,324	1,299

β	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10	0,05
$\frac{a_i}{h}$	1,299	1,277	1,257	1,239	1,222	1,207	1,193	1,180	1,168	1,156

Table 2 Dependence of the ratio of the inflection point and thickness of the upper soil layer on the amount of negative reflection factor

β	–0,95	–0,90	–0,85	–0,80	–0,75	–0,70	–0,65	–0,60	–0,55	–0,50
$\frac{a_i}{h}$	1,015	1,020	1,025	1,030	1,035	1,040	1,046	1,052	1,058	1,064

β	–0,50	–0,45	–0,40	–0,35	–0,30	–0,25	–0,20	–0,15	–0,10	–0,05
$\frac{a_i}{h}$	1,064	1,071	1,078	1,085	1,092	1,100	1,108	1,117	1,126	1,135

The data in Tab. 1 and Tab. 2 show that the ratios of the inflection point distance and the thickness of the upper soil layer (a_i/h) are significantly different for the positive and the negative reflection factor. According to Tab. 1, the changes in the reflection factor from 0,05 to 0,95, result in changes of the ratios a_i/h ranging from 1,156 to 1,898. According to Tab. 2, the changes of the reflection factor from –0,05 to –0,95, result in changes of the ratios a_i/h ranging from 1,135 to 1,015. The mean value of the ratios a_i/h for the positive reflection factor values is 1,527. If we take as an approximation an easily remembered mean value of a_i/h = 1,5, the biggest relative error in a rough estimate of the inflection point position expressed by the ratio of a_i/h would amount to $p\% = 29{,}8 \approx 30\ \%$ – and this would apply to the case when the reflection factor is β = 0,05. In the case when the reflection factor is β = 0,95 the biggest relative error would amount to $p\% = -21 \approx -20$ %. Therefore, in this case the relative error will not be greater than 20 %. The first and very rough estimate of the inflection point position is that it occurs when the distance between neighbouring electrodes is 50 % greater than the thickness of the upper layer of soil, i.e. when $a_i = 1,5h$ with the positive reflection factor. Or, a rough estimate of the inflection point position when the reflection factor is negative is that it occurs when the distance between neighbouring electrodes is 10 % greater than the thickness of the upper layer of soil i.e. when $a_i/h = 1,075 \approx 1,1$. It would be useful to memorize these two values, because it would allow one to rapidly evaluate the validity of the interpretation of measurement data, and these two values are very easy to determine graphically.

6 Equivalent function of apparent soil resistivity

The previously obtained knowledge about the dependency of ratio a_i/h on the factor β needs to connect with electro-geometrical soil parameters. The connection must be such that each soil parameter (ρ_b, ρ_c and h) can be determined individually. Therefore, it is necessary to find a "simple" analytical function, whose form corresponds to the form of apparent soil resistivity, which is obtained by using equations (4) and (5).

Considering that the apparent soil resistivity curve has the typical sigmoid shape, then the shape of the "simple" analytical function can be accurately described by means of the function (8) [12]:

$$\rho(a) = \rho_c - (\rho_c - \rho_b)e^{-\lambda a}(2 - e^{-\lambda a}). \tag{8}$$

Expression in (8), for reasons of mathematical convenience, is more appropriately given in the following form:

$$\rho(a) = \rho_c - 2(\rho_c - \rho_b)e^{-\lambda a} + (\rho_c - \rho_b)e^{-2\lambda a}. \tag{9}$$

The equivalent functions (8) and (9) depend on the electrical parameters, but not on the geometrical parameters of soil, i.e. the thickness of the upper soil layer. But, they depend on the parameter λ (1/m), which is needed to connect with the electro-geometric parameters of soil. To determine the inflection point of equivalent functions (8) and (9), equation (7) is applied. The second derivative of expression (9) by variable a reads as follows:

$$\frac{d^2\rho(a)}{da^2} = -2(\rho_c - \rho_b)\lambda^2 e^{-\lambda a} + (\rho_c - \rho_b)4\lambda^2 e^{-2\lambda a}. \tag{10}$$

Expression (10) at the inflection point is equal to zero, i.e.:

$$-2(\rho_c - \rho_b)\lambda^2 e^{-\lambda a_i} + (\rho_c - \rho_b)4\lambda^2 e^{-2\lambda a_i} = 0. \tag{11}$$

