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Traffic Management
Original Scientific Paper
Accepted: May 7, 2011
Approved: May 22, 2012

BUS TIMETABLING AS A FUZZY MULTIOBJECTIVE OPTIMIZATION PROBLEM USING PREFERENCE-BASED GENETIC ALGORITHM

ABSTRACT

Transportation plays a vital role in the development of a country and the car is the most commonly used means. However, in third world countries long waiting time for public buses is a common problem, especially when people need to switch buses. The problem becomes critical when one considers buses joining different villages and cities. Theoretically this problem can be solved by assigning more buses on the route, which is not possible due to economical problem. Another option is to schedule the buses so that customers who want to switch buses at junction cities need not have to wait long. This paper discusses how to model single frequency routes bus timetabling as a fuzzy multiobjective optimization problem and how to solve it using preference-based genetic algorithm by assigning appropriate fuzzy preference to the need of the customers. The idea will be elaborated with an example.

KEY WORDS

bus timetabling, fuzzy multiobjective optimization, genetic algorithm, fuzzy preference

1. INTRODUCTION

Transportation system is the backbone of a country's economy. Unlike developed countries, the transportation problem is widely spread in the third world countries. Most people use buses to go from one place to another. For instance, if we consider Addis Ababa, the capital of Ethiopia, the public transportation system is mainly provided by buses, which is affordable compared to taxis. It is provided by Anbesa City Bus service, which is run by the government. It is common to see people waiting for long hours at bus stations. This is especially the case if one plans to go from one place to another far place which requires switching buses. In the case of city buses customers will get a bus after a long wait, but in the case of buses joining cities and villages, if a customer misses a bus they might be even forced to spend one more night there. There are also people who use one bus and switch to another bus when they reach one city or village. Basically, this problem can be solved by assigning as many buses as necessary on each route and or assigning another means of transportation. However, this needs time and economical strength of the country. Hence it is unlikely, for the developing countries, to do so in the near future. The other option is to set a timetable for the buses in such a way that the customers who switch buses at junction villages wait as short as possible.

A lot of studies have been conducted on transportation and transit problems. Heuristic solution algorithms, mainly genetic algorithm, have been used in a number of papers. Bin Yu uses genetic algorithm for the bi-level model of bus frequency optimization [1]. Lei Zhon et al. use systematic improved genetic algorithm in public traffic dispatch system [2]. Kidwai et al. use genetic algorithm in allocation of buses in transit network [3]. Stefancic et al. use genetic algorithm in organizing passenger transport [4]. A lot more studies have been conducted regarding this aspect [5, 6, 7]. Furthermore, Gupta et al. used satisfaction as fuzzy input to solve multiobjective fuzzy routing problem [8]; and solving city bus scheduling problem using Eligen-algorithm was proposed by Surapholchoi [9]. A detailed review on the school bus routing problem can also be found in [10]. Ren et al. [11] takes the problem from the customer satisfaction and the bus agency's benefit viewpoint and uses the hybrid genetic and simulated annealing method to solve the problem [11].

Most of the studies model timetabling using graph theory. It is a good idea to explore different possible models and the corresponding solution methods. This paper discusses and shows how to model setting the timetabling of buses joining different cities, with the objective of minimizing the waiting time of customers who want to switch buses at junction villages, as a fuzzy multiobjective optimization problem and how to solve it using the genetic algorithm by assigning appropriate dynamic weights for each objective function. In the next section the basic concepts will be discussed followed by a discussion on formulation of the problem as a fuzzy multiobjective optimization problem. Section 4 discusses how to use a preference-based genetic algorithm by assigning dynamic weights for the modeled fuzzy objective functions and is followed by a hypothetical example. Finally, a conclusion on the study is provided.

2. PROBLEM DEFINITION AND PRELIMINARIES

2.1 The problem

One of the main problems in the developing world is the transportation problem. It is possible to take Ethiopia as an example. The main transportation system for the public service are the public buses. There are city buses in the capital city and there are different buses to transport customers from one city to another. It is common to see people waiting long hours for buses, in the capital. The problem becomes worse when we consider buses joining cities. Unlike city bus service there may not be another bus option if one misses a bus in going from one city to another. Hence, customers missing a bus may need to spend another night. In this paper we consider the frequency of buses on each route to be one, limited resource case. Assigning more buses on the routes would have solved the problem, which is very difficult due to the limitation of resource. But it is possible to set a timetable for buses so that customers switching buses will not wait long at junction villages. This paper focuses on the problem of setting a timetable for buses joining different cities or villages. This is to ensure that customers switching buses at different points need not wait long in the switching process. Even though we mention Ethiopia as an example it is a common problem in the third world.

2.2 Multiobjective optimization

An optimization problem is a problem of optimizing, either minimizing or maximizing, a given function known as the objective function, by choosing a value for the variable from a set known as feasible set. If one has a maximization problem it is easy to change it to a minimization problem by multiplying the objective function by negative one. The same will hold in changing minimization problem to maximization problem. Consider a minimization problem, given in equation (1).

$$\min_{s.t\ x\in S} f(x) \tag{1}$$

Here $f:\Re^n \to \Re$ and $S \subseteq \Re^n$. If $S = \Re^n$ then the problem is known as an unconstrained minimization problem.

A solution, say x^*, for equation (1) should satisfy the following condition: $x^* \in S$ and $f(x^*) \le f(x), \forall x \in S$.

A multiobjective optimization problem is an optimization problem in which the objective function is a vector function as in equation (2).

$$\min_{s.t.} F(x) = (f_1(x), f_2(x), \ldots, f_m(x)) \tag{2}$$

Here $F:\Re^n \to \Re^m$, $f_i:\Re^n \to \Re$ for $i \in \{1,2,\ldots m\}$ and $S \subseteq \Re^n$.

Unlike an optimization problem with single objective, it is not possible to compare all outcomes. For instance, for $m = 2$, consider $F(x') = (2,1)$ and $F(x'') = (0,2)$. Here x' is better in terms of the second function but not in the first function. Hence the functional value is not totally ordered. A member of the feasible region whose outcome is not dominated by any other outcome of the feasible set is known as Pareto optimal. This means, x' is said to be a Pareto solution if and only if there does not exist another x^* in the feasible region so that $f_i(x^*) \le f_i(x')$ for all i and strictly for at least one i. Choosing the best among the set of Pareto solutions depends on the preference of the decision-maker.

A fuzzy optimization problem is an optimization problem with fuzzy objective function and/or fuzzy constraints. This paper considers a multiobjective optimization problem with an objective function and constraint inequalities involving fuzzy and crisp numbers.

2.3 Preference-based genetic algorithm

Genetic Algorithm (GA) is an evolutionary algorithm in which the population is expressed as a chromosome of 0s and 1s. The population-based algorithm becomes the main tool in solving multiobjective optimization problems because within a single run, one can get many possible solutions. Applying the genetic algorithm to solve multiobjective optimization problems has been a hot research topic for different researchers. Incorporating decision-maker's fuzzy preference so that the algorithm converges to the efficient region in which the maker's preference lies has been discussed by different researchers [12, 13, 14]. It has also been discussed on how to generate a random weight for each objective function in order to construct the fitness function using the fuzzy preference of the decision-maker. Basically, a genetic algorithm has the following steps:

1. randomly generate a solution population;
2. choose some members with a probability that depends on their fitness;
3. perform crossover and mutation on the selected solutions;

4. construct a new population by taking the fittest;
5. if termination criteria are met stop, else go back to step 2.

