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FRACTAL DIMENSION OF COASTLINE OF THE CROATIAN ISLAND CRES

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Abstract:

Using a 1 : 100,000 geographic map of the island Cres, its coastline was digitalized into bitmap of 1696 x 5052 pixels. This bitmap was analyzed computationally using our refined box-counting method. It was shown that the log-log diagram separates into two parts: non-selfsimilar and selfsimilar, divided by a critical value of the mesh size. It was found that the power law is extremely well satisfied in selfsimilar section of the log-log diagram, reflecting a high degree of statistical fractality of the coastline, almost without detectable fluctuations. Therefrom the overall fractal dimension of the coastline of Cres was determined as $D_b = 1.118 \pm 0.001$. It was found that partial fractal dimensions of particular parts of the coastline of Cres differ sizeably from the overall value. These results show that the coastline of Cres exhibits a high degree of stable statistical self-similarity. A possible origin of this pattern was suggested along the lines of a recent model of water erosion as a fractal growth process simulated on a lattice which was used to model the stationary state of the river pattern such as a power-law size distribution of the drainage basin area and for the Horton's law. In light of our results for overall fractal dimension we discuss the problem of dependence of the length of coastline on precision of measurement and present the corresponding asymptotic formula.

Keywords:

Island Cres; box-counting method; fractal dimension; power law for the coastline; overall fractal dimension of the coastline; partial geographic fractal dimension of the coastline; water erosion process; relief; lenght of coastline

FRAKTALNA DIMENZIJA OBALNE CRTE HRVATSKOG OTOKA CRESA

Izvadak:

Koristeći geografsku kartu otoka Cresa u mjerilu 1 : 100 000 digitalizirana je njegova obalna crta u bitkartu od 1696 x 5052 piksela. Pokazano je da se log-log dijagram separira na dva područja:

nesamoslično i samoslično, razdijeljena kritičnom vrijednošću duljine kutije. Dobiven je rezultat da je eksponencijalni zakon izvanredno dobro zadovoljen u samosličnom dijelu log-log dijagrama, što ukazuje na visoki stupanj fraktalnosti obalne crte, gotovo bez uočljivih fluktuacija u tom dijelu log-log dijagrama. Na osnovi toga izračunata je vrijednost ukupne fraktalne dimenzije obalne crte otoka Cresa: $D_b = 1,118 \pm 0,001$. Nadalje je otkriveno da parcijalne fraktalne dimenzije pojedinih dijelova otoka Cresa pokazuju znatne razlike od ukupne vrijednosti. Ovi rezultati pokazuju da obalna crta Cresa pokazuje visoku razinu statističke samosličnosti. Moguće objašnjenje za podrijetlo ove pojave sugerirano je na crti novog modela erozije kao procesa s fraktalnim rastom simuliranog na rešetki što je korišteno za modeliranje stacionarnog stanja poriječja kao na primjer eksponencijalnog zakona za poriječje i za Hortonov zakon. U svjetlu ovih rezultata za fraktalnu dimenziju raspravlja se problem ovisnosti procjene za duljinu obale o preciznosti mjerenja i prikazuje se odgovarajuća asimptotska formula.

Ključne riječi:

otok Cres; fraktalna dimenzija, metoda brojanja kutija; eksponencijalni zakon za obalnu crtu; ukupna geografska fraktalna dimenzija; parcijalna geografska fraktalna dimenzija; erozija; reljef; duljina obale

INTRODUCTION

It is known that there exist many fractal geometries in the nature (MANDELBROT, 1983; BARNSLEY, 1988; FEDER, 1988; HIRABAYASI, Ito & YOSHII, 1992; TAKAYASU, 1990; INAOKA & TAKAYASU, 1993; MARITAN, RINALDO, RIGON, GIACOMETTI & RODRIGUEZ-ITURBE, 1996; RODRIGUEZ-ITURBE & RIONALDO, 1996; SOUSA VIEIRA, 1996; PEITGEN, JUERGENS & SAUPE, 1992; SCHROEDER, 1990; VICSEK, 1989). Landscapes such as coastlines and river patterns are familiar examples, and their fractalities are supported by numerical analyses of real topographical data. The fractality of landscapes first was pointed out by Mandelbrot (MANDELBROT, 1983) in the second half of twentieth century. According to Mandelbrot, the Koch curve, which is a typical fractal curve with fractal dimension of 1.26 (MANDELBROT, 1983; BARNSLEY, 1988; FEDER, 1988), presents a rough model of the coastline. However, we note that some origins of the idea of fractal coastlines can be traced already in the work of the eighteenth century scientist and philosopher Bošković (BOSCOVICH, 1758). Bošković wrote: "...Nothing in Nature is mathematically flat and smooth.... So in the river beds, in branches of trees, in edges of salts, crystals, ... according to my theory there is no continuity, because all bodies consist of points... if we imagine that any three of these points are connected by straight lines, a triangle will be formed."

