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HEPTATHLON EVALUATION MODEL USING GREY SYSTEM THEORY

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Original scientific paper

In this paper we investigate the effectiveness of the Grey system theory to determine the ranking of the best women athletes in heptathlon. The scoring method currently used in women's heptathlon needs alternative scoring as it displays unacceptable bias towards some athletic disciplines while deferring others. The term Grey stands for poor, incomplete and uncertain, and is especially related to the information about the system. The Grey relational grade deduced by the Grey theory is used to establish a complete and accurate model for determining the ranking of the heptathletes. The proposed scoring method is accurate and is shown to improve fairness and results' validity in women's heptathlon.

Keywords: *Grey system theory, Grey relational analysis, heptathlon, ranking of heptathletes*

Model vrednovanja sedmoboja upotrebom sive relacijske analize

Izvorni znanstveni članak

U ovom radu istražujemo djelotvornost teorije Sivih sustava za utvrđivanje redoslijeda najboljih atletičarki u sedmoboju. Postojeća metoda bodovanja u ženskom sedmoboju treba alternativni način bodovanja jer pokazuje neprihvatljivu pristranost prema nekim atletskim disciplinama dok druge zanemaruje. Izraz Sivi znači nešto siromašno, nepotpuno i neizvjesno, i posebno se odnosi na informaciju o sustavu. Sivi relacijski stupanj dobiven Sivom teorijom koristi se za uspostavu cjelokupnog i točnog modela za utvrđivanje redoslijeda sedmobojki. Predložena metoda bodovanja je točna i pokazano poboljšava pravednost i ispravnost rezultata ženskog sedmoboja.

Ključne riječi: *Teorija sivih sustava, siva relacijska analiza, sedmoboj, rangiranje sedmobojki*

1 Introduction

The heptathlon is a track and field athletics combined events contest made up of seven events. There is the women's (outdoor) and the men's (indoor) heptathlon.

In this paper, the women's heptathlon rules that are the same as the men's decathlon rules will be analyzed. The heptathlon is a women's sporting event, designed to be the female counterpart for the men's decathlon. Like other combined sports events, the heptathlon tests the speed, strength, mobility and endurance of the heptathletes. The competition consists of seven events held on two consecutive days. The first day's events, in order, include the 100–meter hurdles, the high jump, the shot put and the 200–meter run. The second day's events, in order, include the long jump, the javelin throw and an 800–meter run.

Points are awarded to each heptathlete according to her time or distance score, not her placement in the field, according to pre-set formulas, and the overall point score determines the winner [13, 14].

In 2006 issue of New Studies in Athletics a revision of the decathlon scoring method of the IAAF (International Association of Athletics Federations) has been proposed. An analysis of the world top 100 decathletes showed that decathletes gather far more points in sprinting – based events like 100 m, 110 m hurdles and long jump than in throwing events (shot put, javelin throw, discus) and endurance (1500 m) [14].

The women's heptathlon displays similar anomalies. An analysis of the world top 100 heptathletes showed that heptathletes gather more points in sprinting-based events than in throwing events.

The current scoring method displays unacceptable bias as it favors some of the events and defers others. It lacks fairness and validity, because sprinters benefit disproportionately. Converting the measurement into points and summing up is questionable. There is no way to decide the winner if there is a tie-score.

In order to overcome this defect the Grey relational analysis deduced by the Grey system theory will be used to establish accurate and complete evaluating model for determining the best heptathletes.

This evaluating model might provide IAAF with a score awarding method that displays improved fairness and validity.

The structure of this paper is as follows: after this introduction, section 2 discusses some related work on the Grey theory. Section 3 elaborates the Grey system theory. In section 4, the calculation of the Grey relational grade is presented. Section 5 explains the calculation of the Grey relational grade. Section 6 discusses the traditional score evaluation. Section 7 clarifies how to set up the Grey relation evaluation model for heptathlon. In Section 8 the Grey relational coefficient determination is explained. Finally, conclusion is given in Section 9.