A preference-based GA for a multiobjective optimization problem is a genetic algorithm which incorporates a preference. The preference is used in generating a dynamic weight for each objective function and will be incorporated in the fitness evaluation stage of the GA.

3. FUZZY MULTIOBJECTIVE FORMULATION OF THE PROBLEM

The first step in applying an optimization problem solving technique to a real life problem is to model the problem as an optimization problem. The assumptions are that the frequency of buses on each route is one per day and there is no space limitation which means that the customers switching buses at the junction point can get a place on the bus. It is necessary to identify the decision variables, the objective function and the feasible region and express them mathematically. In our case, the objective is to set a timetable for buses in such a way that the customers who want to switch buses at junction points will not wait long. In other words, we need buses to reach at around the same time the junction points with appropriate waiting times. To do so, we take the initial time of one of the buses as reference. Hence, if we have n buses there will be $n-1$ decision variables without the waiting time. Let t_P^Q be the initial time for bus from P to Q, $\tilde{t}_{NM}$ be the time for a bus from N to reach M ($\tilde{t}_{NM}$ is a fuzzy number) and w_N^{PQ} be the waiting time for a bus from P to Q at station N. Furthermore, let $\mu_{NM}(t)$ be the membership function of $\tilde{t}_{NM}$. It is possible to construct a probability density function based on the membership function by giving high probability for members with high membership function [14]. Hence, let $g_{NM}(t)$ be the probability density function of $\tilde{t}_{NM}$. $g_{NM}(t)$ can be constructed after collecting appropriate data and constructing the fuzzy number. The decision variables are the initial time for all buses except the reference one and the waiting time of buses at junction stations. Furthermore, let bus NM mean a bus with starting station N and destination M. Hence, for bus NM, $(N\ A_1\ A_2\ \ldots\ A_v\ M)$ is the route vector and $(g_{NA_1}(t)\ g_{A_1A_2}(t)\ \ldots\ g_{A_vM}(t))$ is the probability distribution vector of the time needed to go between the junctions. This means bus NM passes through A_i's in the given order to reach station M from N and the time needed to go from any station A_i to A_j is a time with the probability density function $g_{A_iA_j}(t)$, A_i can be N and A_j can be M. This can describe the whole route of the bus. If $g_{AB}(t)$ is the same as $g_{BA}(t)$ for all junctions of bus NM then the same route vector and the probability distribution vector for bus NM can also be used for bus MN. But in some cases $g_{AB}(t)$ is different from $g_{BA}(t)$. The time needed to go from station A to B for two different stations, $\tilde{t}_{AB}$, is generated from the probability density function $g_{AB}(t)$. There are three basic cases to be considered as shown in *Figure 1*.

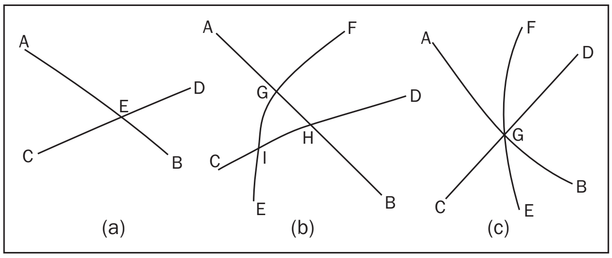

Figure 1 - Possible route scenarios

Consider *Figure 1* (a): In this figure we have four buses AB, BA, CD and DC. Let us take the initial time for bus AB to be zero, as reference. Passengers may want to switch buses at E. For instance, a passenger may want to go from A to D and another from C to B; it is accomplished if both buses AB and CD reach E almost at the same time with appropriate waiting time. To go from A to D, the bus AB should arrive at E first. Even if the bus CD arrives at E first, it needs to wait for bus AB, say half of its waiting time while the remaining half will be used by the passengers to switch. Determining the ratio of waiting time is subjective, but taking half is quite reasonable, the time needed to switch between buses should be less than half of the waiting time. We need to minimize the waiting time which means that we need to minimize f_1 given by:

$$f_1 = \left| \tilde{t}_{AE} + \frac{w_E^{AB}}{2} - t_C^D - \tilde{t}_{CE} - \frac{w_E^{CD}}{2} \right| \tag{3}$$

Similarly, for passengers going from A to C and D to B we will have the following function to minimize:

$$f_2 = \left| \tilde{t}_{AE} + \frac{w_E^{AB}}{2} - t_D^C - \tilde{t}_{DE} - \frac{w_E^{DC}}{2} \right| \tag{4}$$

For those who need to go from B to D and from C to A; and from B to C and from D to A we will have the following two functions to minimize respectively,

$$f_3 = \left| t_B^A + \tilde{t}_{BE} + \frac{w_E^{BA}}{2} - t_C^D - \tilde{t}_{CE} - \frac{w_E^{CD}}{2} \right| \tag{5}$$

$$f_4 = \left| t_B^A + \tilde{t}_{BE} + \frac{w_E^{BA}}{2} - t_D^C - \tilde{t}_{DE} - \frac{w_E^{DC}}{2} \right| \tag{6}$$

Hence, the objective function will be $\phi = (f_1, f_2, f_3, f_4)$. The decision variables are the initial time of the three buses, t_B^A, t_C^D and t_D^C, and the waiting time of the buses, w_E^{AB}, w_E^{BA}, w_E^{CD} and w_E^{DC}.

Next, the limitations on the decision variables need to be set. Suppose the minimum time needed for a customer to switch from bus NM to bus PQ at junction station I be $\delta_I^{NM \to PQ}$, for any route NM and PQ and any common junction I of NM and PQ. Furthermore, bus CD and DC should arrive at E at least $\delta_E^{CD \to AB}$ and

$\delta_E^{DC\to AB}$ unit of time, respectively, before bus AB leaves E. Hence

$$\tilde{t}_{AE} + w_E^{AB} - \delta_E^{DC\to AB} > t_D^C + \tilde{t}_{DE} \text{ and}$$

$$\tilde{t}_{AE} + w_E^{AB} - \delta_E^{CD\to AB} > t_C^D + \tilde{t}_{CE}.$$

Similarly, there are two limitations for bus BA given as

$$t_B^A + \tilde{t}_{BE} + w_E^{BA} - \delta_E^{DC\to BA} > t_D^C + \tilde{t}_{DE} \text{ and}$$

$$t_B^A + \tilde{t}_{BE} + w_E^{BA} - \delta_E^{CD\to BA} > t_C^D + \tilde{t}_{CE}.$$

Similarly, for bus CD and DC we have

$$t_C^D + \tilde{t}_{CE} + w_E^{CD} - \delta_E^{AB\to CD} > \tilde{t}_{AE} \text{ \&}$$
$$t_C^D + \tilde{t}_{CE} + w_E^{CD} - \delta_E^{BA\to CD} > t_B^A + \tilde{t}_{BE},$$

and

$$t_D^C + \tilde{t}_{DE} + w_E^{DC} - \delta_E^{AB\to DC} > \tilde{t}_{AE} \text{ \&}$$
$$t_D^C + \tilde{t}_{DE} + w_E^{DC} - \delta_E^{BA\to DC} > t_B^A + \tilde{t}_{BE},$$

respectively. Furthermore, let w_N^{PQ} be the weighting time of bus PQ at junction N. There are upper and lower bounds for w_N^{PQ}, the lower bound can be taken as the minimum of the switching time means
$\delta = \min\left\{d_i^{NM\to PQ} \,\middle|\, \text{for all routes NM and PQ and junctions i}\right\}$,
and $\left|t_P^Q\right| \le t_{\max}$, for some number $t_{\max}$.