After rearranging of the expression (11), we get the following equation

$$-e^{-\lambda a_i} + 2e^{-2\lambda a_i} = 0, \tag{12}$$

whose solution is $\lambda = \dfrac{\ln 2}{a_i}$. (13)

The term (13) establishes the relation between the parameter λ in equivalent functions given by (8) and (9), which can present the curve of apparent resistance with respect to points of inflection. Inserting the term (13) into the term for the equivalent function (8), i.e. (9) the next equation can be obtained:

$$\rho(a) = \rho_c - (\rho_c - \rho_b)e^{-a\frac{\ln 2}{a_i}}(2 - e^{-\lambda\frac{\ln 2}{a_i}}) \tag{14}$$

or in the form:

$$\rho(a) = \rho_c - 2(\rho_c - \rho_b)e^{-a\frac{\ln 2}{a_i}} + (\rho_c - \rho_b)e^{-a\frac{2\ln 2}{a_i}}. \tag{15}$$

Characteristic points of equivalent functions (14) and (15) must correspond to the characteristic points of the original function or measured data. This means that the first derivative of an equivalent function in the inflection point must match the first derivative of the original function or interpolation function based on measured data. The first derivative of the function (15) at inflection point $a = a_i$ can be determined as follows

$$\left.\frac{d\rho(a)}{da}\right|_{a=a_i} = \frac{2\ln 2}{a_i}(\rho_c - \rho_b)\left(e^{-\ln 2} - e^{-2\ln 2}\right), \tag{16}$$

from where it follows that

$$(\rho_c - \rho_b) = \frac{a_i}{2\ln 2}\frac{1}{e^{-\ln 2} - e^{-2\ln 2}}\left.\frac{d\rho(a)}{da}\right|_{a=a_i}. \tag{17}$$

This allows expression (17) to be written in the form

$$\rho(a) = \rho_c - \left(\frac{a_i}{2\ln 2}\frac{1}{e^{-\ln 2} - e^{-2\ln 2}}\left.\frac{d\rho(a)}{da}\right|_{a=a_i}\right)e^{-a\frac{\ln 2}{a_i}}\left(2 - e^{-a\frac{\ln 2}{a_i}}\right). \tag{18}$$

from which follows the expression for ρ_c:

$$\rho_c = \rho(a) + \left(\frac{a_i}{2\ln 2}\frac{1}{e^{-\ln 2} - e^{-2\ln 2}}\left.\frac{d\rho(a)}{da}\right|_{a=a_i}\right)e^{-a\frac{\ln 2}{a_i}}\left(2 - e^{-a\frac{\ln 2}{a_i}}\right). \tag{19}$$

Although from the mathematical point of view in the expression (19) can take the value of apparent soil resistivity $\rho(a)$ measured at any distance between the electrodes $\rho(a) = \rho(a_m)$, it is best to take the measured value at the largest distance between the electrodes. For an explanation of this, see the commentary following the expression (20).

When the lower soil layer resistivity ρ_c is determined, the upper soil layer resistivity ρ_b is determined as follows:

$$\rho_b = \rho_c - \frac{a_i}{2\ln 2}\frac{1}{e^{-\ln 2} - e^{-2\ln 2}}\left.\frac{d\rho(a)}{da}\right|_{a=a_i}. \tag{20}$$

However, it is important to emphasize that superior results are always obtained by using an alternative polynomial or some other equivalent functions $\rho(a)$ for $a = 0$, rather than the expression (20). The reason for this lies in the fact that in the expression (20) to determine the upper soil layer resistivity (ρ_b) is used indirectly determined the approximate value of the lower soil layer resistivity (ρ_c).

Namely, measurement with the Wenner method is initially done with a small spacing between the electrodes, whereby the penetration of the current into the soil is low and most of the streamlines is in the upper soil layer (Fig. 5a). Therefore, for small distances between the electrodes, the apparent soil resistivity corresponds roughly to the upper soil layer resistivity.