2 Related work

The Grey relational analysis has been recently applied to the field of sports. For example, Chang-Liang et al. [1] applied Grey relational analysis to the decathlon evaluation model. Wang [12] took several factors as attributes to find their influence on the javelin throw. Han [7] used Grey relational analysis for the study of women marathon. Yen [16] used Grey Relational Analysis to study the defence and offense techniques and establishment of training goal for Ku-Tai basketball team. Chen [3] used Grey theory to make predictions for those soccer teams who can enter quarter finals (round of eight) in 2000 World Cup based on the 1998 results of group preliminaries. Sun [10] investigated the influence of body characteristics on Hammer throw players and found six main body characteristics and its ranking order affecting the score by Grey relational analysis. Wang [12] studied which factors influence the relational grade ranking for women javelin throw.

Due to the widespread and successful application of the Grey relational analysis in sports, it is reasonable to expect that applying it to the women's heptathlon will result in an accurate evaluation model.

3 Grey system theory

The information that is either incomplete or undetermined is called Grey [1]. The Grey system provides multidisciplinary approaches for analysis and abstract modelling of systems for which the information is limited, incomplete and characterized by random uncertainty [9].

Main contributions to the Grey system theory came from: Grey systems and control (GSC), Grey relational analysis (GRA), and Gray modelling (GM). GRA uses information from the Grey system to dynamically compare each factor quantitatively [8]. The approach is based on the level of similarity and variability among all factors to establish their relation [17]. GM is developed based on requirements for system modelling with limited data.

The three terms that are typical symbols and features for the Grey system are [3]:

1) The Grey number in Grey system is a number with less complete information,
2) The Grey element represents an element with incomplete information,
3) The Grey relation is the relation with incomplete information.

There are several aspects for the theory of Grey system:

1. Grey generation: This is data processing to supplement information. It is aimed to process those complicate and tedious data to gain a clear rule, which is the whitening of a sequence of numbers.
2. Grey modelling: The modelling is performed in order to establish a set of Grey variation equations and Grey differential equations, which is called the whitening of the model. This is done by step 1 to establish a set of Grey variation equations and Grey differential equations. The Grey model is denoted as GM (*n, h*), which is the *n*-th order differential equation of *h* variables. This Grey differential equitation is used for infinite information. Most of the previous researchers have focussed on GM (1, 1) models because of their computational efficiency. GM (1, 1) model has time – varying coefficients. It means that the model is renewed as the new data become available to the prediction model. A Grey differential equation having *N* variables is called GM (1, N).
3. Grey prediction: By using the Grey model to conduct a qualitative prediction, this is called the whitening of development. Grey models predict the future values of a time series based on a set of the most recent data.
4. Grey decision: A decision is made under imperfect countermeasure and unclear situation, which is called the whitening of status. It is primarily concerned with the Grey strategy of situation, Grey group decision making and Grey programming [5, 6]. The Grey strategy of situation deals with the strategy – making based on multi objects which are contradictory in the ordinary way. It is important to make a satisfactory strategy by means of effect measure maps, which transfer the disconformities samples resulting from different objects into identical scales.
5. Grey relational analysis (GRA): Quantify all influences of various factors and their relation, which is called the whitening of factor relation. It uses information from the Grey system to dynamically compare each factor quantitatively, based on the level of similarity and variability among factors to establish their relation. GRA gives the relational grade for discrete sequences.

This study will adopt the above mentioned research steps to develop an accurate evaluation model based on GRA, for determining the best heptathletes. The generation of the Grey relation takes the following steps:

1. Setting up eigenvalue matrix, input original data
2. Standardized transformation, formulas:

I) bigger-the-better (1),
II) smaller-the-better (2), or
III) nominal-the best (3)

3. Grey relational coefficient determination
4. Calculation of the Grey relational grade:
   - getting absolute difference of compared series and referential series (4)
   - find out minimum and maximum
   - calculation of relational coefficients and relational degree (5,6)
5. Set up the order based on influence factors.

4 Gray relational analysis

In order to pursue the Grey relational analyses the data processing is first performed, which is called the generation of the Grey relation.

Three kinds of influence factors are included:

1. Benefit – type factor (the bigger the better),
2. Defect – type (the smaller the better)
3. Medium – type, or nominal-the-best (the nearer to a certain standard value the better).

It is difficult to compare between different kinds of factors because they exert a different influence.