Hence, the problem will become:

$$\min_{s.t.} \quad \phi = (f_1, f_2, f_3, f_4) \tag{7}$$
$$\begin{aligned}
&\tilde{t}_{AE} + w_E^{AB} - \delta_E^{DC\to AB} > t_D^C + \tilde{t}_{DE}\\
&\tilde{t}_{AE} + w_E^{AB} - \delta_E^{CD\to AB} > t_C^D + \tilde{t}_{CE}\\
&t_B^A + \tilde{t}_{BE} + w_E^{BA} - \delta_E^{DC\to BA} > t_D^C + \tilde{t}_{DE}\\
&t_B^A + \tilde{t}_{BE} + w_E^{BA} - \delta_E^{CD\to BA} > t_C^D + \tilde{t}_{CE}\\
&t_C^D + \tilde{t}_{CE} + w_E^{CD} - \delta_E^{AB\to CD} > \tilde{t}_{AE}\\
&t_C^D + \tilde{t}_{CE} + w_E^{CD} - \delta_E^{BA\to CD} > t_B^A + \tilde{t}_{BE}\\
&t_D^C + \tilde{t}_{DE} + w_E^{DC} - \delta_E^{AB\to DC} > \tilde{t}_{AE}\\
&t_D^C + \tilde{t}_{DE} + w_E^{DC} - \delta_E^{BA\to DC} > t_B^A + \tilde{t}_{BE}\\
&\left|t_P^Q\right| \le t_{\max} \forall PQ\\
&\delta \le w_N^{PQ} \le w_{\max} \forall PQ \text{ and } N
\end{aligned}$$

The second basic case is as shown in *Figure 1* (b): In this case there are six buses. So, without waiting time there are five decision variables, $t_A^B = 0$. Furthermore, one needs to be careful in modelling the problem so as not to face a contradiction. For example, if we model it in such a way that bus EF, BA and FE arrives almost at the same time as bus AB at junction G (EF and BA should start earlier than AB and FE); bus CD arrives almost at the same time with bus EF at junction I (CD should also start earlier) contradicts with bus BA and bus CD reaches almost at the same time junction H. So, it is necessary to eliminate these kinds of self-contradicting cases. This kind of criteria or limitations can be expressed in the constraint set.

In the current case for junction G, bus AB and FE need to reach it almost at the same time; hence

$$f_5 = \left| \tilde{t}_{AG} + \frac{w_G^{AB}}{2} - t_F^E - \tilde{t}_{FG} - \frac{w_G^{FE}}{2} \right| \tag{8}$$

So does bus EF and BA:

$$f_6 = \left| t_B^A + \tilde{t}_{BH} + w_H^{BA} + \tilde{t}_{HG} + \frac{w_G^{BA}}{2} - \right.$$
$$\left. - t_E^F - \tilde{t}_{EI} - w_I^{EF} - \tilde{t}_{IG} - \frac{w_G^{EF}}{2} \right| \tag{9}$$

And at junction I there will be:

$$f_7 = \left| t_C^D + \tilde{t}_{CI} + \frac{w_I^{CD}}{2} - t_E^F - \tilde{t}_{EI} - \frac{w_I^{EF}}{2} \right| \tag{10}$$

$$f_8 = \left| t_D^C + \tilde{t}_{DH} + w_H^{DC} + \tilde{t}_{HI} + \frac{w_I^{DC}}{2} - \right.$$
$$\left. - t_F^E - \tilde{t}_{FG} - w_G^{FE} - \tilde{t}_{GI} - \frac{w_I^{EF}}{2} \right| \tag{11}$$

Similarly for junction H:

$$f_9 = \left| t_B^A + \tilde{t}_{BH} + \frac{w_H^{BA}}{2} - t_D^C - \tilde{t}_{DH} - \frac{w_H^{DC}}{2} \right| \tag{12}$$

$$f_{10} = \left| \tilde{t}_{AG} + w_G^{AB} + \tilde{t}_{GH} + \frac{w_H^{AB}}{2} - \right.$$
$$\left. - t_C^D - \tilde{t}_{CI} - w_I^{CD} - \tilde{t}_{IH} - \frac{w_H^{CD}}{2} \right| \tag{13}$$

Hence, the objective function will be

$$\varphi = (f_5, f_6, f_7, f_8, f_9, f_{10}).$$

And the constraint set can be constructed in the same way as in the previous case. After all, the problem will be:

$$\min_{s.t.} \quad \varphi = (f_5, f_6, f_7, f_8, f_9, f_{10}) \tag{14}$$
$$\begin{aligned}
&\tilde{t}_{AG} + w_G^{AB} - \delta_G^{FE\to AB} > t_F^E + \tilde{t}_{FG}\\
&t_F^E + \tilde{t}_{FG} + w_G^{FE} - \delta_G^{AB\to FE} > \tilde{t}_{AG}\\
&t_B^A + \tilde{t}_{BH} + w_H^{BA} + \tilde{t}_{HG} + w_G^{BA} - \delta_G^{EF\to BA} > t_F^E + \tilde{t}_{EI} + w_I^{EF} + \tilde{t}_{IG}\\
&t_E^F + \tilde{t}_{EI} + w_I^{EF} + \tilde{t}_{IG} + w_G^{EF} - \delta_G^{BA\to EF} > t_B^A + \tilde{t}_{BH} + w_H^{BA} + \tilde{t}_{HG}\\
&t_B^A + \tilde{t}_{BH} + w_H^{BA} - \delta_H^{DC\to BA} > t_D^C + \tilde{t}_{DH}\\
&t_D^C + \tilde{t}_{DH} + w_H^{DC} - \delta_H^{BA\to DC} > t_B^A + \tilde{t}_{BH}\\
&\tilde{t}_{AG} + w_G^{AB} + \tilde{t}_{GH} + w_H^{AB} - \delta_H^{CD\to AB} > t_C^D + \tilde{t}_{CI} + w_I^{CD} + \tilde{t}_{IH}\\
&t_C^D + \tilde{t}_{CI} + w_I^{CD} + \tilde{t}_{IH} + w_H^{CD} - \delta_H^{AB\to CD} > \tilde{t}_{AG} + w_G^{AB} + \tilde{t}_{GH}\\
&t_C^D + \tilde{t}_{CI} + w_I^{CD} - \delta_I^{EF\to CD} > t_E^F + \tilde{t}_{EI}\\
&t_E^F + \tilde{t}_{EI} + w_I^{EF} - \delta_I^{CD\to EF} > t_C^D + \tilde{t}_{CI}\\
&t_D^C + \tilde{t}_{DH} + w_H^{DC} + \tilde{t}_{HI} + w_I^{DC} - \delta_I^{FE\to DC} > t_F^E + \tilde{t}_{FG} + w_G^{FE} + \tilde{t}_{GI}\\
&t_F^E + \tilde{t}_{FG} + w_G^{FE} + \tilde{t}_{GI} + w_I^{FE} - \delta_I^{DC\to FE} > t_D^C + \tilde{t}_{DH} + w_H^{DC} + \tilde{t}_{HI}\\
&\left|t_P^Q\right| \le t_{\max} \forall PQ\\
&\delta \le w_N^{PQ} \le w_{\max} \forall PQ \text{ and } N
\end{aligned}$$