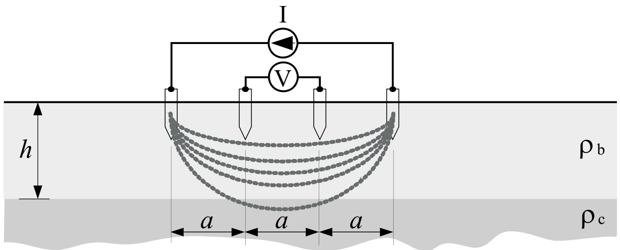

Figure 5a Influence of the spacing between the current electrodes on the depth of current penetration into the soil

Increasing the spacing between the current electrodes during the measurement, increases penetration depth of streamlines in the soil (Fig. 5b), which depending on the electro-geometrical soil parameters reflects on the measured value of voltage between voltage electrodes. Theoretically, if this could be done in practice, when the current electrodes were spaced so that is $3a > h$, the streamlines would pass a considerably larger part of the path through the lower layer of soil, and the voltmeter would measure the potential difference caused by the voltage drop due to the current passing through the lower layers of soil. For this reason, the resulting apparent soil resistivity would correspond to the lower soil layer resistivity. This is also the reason why in the expression (19) for $\rho(a) = \rho(a_m)$ it is best to take the result of measurement obtained at the greatest electrode spacing.

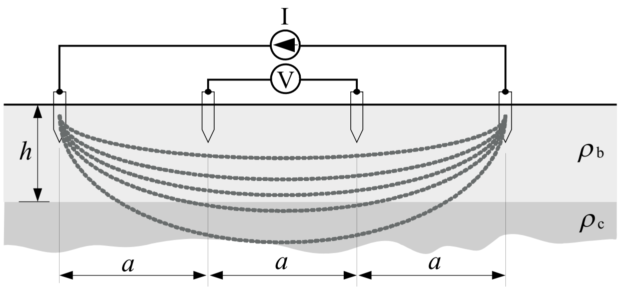

Figure 5b Influence of the spacing between the current electrodes on the depth of current penetration into the soil

Of course, since this is not feasible, as opposed to the upper soil layer, where the resistivity is simply determined measuring at a small distance between the current electrodes, information about the lower soil layer resistivity is harder to obtain, and thus its value must be determined indirectly.

Knowing the upper and lower soil layer resistivity can be determined soil reflection coefficient by means of expression (5). When its value is determined, using the Tab. 1 or Tab. 2, depending on whether its value is positive or negative, thickness of the upper soil layer can be determined. In order to make the obtained expressions understandable to a wider audience, it is appropriate to show the sequence of actions during the simplified determining of electro-geometrical soil parameters.

7 The sequence of actions during the simplified determining of electro-geometrical soil parameters

Procedure for simplified determining of electro-geometrical two-layered soil parameters according to the previously derived equations and tables is performed as described in the following seven steps (from S1 to S7).

S1 For the measured data on the apparent soil resistivity provide spreadsheet view in a series against growing variable a.

S2 So arranged measurement data from the step S1 is necessary to be described with a continuous smooth curve (polynomial) using a suitable interpolation polynomial (Lagrange, Newton, ...) or third-degree polynomial obtained by the method of least squares.

S3 Determine the inflection point (a_i) of the apparent soil resistivity curves (obtained in the step S2) by means of the numerical procedure.

S4 Numerically determine the value of the first derivation of the corresponding polynomial (obtained in step S2) at the point of inflection.

S5 Once you determine the amount of the first derivative of the equivalent functions (corresponding polynomials) at the point of inflection, the lower soil layer resistivity is determined by means of the expression (19). In doing so, the measured soil resistivity value $\rho(a)$ should be obtained at the largest distance between the electrodes.

S6 Having known the lower soil layer resistivity (ρ_c), the upper soil layer resistivity (ρ_b) is determined by the expression (20), or by the equivalent function of the apparent soil resistivity for $a = 0$. In another case, the error will be smaller!

S7 After determining the inflection point, the upper (ρ_b) and lower soil layers resistivity (ρ_c), are determined:

- reflection factor, according to the formula (5) and
- the thickness of the upper layer using Tab. 1 (if $\beta > 0$), or Tab. 2 (if $\beta < 0$), too.

After implementing the actions described in the step (S7), it can be concluded that all electro-geometrical parameters of two-layer soil are determined. For a better understanding of the previously described sequence of actions when determining the parameters of a two-layer soil by means of the proposed procedure, its application is shown by numerical example.