Therefore, the standardized transformation of these factors has to be performed. Three formulas can be used [16]:

$$x_i(k) = \frac{x_i(k) - \min x_i(k)}{\max x_i(k) - \min x_i(k)}. \tag{1}$$

The first standardized formula is suitable for the benefit – type factor (the bigger the better).

$$x_i(k) = \frac{\max x_i(k) - x_i(k)}{\max x_i(k) - \min x_i(k)}. \tag{2}$$

The second standardized formula is suitable for defect – type factor (the smaller the better).

$$x_i(k) = \frac{1 - |x_i(k) - u_i|}{|\max x_i(k) - u_i|}. \tag{3}$$

The third standardized formula is suitable for the medium – type factor (the nearer to a certain standard value the better).

5 Calculation of the Grey Relational Grade

The Grey relation grade represents the correlation between two series. When one determines Grey relation and takes only one series, $x_0(x)$, as a referenced series, it is called the grade of local Grey relation [10].

If any of the series, $x_i(x)$, is referenced series, it is called the grade of the global Grey relation [1]. The Grey relational coefficient must be determined prior to the Grey relational grade. The Grey relational coefficient can be calculated by steps as follows:

The absolute difference of the compared series and the referential series should be obtained using the following formula [2]:

$$\Delta x_i(k) = |x_0(k) - x_i(k)|, \tag{4}$$

and the maximum and the minimum difference should be found.

If the grade of local Grey relation is brought to define the Grey relational coefficient, $\gamma(x_i(k), x_j(k))$, the Grey relational coefficient can be expressed as follows:

$$\gamma(x_i(k), x_j(k)) = \frac{\Delta_{\min} + \Delta_{\max}}{\Delta_{0i}(k) + \Delta_{\max}}. \tag{5}$$

After obtaining the Grey relational coefficient we take the arithmetic mean of the Grey relational coefficient as the Grey relational grade:

$$\Gamma = \gamma(x_i, x_j) = \frac{1}{n}\sum \gamma(x_i(k), x_j(k)).$$

6 Traditional score evaluation

The results for the women's heptathlon are calculated through official scoring tables that convert the separate performances in various jumping, throwing and running events into points to allow simple addition. The points are awarded to each heptathlete and the overall score determines the winner. The current scoring method favors some of the events and defers others.

The women's heptathlons rules display anomalies and would need alternative ways of converting performances into scores. The results for the women's heptathlon are calculated through official scoring tables that convert the separate performances in various jumping, and throwing events into points to allow simple addition. This scoring method seems to be quite impenetrable. The current scoring method for each discipline is covered by a mathematical expression:

$$S(P) = A(P - B)^C, \tag{7}$$

where P is the performance (i.e. the achieved distance in the long jump, etc.); A, B and C are event-dependent parameters that define the nature of the scoring table; and S is the score (the number of assigned points).

For running events $(P - B)$ has to be replaced with $(B - P)$ because of the descending nature of performance with time.

The performance assessment method is comprised of two stages:

1. the performance P is measured (in units of time or distance),
2. the performances are converted to a score S in order to allow addition.

According to the scoring table, each event reaching certain distance or height measured in metres, and time measured in seconds is converted into scores. Total points are obtained by summing up each individual event score.

The measurement unit is the same for some events, but class intervals in conversion table are not the same. The class interval is the number of points for difference in metres, for example, for javelin throw, 101,14 m gives 1373 points, and 101,20 m gives 1374 points. Simply converting the measurement into points and summing up is questionable. There is no way to decide the winner if there is a tie-score (the winner is the heptathlete with the highest number of points).

The women's heptathlon would need alternative ways of converting performances into scores. For these alternative models, the following requirements have been expressed [14]:

- allow a fair comparison between events,
- be uniform over all events (this follows from the starting points of the decathlon),
- use objective standards (distance and time measurements),
- be grounded in empirical evidence (practical significance),
- be based on sound principles and logic (consistent, transparent and substantiated),
- be stable over time and thus possess self – stabilizing characteristics,
- allow smooth transitions from the existing model (acceptability).

We argue that the general Grey model that we have clarified in Sections 4 and 5 satisfies all of the above stated requirements.

In the Grey relation analysis, in order to perform overall evaluation of multiple attributes, the measurement unit need not necessarily be the same. The method can determine the overall ranking without consulting the utility function. It is therefore appropriate to adopt the Grey relational analysis for women's heptathlon in order to determine the ranking for heptathletes.

7 Setting up the Grey relation evaluation model of the heptathlon

In the following example, it is assumed that five heptathletes attend a heptathlon competition. The results of the heptathlon competition are shown in Tab. 1.