And consider *Figure 1*(c): Here, there are six buses passing through a common station G. Note that if the problem is modelled as bus AB and CD arriving almost at the same time at G and bus CD and EF reaching almost at the same time G, it means that bus AB and EF will reach almost at the same time G. Hence, we can consider two bus lines separately. First, consider route AB and CD, and set $t_A^B = 0$, as reference.

$$f_{11} = \left| \tilde{t}_{AG} + \frac{w_G^{AB}}{2} - t_C^D - \tilde{t}_{CG} - \frac{w_G^{CD}}{2} \right| \tag{15}$$

$$f_{12} = \left| \tilde{t}_{AG} + \frac{w_G^{AB}}{2} - t_D^C - \tilde{t}_{DG} - \frac{w_G^{DC}}{2} \right| \tag{16}$$

$$f_{13} = \left| t_B^A + \tilde{t}_{BG} + \frac{w_G^{BA}}{2} - t_C^D - \tilde{t}_{CG} - \frac{w_G^{CD}}{2} \right| \tag{17}$$

$$f_{14} = \left| t_B^A + \tilde{t}_{BG} + \frac{w_G^{BA}}{2} - t_D^C - \tilde{t}_{DG} - \frac{w_G^{DC}}{2} \right| \tag{18}$$

and by considering route CD and EF:

$$f_{15} = \left| t_C^D + \tilde{t}_{CG} + \frac{w_G^{CD}}{2} - t_E^F - \tilde{t}_{EG} - \frac{w_G^{EF}}{2} \right| \tag{19}$$

$$f_{16} = \left| t_C^D + \tilde{t}_{CG} + \frac{w_G^{CD}}{2} - t_F^E - \tilde{t}_{FG} - \frac{w_G^{FE}}{2} \right| \tag{20}$$

$$f_{17} = \left| t_D^C + \tilde{t}_{DG} + \frac{w_G^{DC}}{2} - t_E^F - \tilde{t}_{EG} - \frac{w_G^{EF}}{2} \right| \tag{21}$$

$$f_{18} = \left| t_D^C + \tilde{t}_{DG} + \frac{w_G^{DC}}{2} - t_F^E - \tilde{t}_{FG} - \frac{w_G^{FE}}{2} \right| \tag{22}$$

Hence, the objective function will be

$$\gamma = (f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}, f_{18}).$$

In constructing the constraint set we need to consider all combinations of routes. And finally the problem becomes:

$$\begin{aligned}
\min_{s.t.} \gamma = (f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}, f_{18}) \quad (23)\\
&\tilde{t}_{AG} + w_G^{AB} - \delta_G^{CD\to AB} > t_C^D + \tilde{t}_{CG}\\
&\tilde{t}_{AG} + w_G^{AB} - \delta_G^{DC\to AB} > t_D^C + \tilde{t}_{DG}\\
&\tilde{t}_{AG} + w_G^{AB} - \delta_G^{EF\to AB} > t_E^F + \tilde{t}_{EG}\\
&\tilde{t}_{AG} + w_G^{AB} - \delta_G^{FE\to AB} > t_F^E + \tilde{t}_{FG}\\
&t_B^A + \tilde{t}_{BG} + w_G^{BA} - \delta_G^{CD\to BA} > t_C^D + \tilde{t}_{CG}\\
&t_B^A + \tilde{t}_{BG} + w_G^{BA} - \delta_G^{DC\to BA} > t_D^C + \tilde{t}_{DG}\\
&t_B^A + \tilde{t}_{BG} + w_G^{BA} - \delta_G^{EF\to BA} > t_E^F + \tilde{t}_{EG}\\
&t_B^A + \tilde{t}_{BG} + w_G^{BA} - \delta_G^{FE\to BA} > t_F^E + \tilde{t}_{FG}\\
&t_C^D + \tilde{t}_{CG} + w_G^{CD} - \delta_G^{AB\to CD} > \tilde{t}_{AG}\\
&t_C^D + \tilde{t}_{CG} + w_G^{CD} - \delta_G^{BA\to CD} > t_B^A + \tilde{t}_{BG}\\
&t_C^D + \tilde{t}_{CG} + w_G^{CD} - \delta_G^{EF\to CD} > t_E^F + \tilde{t}_{EG}\\
&t_C^D + \tilde{t}_{CG} + w_G^{CD} - \delta_G^{FE\to CD} > t_F^E + \tilde{t}_{FG}\\
&t_D^C + \tilde{t}_{DG} + w_G^{DC} - \delta_G^{AB\to DC} > \tilde{t}_{AG}\\
&t_D^C + \tilde{t}_{DG} + w_G^{DC} - \delta_G^{BA\to DC} > t_B^A + \tilde{t}_{BG}\\
&t_D^C + \tilde{t}_{DG} + w_G^{DC} - \delta_G^{EF\to DC} > t_E^F + \tilde{t}_{EG}\\
&t_D^C + \tilde{t}_{DG} + w_G^{DC} - \delta_G^{FE\to DC} > t_F^E + \tilde{t}_{FG}\\
&t_E^F + \tilde{t}_{EG} + w_G^{EF} - \delta_G^{AB\to EF} > \tilde{t}_{AG}\\
&t_E^F + \tilde{t}_{EG} + w_G^{EF} - \delta_G^{BA\to EF} > t_B^A + \tilde{t}_{BG}\\
&t_E^F + \tilde{t}_{EG} + w_G^{EF} - \delta_G^{CD\to EF} > t_C^D + \tilde{t}_{CG}\\
&t_E^F + \tilde{t}_{EG} + w_G^{EF} - \delta_G^{DC\to EF} > t_D^C + \tilde{t}_{DG}\\
&t_F^E + \tilde{t}_{FG} + w_G^{FE} - \delta_G^{AB\to FE} > \tilde{t}_{AG}\\
&t_F^E + \tilde{t}_{FG} + w_G^{FE} - \delta_G^{BA\to FE} > t_B^A + \tilde{t}_{BG}\\
&t_F^E + \tilde{t}_{FG} + w_G^{FE} - \delta_G^{CD\to FE} > t_C^D + \tilde{t}_{CG}\\
&t_F^E + \tilde{t}_{FG} + w_G^{FE} - \delta_G^{DC\to FE} > t_D^C + \tilde{t}_{DG}\\
&\left| t_P^Q \right| \le t_{\max} \forall PQ\\
&\delta \le w_N^{PQ} \le w_{\max} \forall PQ \text{ and } N
\end{aligned}$$

By combining these basic cases it is possible to model a complex system with many routes.