8 Numerical example

Let's take that the upper soil layer resistivity is $\rho_b = 100$ Ω·m and that its thickness is $h_b = 10$ m, as well as that the lower soil layer resistivity is $\rho_c = 200$ Ω·m. The use of the equations (4) and (5) gives us the information, summarized in Tab. 3, about dependence of the apparent soil resistivity with respect to the spacing between adjacent electrodes.

Table 3 Dependence of apparent soil resistivity on the spacing between adjacent electrodes

a /m	1	3	5	10	20	30
$\rho(a)$/ Ω·m	100,026	100,653	102,662	113,439	139,506	157,574

Suppose that these data are also the measurement data obtained from the measurement field, i.e. perfect measurement data. If the aforementioned steps are correctly applied, these data should give us the approximate values of the electro-geometrical parameters. The data in Tab. 3, have been arranged in a sequence according to the growing variable *a*, thereby the step S1 is performed. Now follows the step S2: the data from Tab. 3 must be interpolated using the interpolation polynomial of *N*-th degree, [19]. Let us apply for this purpose the Lagrange form of the interpolation polynomial [19]:

$$\rho(a)=\sum_{k=0}^{N} S_k(a)\rho_k, \tag{21}$$

$$S_k(a)=\prod_{m=1}^{N}\frac{(a-a_m)\text{ omitting }(a-a_k)}{(a_k-a_m)\text{ omitting }(a_k-a_k)}, \tag{22}$$

where $S_k(a)$ denotes a polynomial, which represents the *k*-th formative function, ρ_k *k*-th measurement value in the point a_k. In the equation (22), the m index represents the number of the measurement data, for this reason that there is no confusion measurement data reported ordinal number of measurements are summarized in Tab. 4.

Table 4 Dependence of apparent soil resistivity on the spacing between adjacent electrodes

m	1	2	3	4	5	6
a_m/m	1	3	5	10	20	30
$\rho(a_m)$/ Ω·m	100,026	100,653	102,662	113,439	139,506	157,574

Applying the interpolating polynomial expressions described in (21) and (22), to the data in Tab. 4, we get a polynomial whose degree is determined by the number of measurement data that is reduced by one $N-1$. That is, in this case, the following fifth degree polynomial:

$$\rho(a)=100{,}13-0{,}212a+9{,}157\times10^{-2}a^2+ +1{,}55\times10^{-2}a^3-1{,}11\times10^{-3}a^4+1{,}917\times10^{-5}a^5. \tag{23}$$

The inflection point is determined by the iterative numerical procedure in [19], using one of the proposed software packages [9, 10, 11], and its value is a_i = 12,72 m. The first derivation of the interpolating polynomial (23) at the inflection point is given by

$$\left.\frac{\mathrm{d}\rho(a)}{\mathrm{d}a}\right|_{a_i=12{,}72}=3{,}013. \tag{24}$$

The lower layer of soil resistivity (ρ_c), according to (19) is given by

$$\rho_c=157{,}5+ +\left(\frac{12{,}72}{2\ln2}\frac{3{,}013}{e^{-\ln2}-e^{-2\ln2}}\right)e^{-\frac{30\cdot\ln2}{12{,}72}}\left(2-e^{-\frac{30\cdot\ln2}{12{,}72}}\right)= =196{,}422\ \Omega\cdot\text{m}. \tag{25}$$

Having determined the lower soil layer resistivity (ρ_c), the next step is to determine the upper soil layer resistivity (ρ_b), according to expression (20):

$$\rho_b=196{,}422-\left(\frac{12{,}72}{2\ln2}\frac{3{,}013}{e^{-\ln2}-e^{-2\cdot\ln2}}\right)=85{,}838\ \Omega\cdot\text{m} \tag{26}$$

That is, according to expression (23), for a = 0

$$\rho(a=0)=\rho_b=100{,}13\ \Omega\cdot\text{m}. \tag{27}$$

Once the upper and lower soil layer resistivity are defined, the reflection factor is determined according to (5)

$$\beta=\frac{196{,}422-85{,}838}{196{,}422+85{,}838}=0{,}392. \tag{28}$$

As the reflection factor is positive, Tab. 1 is used to determine the a_i/h ratios. According to Tab. 1, for $\beta\approx0{,}4$, a_i/h = 1,257. If the amount of inflection point is a_i = 12,72 m, it follows that the thickness of the upper soil layer is h = 12,72/1,257 = 10,119 m.