The codes of the heptathletes are X1, X2, X3, X4, and X5. The codes of the events are S1, S2, S3, S4, S5, S6, and S7.

The results of the heptathlon competition are used to form the evaluation matrix with the event as attribute column, and heptathletes as comparative sequences.

The following assumptions can be made:

1. For the events: high jump, shot put, long jump, and javelin throw, the expectancy is the longer-the-beter for the distance, or the benefit – type factor, which can be determined by equation (1).

2. For the events: 100–meter hurdles, 200–meter, and 800–meter, the expectancy is the shorter-the-better for the time, or defect – type factor, which can be determined by the equation (2).

The referenced series is therefore X0 = (12,87; 197,00; 17,09; 23,88; 7,20; 60,00; 123,00).

The above equations (1), and (2) can be used to do the standardized transformation of this sample. The scoring points for each attribute are normalized in order to obtain matrix table of comparative series based on the expectancy of each individual event, as shown in Tab. 2. An ideal standard series is established (X0=1) in the last line in Tab. 2.

Table 1 Results of the hepthatlon competition - original data

Event / Heptathlete	100–meter hurdles /s S1	high jump /cm S2	shot put /m S3	200–meter /s S4	long jump /m S5	javelin throw /m S6	800–meter /s S7
X1	13,86	186,00	17,07	24,10	6,98	55,60	123,00
X2	12,87	195,00	16,55	23,90	6,55	57,20	127,70
X3	14,00	197,00	17,00	23,85	7,20	58,00	129,00
X4	12,90	182,00	17,09	24,25	6,50	59,20	127,30
X5	14,75	176,00	16,02	23,88	6,48	60,00	127,10

Table 2 The compared series and the referenced series

Event / Heptathlete	S1	S2	S3	S4	S5	S6	S7
X1	0,473	0,476	0,981	0,366	0,694	0	1,000
X2	1,000	0,905	0,495	0,854	0,097	0,364	0,216
X3	0,398	1,000	0,916	0,976	1,000	0,545	0
X4	0,984	0,285	1,000	0	0,028	0,818	0,283
X5	0	0	0	1,000	0	1,00	0,316
Standard series (X0)	1	1	1	1	1	1	1

Table 3 The Grey relational coefficient

Event / Heptathlete	S1	S2	S3	S4	S5	S6	S7
X1	0,655	0,656	0,981	0,612	0,766	0,500	1,000
X2	1,000	0,952	0,664	0,873	0,525	0,611	0,560
X3	0,624	1,000	0,922	0,976	1,00	0,687	0,500
X4	0,984	0,583	1,000	0,500	0,583	0,846	0,582
X5	0,500	0,500	0,500	1,000	0,500	1,000	0,594

8 The Grey relational grade determination and ranking

The next step is to calculate the absolute difference of the compared series and the referential series using equation (4), and find out the maximum and minimum.

The Grey relational coefficient can be obtained by equation (5), where $n = 7$, Tab. 3. The Grey relational grade is then obtained using equation (6). The final results with ranking are given in Tab. 4.

From Tab. 4 it can be seen that the ranking of five women heptathletes is: X3, X2, X1, X4, X5.

Table 4 The Grey relational grade and ranking

Heptathlete	Relational grade	Ranking
X1	0,739	3
X2	0,803	2
X3	0,815	1
X4	0,725	4
X5	0,656	5

The women heptathletes X1 and X5 are of the same rank. When the calculation is made according to the scoring

Table 5 The results according to the currently used method

Event / Heptathlete	100–meter hurdles /s S1	high jump /cm S2	shot put /m S3	200–meter /s S4	long jump /m S5	javelin throw /m S6	800–meter /s S7	Total score
X1	13,86 **998**	186,00 **1044692**	17,07 **1000**	24,10 **971**	6,98 **1165**	55,60 **969**	123,00 **1070**	**1050865**
X2	12,87 **1144**	195,00 **1113681**	16,55 **965**	23,90 **990**	6,55 **1023**	57,20 **1000**	127,70 **999**	**1119802**
X3	14,00 **978**	197,00 **1113681**	17,00 **995**	23,85 **995**	7,20 **1240**	58,00 **1016**	129,00 **979**	**1135369**
X4	12,90 **1140**	182,00 **1014400**	17,09 **1001**	24,25 **957**	6,50 **1007**	59,20 **1039**	127,30 **1005**	**1020549**
X5	14,75 **875**	176,00 **969396**	16,02 **929**	23,84 **996**	6,48 **1001**	60,00 **1055**	127,10 **1007**	**975259**

method currently used in women's heptathlon (www.usatf.org), the result is shown in Tab. 5. The obtained ranking is X3>X2>X1>X4>X5; which is the same as when the calculation is done using the Grey system theory.