Note that the tea time, lunch time, etc., are not taken as a variable. For instance, on the route from N to M the bus may stop for tea. It is possible to consider the tea time as a decision variable using appropriate lower and upper limits. But in this paper it is not considered as a variable and simply included in $\tilde{t}_{NM}$. And for all the above formulations $\tilde{t}_{IJ}$ is generated from the probability density function $g_{IJ}(t)$, which gives high probability to members with high membership function value.

4. SOLVING THE PROBLEM BY USING THE PREFERENCE-BASED GENETIC ALGORITHM

In this section, the method to solve the modelled fuzzy multi-objective optimization problem using preference-based genetic algorithm will be discussed. To solve the problem using a preference-based genetic algorithm a preference should be given in order to update the fitness function. A sound preference can be obtained by collecting data on the number of passengers who want to switch buses at junction villages. Consider village L; the number of customers who will switch from bus PQ to bus NM and from bus NM to bus PQ at this village can be described as a fuzzy number in which the membership value is higher around $w_L^{PQ\leftrightarrow NM}$ and keeps on decreasing as one gets further away from $w_L^{PQ\leftrightarrow NM}$. Furthermore, the membership value will be zero outside the interval $[w_L^{PQ\leftrightarrow NM} - a_L^{PQ\leftrightarrow NM}, w_L^{PQ\leftrightarrow NM} + b_L^{PQ\leftrightarrow NM}]$. Define $a_L^{PQ\leftrightarrow NM}$ and $b_L^{PQ\leftrightarrow NM}$ to be the left and right width, respectively. One needs to collect data on the number of passengers for possible bus switches according to the objective functions. Once the necessary data on the number of passengers are collected, it is possible to take the average fuzzy number and generate a dynamic weight for each objective function. Suppose the j^{th} objective function is to minimize the arriving time difference of bus NM and PQ at village L. The number of passengers benefiting from this can be expressed fuzzily on the interval $[w_L^{PQ\leftrightarrow NM} - a_L^{PQ\leftrightarrow NM}, w_L^{PQ\leftrightarrow NM} + b_L^{PQ\leftrightarrow NM}]$ or simply $[w_j - a_j, w_j + b_j]$. Consequently, the average fuzzy number, $[\bar{w}_j - \bar{a}_j, \bar{w}_j + \bar{b}_j]$ can be found as follows:

$$\bar{w}_j = \frac{w_j}{\sum_{i=1}^{m} w_i} \tag{24}$$

$$\bar{a}_j = \frac{a_j}{\sum_{i=1}^{m} a_i} \tag{25}$$

$$\bar{b}_j = \frac{b_j}{\sum_{i=1}^{m} b_i} \tag{26}$$

From these it is possible to determine the appropriate probability distribution, say $h_j(w)$, to generate a random weight, which will be incorporated in the fitness evaluation step of GA [15]. The algorithm can be generalized as shown in *Table 1*.

5. HYPOTHETICAL EXAMPLE

Consider a bus network connecting different villages or cities. Suppose the network has ten bus routes with frequency of buses one per day and with given route vector and probability density vector. The time needed to go from one village to another can be expressed using a Gaussian probability distribution with the given average and standard deviation of 6 minutes or 0.1 hours. Let the network be described by the following route and probability density vector. (A I B) is a route vector of bus AB with probability distribution vector is $(\mathcal{N}(4, 0.1^2)\ \ \mathcal{N}(3, 0.1^2))$, which means the bus will go from A to B through I and the time to go from A to I is normally distributed with mean 4 hours with standard deviation 0.1 and to go from I to B with mean 3 hours with standard deviation 0.1. The time unit in this example is in hours. Furthermore, suppose the probability distribution to go from any villages or cities N to M is the same as going from M to N, hence for bus BA we will have (B I A)

Table 1 – Preference-based GA

Input

$f_j(x), x \in \Re^n, j \in \{1,2,\ldots,m\} \leftarrow$ Objective functions and decision variables

$P_r, P_m \leftarrow$ Probability of reproduction and mutation

$h_j(w) \leftarrow$ Probability density function of the dynamic weight for each of the objective functions

$g_{NM}(t) \leftarrow$ Probability density function for time needed to go from N to M

Output

$x_i, i \in \{1,2,\ldots,k\} \leftarrow$ solutions for the minimization problem from the feasible set

Begin

  for $i = 1:k$

    $x_i \leftarrow$ Generate initial population from the feasible region

  end for

  Repeat

    for $j = 1:m$

      $w_j \leftarrow$ Generate weight using the $h_i(w)$

    end for

    $\tilde{t}_{NM} \leftarrow$ using $g_{NM}(t)$, for all routes NM

    for $i = 1:k$

      $fun_i = \sum_{j=1}^{m} w_j f_j(x_i) \leftarrow$ Calculate fitness of each member of the population solution

    end for

    count = 1

    do while $(2 * count < k)$

      $x_n, x_m \leftarrow$ Choose two parents with a probability associated with the fitness

      if$(rand \le P_r)$

        $y_{2*count-1}, y_{2*count} \leftarrow$ reproduction

      else if

        $y_{2*count-1} = x_n; y_{2*count} = x_m$

      end if

      for $i = 1:2$

        if$(rand \le P_m)$

          $y_{2*count+1-i} \leftarrow$ mutation

        end if

        if$(y_{2*count+1-i} \notin S) \leftarrow$ check feasibility

          $y_{2*count+1-i} = x_i$

        end if

      end for

      count = *count* + 1

    end do while

    $z = sort(x,y)$

    for $i = 1:k$

      $x_i = z_i$

    end for

  until *condition is met*

End

and $(\mathcal{N}(3,0.1^2)\ \mathcal{N}(4,0.1^2))$. Similarly we can have, for bus AC (A J C) $(\mathcal{N}(3.75,0.1^2)\ \mathcal{N}(3.25,0.1^2))$; for bus AD (A L K D) $(\mathcal{N}(3.25,0.1^2)\ \mathcal{N}(4,0.1^2)\ \mathcal{N}(1.75,0.1^2))$; for bus EF (E K J I F) $(\mathcal{N}(1,0.1^2)\ \mathcal{N}(3,0.1^2)\ \mathcal{N}(3.5,0.1^2)\ \mathcal{N}(2.5,0.1^2))$ and for bus GH (G L J H) $(\mathcal{N}(1.75,0.1^2)\ \mathcal{N}(2.75,0.1^2)\ \mathcal{N}(3.5,0.1^2))$. The routes are shown in *Figure 2*.