The electro-geometrical soil parameters estimated by the application of the procedure described above, as well as their percentage measurement error compared to the exact values, are given in Tab. 5, which is an excellent approximation. The numerical example with the real data has shown that the proposed approximation procedure is valid.

Table 5 The calculated electro-geometrical soil parameters and errors

Value	Exact value	The calculated value	Percentage error
ρ_b/Ω·m	100,00	85,838 (Eq. 20, 26)	−14,162
		100,130 (Eq. 23)	0,1300
ρ_c/Ω·m	200,00	196,422	−1,7989
h/m	10,00	10,119	1,1900

For most engineers the accuracy of the described procedure will be satisfactory. However, due to various electrical noises, errors in measurement, the local soil inhomogeneities, circumstances may arise where the measurement data contain a significant proportion of uncertainty. For this reason, we can expect that the previously described procedure will not provide valid results, i.e. the deviation from the actual electro-geometrical soil parameters will be considerable. One of methods which can in some specific measure decrease influence of measurement uncertainties is to use the interpolating polynomial obtained via the method of least squares (least squares method) [19, 20], instead of the interpolating polynomial whose value is equal to the measured values at interpolation points. In other words, step two is changed in this case. To better understand the advantages of using the least squares method instead of the direct application of the interpolation polynomial the following explanation is given in brief.

9 Least squares method and interpolating polynomial

The above described procedure, although very easy to use, has an inherent weakness. Namely, the interpolating polynomial describes the measurement data in such a way that the form of the resulting polynomial passes through the measuring points. Since, in reality, the measurement is subject to measurement noise and local variation of soil parameters, these data contain measurement uncertainties. For this reason, the variation of measurement data will be strongly reflected in the form of the polynomial interpolation. A significantly lower sensitivity is achieved when the equivalent curve passes between the points of measurement data. One such approach is called the method of least squares [19, 20]. Fig. 6 shows three curves of the apparent soil resistivity. The curve shown with a solid line is the curve that represents the ideal case, i.e. theoretically perfect measurements. This curve passes through the points representing perfect measurement data (shown by circles). The impact of measurement noise, inhomogeneity of local soil and errors is such that the data collected by measurement will be scattered around perfect measurement data (shown by small crosses). Interpolation of real measurement data by means of an interpolating polynomial of n-th degree, gives a smooth curve (shown by dots), whose shape deviates from the previously described curve with which perfect measurements have been interpolated. The deviation is usually not large in amount, but can significantly affect the position of inflection point. Especially if the interpolating polynomial is of a high degree, then can appear multiple inflection points (Fig. 6). Using the least squares method, the resulting polynomial will not necessarily pass through the points representing the actual measurement data, but mostly between them. In this way the scattering of values of real measurement data from the theoretically perfect measurement will have a lesser effect on the shape of the obtained function, and thus on the point of inflection, too.

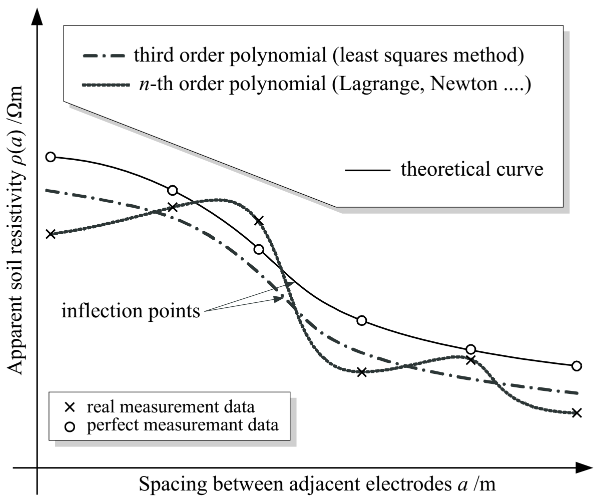

Figure 6 Different approaches to the interpolation of measurement data and resultant deviation of the inflection point position (imaginary case)

In describing the results of measurements using the least squares method, it is advisable to avoid high degree polynomials, as the advantage of this technique could be lost. In fact, in this case multiple inflection points can occur, as well as in the case of the direct application of the interpolating polynomial. It is reasonable to assume that the cubic polynomial will describe measurement results pretty accurately, since it can have ***only one point of inflection,*** and its shape can be adjusted to the measured data. Not completely, but it is still desirable to mitigate deviations due to the scattering of measured values from the actual values.