9 Conclusion

This paper discusses the scoring method that is currently being used in women's heptathlon. As is the case with decathlon, the results for women's heptathlon are calculated through official scoring tables that convert the separate performances in various running, jumping and throwing events into points to allow addition. The heptathletes profit from the long jump and 100 m hurdles, while the javelin and discus throw are unfavorable. The score distribution should be uniform over the disciplines. The current scoring method lacks fairness and validity as it favors some of the events and defers others.

According to the analysis in this paper it is shown that the Grey relational grade deduced by the Grey theory can be used to establish an accurate, precise, and fair evaluating model for detecting the best heptathletes. The Grey relational analysis possesses an overwhelming advantage to solve the problems that the traditional method could not overcome when there is a tie score dispute. The proposed method may provide IAAF with an improved score awarding method for heptathlon disciplines.

10 References

[1] Chang, C. L; Tsai, C. H.; Chen, L. Applying Grey Relational Analysis to the Decathlon Evaluation Model. // International Journal of The Computer, The Internet and Management, 11, 3(2003), pp. 54-62.

[2] Chen, S.; Li, Z; Xu, Q. Grey Target Theory Based Equipment Condition Monitoring and Wear Moode Recognition. // Wear, 260, 4-5(2006), pp. 438-449.

[3] Chen, P. R. Application of Grey Relational Analysis to the Prediction of FIFA World Cup. // The Journal of Grey System, 10, 3(1998), pp. 253-256.

[4] Chih Chih-Hung, T.; Ching-Liang, C.; Lieh, C. Applying Grey Relational Analysis to the Vendor Evaluation Model. // International Journal of The Computer, The Internet and Management, 11, 3(2003), pp. 45-53.

[5] Deng, J. L. Control problems of grey systems. // Systems & Control Letters, 1, 5(1982), pp. 288-294.

[6] Deng, J. L. The introduction of grey system. // The Journal of Grey System, 1, 1(1989), pp. 1-24.

[7] Han, D. The Grey Relational Analysis for Distinguished Women Marathon. // China Physical Education in Science and Technology, 32, 9(1996).

[8] Kayacan, E.; Ulutas, B.; Kaynak, O. Grey System Theory – Based Models in Time Series Prediction. // Expert systems with Applications, 37(2010), pp. 1784-1789.

[9] Liu, S.; Lin, Y. An Introduction to Grey Systems: Foundations, Methodology and Applications, Slippery Rock PA, USA : IIGS Academic Publisher, 1998.

[10] Sifen, L.; Forrest, J. The Current Developing Status on Grey System Theory. // The Journal of Grey System, 2(2007), pp. 111-123.

[11] Sun, Y. P. The Investigation of the Influence of Body Characteristics on PRC Hammer Throw Player. // Peking Sport University Press, 20, 4(1997), pp. 90-94

[12] Wang, L. The Study of Last Force Exertion on the Left Leg for PRC Javelin Throw Athlete. // Peking Sport University Press, 3, 3(1997), pp. 73-78.

[13] Westera, W. Decathlon, towards a balanced and sustainable performance assessment method. // New Studies in Athletics, IAAF. 21, 1(2006), pp. 39-51.

[14] Westera, W. Under attack: the hepthatlon scoring method. // Canadian Athletic Coaching Centre. e-paper available at: http://www.athleticscoaching.ca, 2007.

[15] Yuan, X. Grey Relational Evaluation of Financial Situation of Listed Company. // Journal of Modern Accounting and Auditing, 3, 3(2007), pp. 41-44.

[16] Yen, M. Y. Using Grey Relational Analysis to Study the Defense and Offense Techniques and Establishment of Training Goal for Ku-Tai Basketball Team. // Physical Education and Sport, 88(1993), pp. 38-45.

[17] Yu, M. L.; Wen, K. L. The Error Analysis in Gm (1,1) Model. // Journal of Chinese Grey System Association, 3, 1(2000), pp. 63-70.

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