Let us set the initial time for bus AB as reference, which means put $t_A^B = 0$, the decision variables are then $t_A^C, t_A^D, t_A^B, t_B^A, t_C^A, t_D^A, t_E^F, t_F^E, t_G^H, t_H^G$ and the waiting times at junction villages. By eliminating possible contradictions the objective functions can be formulated as follows:

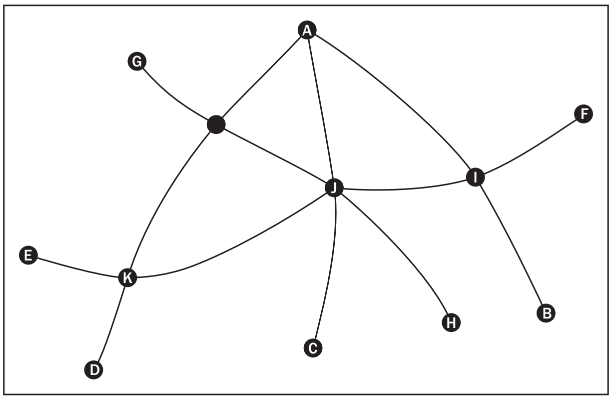

Figure 2 - Bus routes of the example

$$f_1 = \left| \tilde{t}_{AI} + \frac{w_I^{AB}}{2} - t_F^E - \tilde{t}_{FI} - \frac{w_I^{FE}}{2} \right|,$$

$$f_2 = \left| t_A^C + \tilde{t}_{AJ} + \frac{w_J^{AC}}{2} - t_G^H - \tilde{t}_{GL} - w_L^{GH} - \tilde{t}_{LJ} - \frac{w_J^{GH}}{2} \right|,$$

$$f_3 = \left| t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ} + \frac{w_J^{FE}}{2} - t_A^C - \tilde{t}_{AJ} - \frac{w_J^{AC}}{2} \right|,$$

$$f_4 = \left| t_B^A + \tilde{t}_{BI} + \frac{w_I^{BA}}{2} - t_F^E - \tilde{t}_{FI} - \frac{w_I^{FE}}{2} \right|$$

$$f_5 = \left| t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ} + w_J^{FE} + \tilde{t}_{JK} + \frac{w_K^{FE}}{2} - t_A^D - \right.$$
$$\left. - \tilde{t}_{AL} - w_L^{AD} - \tilde{t}_{LK} - \frac{w_K^{AD}}{2} \right|$$

$$f_6 = \left| t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ} + \frac{w_J^{FE}}{2} - t_H^G - \tilde{t}_{HJ} - \frac{w_J^{HG}}{2} \right|$$

$$f_7 = \left| t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ} + \frac{w_J^{FE}}{2} - t_G^H - \tilde{t}_{GL} - w_L^{GH} - \tilde{t}_{LJ} - \frac{w_J^{GH}}{2} \right|$$

$$f_8 = \left| t_C^A + \tilde{t}_{CJ} + \frac{w_J^{CA}}{2} - t_E^F - \tilde{t}_{EK} - w_K^{EF} - \tilde{t}_{KJ} - \frac{w_J^{EF}}{2} \right|$$

$$f_9 = \left| t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + w_J^{EF} + \tilde{t}_{JI} + \frac{w_I^{EF}}{2} - \tilde{t}_{AI} - \frac{w_I^{AB}}{2} \right|$$

$$f_{10} = \left| t_C^A + \tilde{t}_{CJ} + \frac{w_J^{CA}}{2} - t_F^E - \tilde{t}_{FI} - w_I^{FE} - \tilde{t}_{IJ} - \frac{w_J^{FE}}{2} \right|$$

$$f_{11} = \left| t_C^A + \tilde{t}_{CJ} + \frac{w_J^{CA}}{2} - t_G^H - \tilde{t}_{GL} - w_L^{GH} - \tilde{t}_{LJ} - \frac{w_J^{GH}}{2} \right|$$

$$f_{12} = \left| t_D^A + \tilde{t}_{DK} + \frac{w_K^{DA}}{2} - t_E^F - \tilde{t}_{EK} - \frac{w_K^{EF}}{2} \right|$$

$$f_{13} = \left| t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + \frac{w_J^{EF}}{2} - t_A^C - \tilde{t}_{AJ} - \frac{w_J^{AC}}{2} \right|$$

$$f_{14} = \left| t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + \frac{w_J^{EF}}{2} - t_G^H - \tilde{t}_{GL} - w_L^{GH} - \tilde{t}_{IJ} - \frac{w_J^{GH}}{2} \right|$$

$$f_{15} = \left| t_A^C + \tilde{t}_{AJ} + \frac{w_J^{AC}}{2} - t_H^G - \tilde{t}_{HJ} - \frac{w_J^{HG}}{2} \right|$$

$$f_{16} = \left| t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + \frac{w_J^{EF}}{2} - t_H^G - \tilde{t}_{HJ} - \frac{w_J^{HG}}{2} \right|$$

$$f_{17} = \left| t_C^A + \tilde{t}_{CJ} + \frac{w_J^{CA}}{2} - t_H^G - \tilde{t}_{HJ} - \frac{w_J^{HG}}{2} \right|$$

For instance, to go from D to E, the passenger has to wait for bus FE at junction K, which can be put under the constraint set. If we try to put it in the objective function it may result in a contradiction that bus FE and DK should reach almost at the same time village K, but FE to reach K will take more time than bus DA to reach junction K. So, this kind of criteria can be put in the constraint set. So that customers from bus DA should reach K at least 0.2 hours before bus FE arrives. The objective function will be then

$$F = (f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}) \quad (27)$$

The next step in mathematical formulation is formulating the limitation or the formulating the constraint set.