If a polynomial were of a higher degree, it would have more freedom to follow the measured values and thus could have multiple extremes (maxima, minima, and point of inflection). A detailed description of the least squares method is beyond the scope of this work, and the reader is referred to the list of references for further study [19, 20].

The following example illustrates the process of applying the least squares method to determine the equivalent functions of apparent soil resistivity, as well as its inherent noise suppression capability.

Electro-geometrical parameters of the soil are the same as in the previous example, whose using in the expressions (4) and (5) gives the data for $\rho(a_m)$, which are shown in the third rows in Tab. 6 and represent the data to get an ideal measurement.

To these data, we have added random noise in the amount of ±5 % of the amplitude, and thus is obtained data row for $\rho_{\text{noise}}(a_m)$ in Tab. 6, representing a realistic measurement.

Table 6 Dependence of apparent soil resistivity on the spacing between adjacent electrodes

m	1	2	3	4	5	6
a_m/m	1	3	5	10	20	30
$\rho(a_m)$/ Ω·m	100,026	100,653	102,662	113,439	139,506	157,574
$\rho_{\text{noise}}(a_m)$/ Ω·m	95,247	105,686	105,742	110,036	145,086	156,954
Noise /%	–5	+5	+3	–3	+4	–4

The interpolating polynomial which describes the apparent soil resistivity according to Tab. 6, the row for $\rho_{\text{noise}}(a_m)$ is obtained using the expressions (21) and (22) as follows:

$$\rho(a) = 81,78 + 17,33a - 4,282a^2 + 0,4397a^3 - \\ - 0,1825 \times 10^{-1} a^4 + 0,2602 \times 10^{-3} a^5. \tag{29}$$

The polynomial is of the fifth degree, and as such, it can have three inflection points. In this case there are three inflection points present, viz.: a_{i1} = 5,248 m, a_{i2} = 13,345 m, a_{i3} = 23,495 m, which is easily observed in Fig. 7, i.e. in the curve shown by the dot technique. Since there is more than one inflection point, after this step according to the above described algorithm, it is not possible to determine the electro-geometrical parameters of two-layer soil. Thus, the previous step must be repeated, but so that using the measured data (Tab. 6, the $\rho_{\text{noise}}(a_m)$ line), the cubic polynomial is determined using the least squares method. The resulting cubic polynomial, in terms of the least squares method, is obtained using MathCAD 14 ready-made routines [10] and equals:

$$\rho(a) = 99,275 - 0,119a + 0,213a^2 - 4,803 \times 10^{-3} a^3. \tag{30}$$

The polynomial is third degree, and as such can only have one point of inflection, which is: a_i = 14,782 m, and it is consistent with the curves shown in Fig. 7 and represented by point-line technique. The first derivation of expression (30) gives

$$\frac{\mathrm{d}}{\mathrm{d}a}\rho(a) = -0{,}119 + 0{,}416a - 14{,}409\times10^{-3}a^{2}. \tag{31}$$

The first derivation of expression (31) at the inflection point equals

$$\left.\frac{\mathrm{d}\rho(a)}{\mathrm{d}a}\right|_{a_{\mathrm{i}}=14{,}782} = 2{,}882. \tag{32}$$

According to expression (19) is

$$\rho_{\mathrm{c}} = 156{,}954 + \left(\frac{14{,}782}{2\ln2}\frac{2{,}882}{\mathrm{e}^{-\ln2}-\mathrm{e}^{-2\ln2}}\right)\mathrm{e}^{-156{,}954\frac{\ln2}{14{,}782}}\left(2-\mathrm{e}^{-156{,}954\frac{\ln2}{14{,}782}}\right) = 157{,}11\ \Omega\cdot\mathrm{m}. \tag{33}$$

After the lower soil layer resistivity (ρ_c) has been determined, one proceeds to calculating the upper soil layer resistivity (ρ_b). At this point, let us take an opportunity to show that it is preferable to determine the upper soil layer resistivity using the equivalent function, i.e. expression (30), not expression (20). The upper soil layer resistivity determined using expression (30) and for $a = 0$ equals ρ_b = 99,275 Ω·m, which is considerably more accurate value than the following value (obtained using expression (20)):

$$\rho_{\mathrm{b}} = 157{,}11 - \frac{14{,}782}{2\ln2}\frac{2{,}882}{\mathrm{e}^{-\ln2}-\mathrm{e}^{-2\ln2}} = 34{,}187\ \Omega\cdot\mathrm{m}. \tag{34}$$

Once the upper and lower soil layer resistivities have been obtained, the reflection factor (5) is determined:

$$\beta = \frac{157{,}11-99{,}275}{157{,}11+99{,}275} = 0{,}226. \tag{35}$$

Since the reflection factor is positive, Tab. 1 is used to determine the a_i/h ratio. From Tab. 1, it can be seen that ratio a_i/h = 1,207 corresponds to the reflection coefficient $\beta \approx 25$. Being that the amount of the inflection point is known and equals a_i = 14,782 m, the following thickness of the upper soil layer is determined h = 14,782/1,207 = 12,25 m.

The electro-geometrical soil parameters, which are obtained by means of the presented calculating technique and describing apparent soil resistivity with the third degree polynomial equivalent function in the least squares manner (LSM), are summarized in Tab. 7. Real and ideal measurement data together with the corresponding curves are shown in Fig. 7. In the description shown in Fig. 7, the expressions which are used to obtain curves of apparent soil resistivity are given.

Table 7 Actual and calculated values of electro-geometrical soil parameters and their errors

Value	Exact value x_t	The calculated value x_{mes}	Percentage error $p_{\%}$
ρ_b/Ω·m	100,00	99,275	–0,7250
ρ_c/Ω·m	200,00	157,11	–21,445
h/m	10,000	12,250	22,500

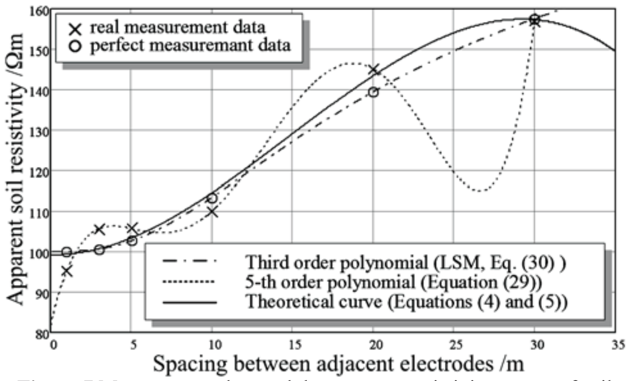

Figure 7 Measurement data and the apparent resistivity curves of soil obtained by different techniques

10 Conclusion

The simplified procedure for approximate determination of electro-geometrical soil parameters that we have proposed and illustrated in this paper is based on very simple mathematical instruments. For this reason it is easily comprehensible to a wider engineering population, as well as to students of geophysics, agriculture and electrical engineering. Moreover, it is easily transferrable into program lines, as it is usable in a number of the more widespread and affordable mathematical and engineering programs of general application such as MathCAD, Mathematica, MATLAB, etc. It should be taken into account that in the last decade these programs have been used in education in the final years of various higher education institutions, and thus students and therefore future engineers are familiar with them.

In cases where due to the different electrical noises, the errors in measurement, the local soil inhomogeneity, or due to the other factors may arise circumstances in which the measurement data would contain a considerable proportion of uncertainty, thereby reducing the accuracy of the data obtained by the proposed procedure, in some cases meaningless results would be obtained. However, in most cases the results obtained by means of the proposed procedure showed acceptable accuracy. When using the above procedure it is recommended to check graphic meaningfulness of measurement data, as is the case with all other procedures. Graphic verification of the meaningfulness of the obtained data concerning the soil resistivity can be made in the above-mentioned mathematical and engineering programs for general use, with just a few clicks on the keyboard, by calling up the subroutines for the graphical presentation of results, as is Fig. 7, which is obtained using the program MathCAD 14 [10]. Numerical examples show that the users of this procedure are not recommended to use polynomial interpolation to describe the measured results but are recommended to use the third-degree polynomial obtained using the method of least squares.

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