$$S = \{(t_A^C, t_A^D, t_B^A, t_C^A, t_D^A, t_E^F, t_F^E, t_G^H, t_H^G, w_I^{AB}, w_I^{BA}, w_I^{EF}, w_I^{FE}, w_J^{AC},$$
$$w_J^{CA}, w_J^{EF}, w_J^{FE}, w_J^{GH}, w_J^{HG}, w_K^{AD}, w_K^{DA}, w_K^{EF}, w_K^{FE}, w_L^{AD}, w_L^{DA},$$
$$w_L^{GH}) \in \Re^{27}: t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + w_J^{EF} + \tilde{t}_{JI} + w_I^{EF} -$$
$$- \delta_I^{AB \to EF} > \tilde{t}_{AI}, t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + w_J^{EF} + \tilde{t}_{JI} + w_I^{EF} -$$
$$- \delta_I^{BA \to EF} > t_B^A + \tilde{t}_{BI}, \tilde{t}_{AI} + w_I^{AB} - \delta_I^{FE \to AB} > t_F^E + \tilde{t}_{FI}, t_F^E +$$
$$+ \tilde{t}_{FI} + w_I^{FE} - \delta_I^{BA \to FE} > t_B^A + \tilde{t}_{BI}, \tilde{t}_{AI} + w_I^{AB} - \delta_I^{EF \to AB} >$$
$$> t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + w_J^{EF} + \tilde{t}_{JI}, t_B^A + \tilde{t}_{BI} + w_I^{BA} -$$
$$- \delta_I^{FE \to BA} > t_F^E + \tilde{t}_{FI}, t_D^A + \tilde{t}_{DK} + w_K^{DA} - \delta_K^{EF \to DA} > t_E^F + \tilde{t}_{EK},$$
$$t_D^A + \tilde{t}_{AL} + w_L^{AD} + \tilde{t}_{LK} + w_K^{AD} - \delta_K^{EF \to AD} > t_E^F + \tilde{t}_{EK}, t_E^F +$$
$$+ \tilde{t}_{EK} + w_K^{EF} - \delta_K^{DA \to EF} > t_D^A + \tilde{t}_{DK}, t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ} +$$
$$+ w_J^{FE} + \tilde{t}_{JK} + w_K^{FE} - \delta_K^{DA \to FE} > t_D^A + \tilde{t}_{DK}, t_A^D + \tilde{t}_{AL} + w_L^{AD} +$$
$$+ \tilde{t}_{LK} + w_K^{AD} - \delta_K^{FE \to AD} > t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ} + w_J^{FE} + \tilde{t}_{JK},$$
$$t_D^A + \tilde{t}_{DK} + w_K^{DA} + \tilde{t}_{KL} + w_L^{DA} - \delta_L^{GH \to DA} > t_G^H + \tilde{t}_{GL}, t_G^H +$$
$$+ \tilde{t}_{GL} + w_L^{GH} + \tilde{t}_{IJ} + w_J^{GH} - \delta_J^{AC \to GH} > t_A^C + \tilde{t}_{AJ}, t_F^E + \tilde{t}_{FI} +$$
$$+ w_I^{FE} + \tilde{t}_{IJ} + w_J^{FE} - \delta_J^{AC \to FE} > t_A^C + \tilde{t}_{AJ}, t_H^G + \tilde{t}_{HJ} + w_J^{HG} -$$
$$- \delta_J^{AC \to HG} > t_A^C + \tilde{t}_{AJ}, t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + w_J^{EF} -$$
$$- \delta_J^{CA \to EF} > t_C^A + \tilde{t}_{CJ}, t_G^H + \tilde{t}_{GL} + w_L^{GH} + \tilde{t}_{LJ} + w_J^{GH} -$$
$$- \delta_J^{CA \to GH} > t_C^A + \tilde{t}_{CJ}, t_H^G + \tilde{t}_{HJ} + w_J^{HG} - \delta_J^{CA \to HG} > t_C^A + \tilde{t}_{CJ},$$
$$t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ} + w_J^{FE} - \delta_J^{CA \to FE} > t_C^A + \tilde{t}_{CJ}, t_G^H + \tilde{t}_{GL} +$$
$$+ w_L^{GH} + \tilde{t}_{IJ} + w_J^{GH} - \delta_J^{EF \to GH} > t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ}, t_A^C +$$
$$+ \tilde{t}_{AJ} + w_J^{AC} - \delta_J^{EF \to AC} > t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ}, t_G^H + \tilde{t}_{GL} +$$
$$+ w_L^{GH} + \tilde{t}_{IJ} + w_J^{GH} - \delta_J^{FE \to GH} > t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ}, t_A^C +$$
$$+ \tilde{t}_{AJ} + w_J^{AC} - \delta_J^{FE \to AC} > t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ}, t_H^G + \tilde{t}_{HJ} +$$
$$+ w_J^{HG} - \delta_J^{FE \to HG} > t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ}, t_E^F + \tilde{t}_{EK} + w_K^{EF} +$$
$$+ \tilde{t}_{KJ} + w_J^{EF} - \delta_J^{GH \to EF} > t_G^H + \tilde{t}_{GL} + w_L^{GH} + \tilde{t}_{IJ}, t_A^C + \tilde{t}_{AJ} +$$
$$+ w_J^{AC} - \delta_J^{GH \to AC} > t_G^H + \tilde{t}_{GL} + w_L^{GH} + \tilde{t}_{IJ}, t_F^E + \tilde{t}_{FI} + w_I^{FE} +$$
$$+ \tilde{t}_{IJ} + w_J^{FE} - \delta_J^{GH \to FE} > t_G^H + \tilde{t}_{GL} + w_L^{GH} + \tilde{t}_{IJ}, t_C^A + \tilde{t}_{CJ} +$$
$$+ w_J^{CA} - \delta_J^{HG \to CA} > t_H^G + \tilde{t}_{HJ}, t_E^F + \tilde{t}_{EK} + w_K^{EF} + \tilde{t}_{KJ} + w_J^{EF} -$$
$$- \delta_J^{HG \to EF} > t_H^G + \tilde{t}_{HJ}, t_A^C + \tilde{t}_{AJ} + w_J^{AC} - \delta_J^{HG \to AC} > t_H^G +$$
$$+ \tilde{t}_{HJ},,, t_F^E + \tilde{t}_{FI} + w_I^{FE} + \tilde{t}_{IJ} + w_J^{FE} - \delta_J^{HG \to FE} > t_H^G + \tilde{t}_{HJ},$$
$$\left| t_N^M \right| \leq 6, 0.2 \leq w_I^{NM} \leq 6 \text{ for all final or initial}$$
$$\text{stations N \& M and junction station } I \} \quad (28)$$

Hence, the problem will be

$$\min_{\substack{s.t. \\ (t_A^C, t_A^D, t_B^A, t_C^A, t_D^A, t_F^E, t_G^H, t_H^G, w_I^{AB}, w_I^{BA}, w_I^{EF}, w_I^{FE}, w_J^{AC}, w_J^{CA}, \\ w_J^{EF}, w_J^{FE}, w_J^{GH}, w_J^{HG}, w_K^{AD}, w_K^{DA}, w_K^{EF}, w_K^{FE}, w_L^{AD}, w_L^{DA}, w_L^{GH}) \in S}} F = (f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, \ldots, f_{16}, f_{17}) \quad (29)$$

In order to apply the preference-based genetic algorithm to solve the problem it necessary to have a

Table 2 - The optimum result after running the Matlab code for n=12 number of population: to make all the entries positive the biggest negative (-5.3) has been subtracted from all entries, and all the entries are in hours.

Bus	Initial or final	L		I		J		K		Initial or final	Bus
		Arrive or leave	Arrive or leave	Arrive or leave	Arrive or leave	Arrive or leave	Arrive or leave	Arrive or leave	Arrive or leave		
AB	5.3	-	-	9.2	11.6	-	-	-	-	14.6	
	8.8	-	-	4.8	3.9	-	-	-	-	0.9	BA
AC	2.4	-	-	-	-	6.2	12.1	-	-	15.3	
	13.9	-	-	-	-	10.1	8.1	-	-	4.9	CA
AD	2.8	6.0	8.5	-	-	-	-	12.5	13.9	15.6	
	13.8	10.6	9.2	-	-	-	-	5.2	2.8	1.0	DA
FE	1.4	-	-	3.9	4.8	8.3	10.0	13.0	13.4	14.4	
	15.7	-	-	13.2	13.0	9.5	8.8	5.8	4.2	3.2	EF
GH	0	1.8	4.5	-	-	7.3	9.0	-	-	12.5	
	15.2	13.5	12.7	-	-	9.9	7.3	-	-	3.8	HG

preference. Suppose we have the same number, fuzzy number, of customers who want to switch buses at junction villages specified by the objective functions. For the simulation we take the weight for each objective to be normally distributed with the mean 1 and standard deviation 0.2.

We run the code of preference-based genetic algorithm for the modelled fuzzy multi-objective optimization problem in Matlab with the size of initial population 12. The program runs for 50 iterations and the best result among the 12 members of the population is recorded as shown in Table (2).

In Table (2) the direction of buses are specified either from right to left or left to right, for example, the row corresponding to AB shows that it will reach station I at 9.2 and leave from I at 11.6 and so on; and for BA it will reach station I at 3.9 and leave I at 4.8 (from right to left) and so on.

6. CONCLUSION

In this paper we discussed how to model a single frequency daily bus timetabling problem as a fuzzy non-linear constrained multi-objective optimization problem. We have also shown how to use a genetic algorithm with a specified preference determined from the input data on the need of customers and assigning an appropriate probability density function for each objective function as a dynamic weight. A hypothetical example is discussed to show how it works. The result shows that it is promising to model the problem as a fuzzy multi-objective optimization problem and to use

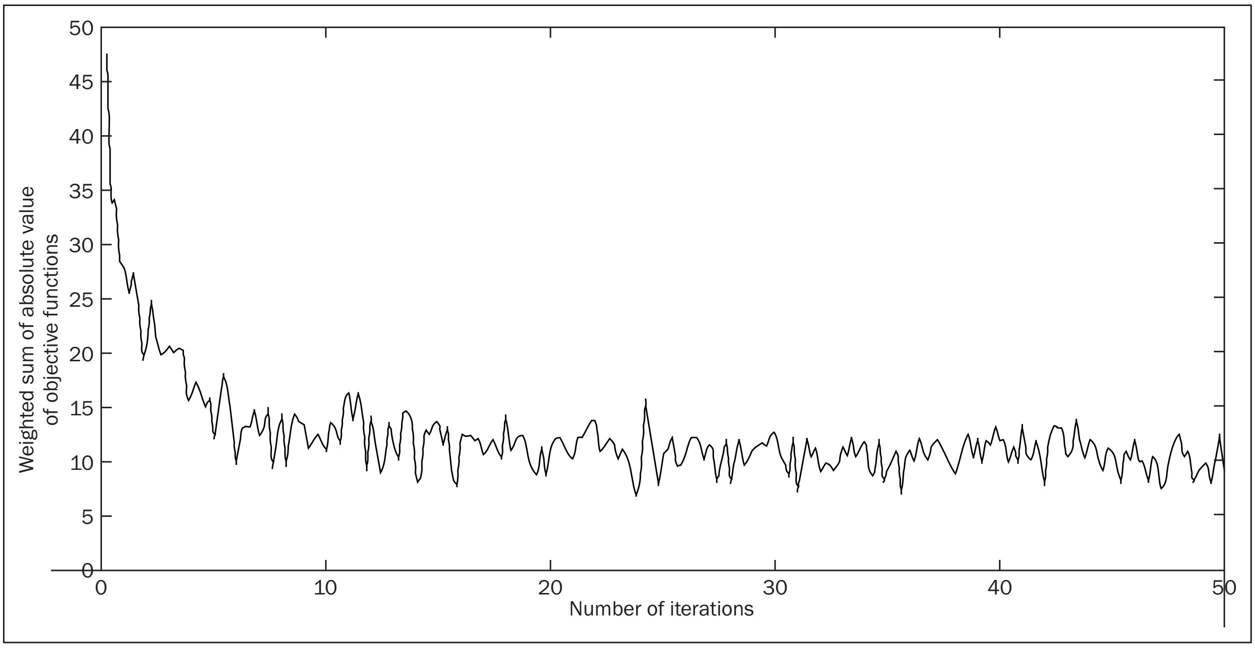

Figure 3 - Performance graph of the algorithm regarding example

preference-based GA in order to solve the modelled problem.

ACKNOWLEDGEMENT

This work is partially supported by the Universiti Sains Malaysia short-term grant number 304/PMATHS/6311126. The first author would like to acknowledge support from TWAS-USM Fellowship 2008.

ሱራፌል ልዑልሰገድ ጥላሁን እና
Hong Choon Ong

የክፍለ ሃገር አውቶብሶችን የመነሻ ሰዓት መወሰን በ”መልቲኦብጀክቲቭ ኦፕቲማይዜሽን ፕሮብሌም” ሞዴል ማድረግና ፍላጎትን መሰረት ባደረገ “ጀነቲክ አልጎሪዝም” ጥሩ የሆነውን የመነሻ ሰዓትን ማግኘት

ጽንሰ ሃሳብ: ለአንድ ሃገር እድገት አስተዋጽዖ ከሚያደርጉት ዐበይት ነገሮች አንዱ ትራንስፖርት ነው:: ከትራንስፖርት ዘርፎች መኪና ትልቁ ሚና ይጫወታል:: በታዳጊ ሃገራት ውስጥ በሕዝብ አውቶብስ መጠበቅ ረጅም ጊዜ ማጥፋት የተለመደ ነገር ነው, በተለይም አውቶብስ በመቀያየር መሄድ ሲያስፈልግ:: የክፍለ ሃገር አውቶብሶችን ስንመለከት ደግሞ ችግሩ የበለጠ የጎላ ነው:: በየትራንስፖርት መስመሩ አውቶብስ በብዛት መመደብ በፅንሰ ሃሳብ ደረጃ ችግሩን ቢቀርፍም በኢኮኖሚ ችግር ምክንያት ሊተገበር አይችልም:: ሌላው አማራጭ የአውቶብሶችን የመነሻ ሰዓትን በአግባቡ በመወሰን ቢያንስ አውቶብሶችን በተለያዩ ከተሞች መቀየር የሚፈልጉትን ተጠቃሚዎች ብዙ እንዳይጠብቁ ማድረግ ነው:: ይሄ ዶሴ ይህንን የአንድ የክፍለሃገር አውቶብስ መስመሮች የመነሻ ሰዓትን በ”መልቲኦብጀክቲቭ ኦፕቲማይዜሽን ፕሮብሌም” ሞዴል ማድረግንና ፍላጎትን መሰረት ባደረገ “ጀነቲክ አልጎሪዝም” ጥሩ የሆነውን የመነሻ ሰዓትን ማግኘትን ያብራራል:: በመጨረሻም ጽንሰ ሃሳቡን በምሳሌ ያስረዳል::

ቁልፍ ቃላት: የአውቶብሶችን የመነሻ ሰዓት, መልቲኦብጀክቲቭ ኦፕቲማይዜሽን ፕሮብሌም, ጀነቲክ አልጎሪዝም

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