Mandelbrot (1983) has proposed a simple model which creates the fractal surfaces, so called Brownian surfaces, the product of which is regarded as a model of the earth's relief. However, the processes of the landform creation in this model are far removed from real processes of landform evolution. The fact that fractals on the earth's relief exist over such an extensive region implies that fractals are probably created by a kind of fractal growth process from nonfractal surfaces.

There are many factors causing the changes of landscapes, for example tectonic movements, water erosion, sedimentation, weathering, and so on (MANDELBROT, 1983; INAOKA & TAKAYASU, 1993; RODRIGUEZ-ITURBE & RIONALDO, 1996; PEITGEN, JUERGENS and SAUPE, 1992). Recently, a minimal model of water erosion was proposed for landform evolution in relatively large systems (INAOKA & TAKAYASU, 1993). In this way the time evolution of river patterns and earth's relief are simulated on a lattice, with an initial land form being a flat surface perturbed by a very slight white noise.

The geometry of mathematical fractal, like for example Koch curve and Koch island, are characterized by self-similarity (KAPLAN & GLASS, 1995; OTT, 1993; PEITGEN, JUERGENS & SAUPE, 1992; SCHROEDER, 1990). The term self-similar describes the geometry of objects in which a small part when expanded looks like a whole. Thus, the structure is said to be self-similar if it can be broken down into arbitrarily small pieces, each of which is a small replica of the entire structure.

In fact, in the cases of fractal sets arising in typical dynamical systems, a strict self-similarity rarely holds. Many objects encountered in nature are to some extent self-similar. Examples of this type are treelike shapes like river networks, vascular system, the branching system of bronchi in the lungs and a tree with its branches (BARNSLEY, 1988; BUNDLE & HAVLIN, 1995; FEDER, 1988; MANDELBROT, 1977; MANDELBROT, 1983; PEITGEN, JUERGENS, & SAUPE, 1992; SCHROEDER,1990; VICSEK, 1989). But self-similarity is not limited to objects with treelike geometry. For example, coastlines, clouds and mountains can exhibit self-similar patterns. Mountains often have small outcrops that resemble the mountain as a whole.

We note that in geometry the word "similar" means "not differing in shape but only in size or position". The mathematical fractals are self-similar in this geometrical sense. On the other hand, in everyday language the word "similar" means "alike" and does not have as narrow a meaning as in geometry. Two things can be alike even if they are slightly different. The self-similarity of real trees in nature, for example, makes use of this common meaning of the term "similar". One does not expect that all branches of a tree will look the same, just that they will look somewhat like the tree as a whole.

Therefore, it is often worthwhile to consider self-similarity in a statisical sense, saying that an object is self-similar if its parts, on average, are similar to the whole. Such a pattern of statistical self-similarity appears for the coastline. The coastline contains gulfs and bays, and gulfs and bays themselves contain smaller bays and inlets, which themselves contain coves and other small structures, and so on. Thus, the objects themselves are not similar to each other, but their statistical pattern remains the same with changing the scale. An example of such a statistical self-similarity is the well-known Brownian motion. Enlargement of this first under microscope observed features are today used in computer graphic to generate the pattern landscapes in nature.

TWO VERSIONS OF FRACTAL DIMENSION

1. Compass dimension - a form of fractal dimension

Fractal nature of the coastline is reflected in measuring its length on a geographical map. In order to measure the length of the coastline, we can take compasses set at a certain distance. For example, if a map is in the scale 1 : 1,000,000 and the compass setting is 5 cm, the corresponding true distance is 50 km. Now we carefully walk the compasses along the coast counting the number of steps. In this way we obtain a polygonal representation of a particular coast. Using different compass settings, one obtains different measured values for the coastal length. For example, for the length of the coast of Britain, compass settings of 500 km, 100 km, and 17 km result in the values of 2600 km, 3800 km, and 8640 km, respectively (PEITGEN, JUERGENS & SAUPE, 1992). With one compass setting many of smaller bays are still not accounted for, while, in the next smaller one they are, while still smaller bays are still ignored at that setting, and so on.

In such a situation one usually passes to a log-log diagram for a polygonal representation of the coast. On the horizontal axis the logarithm of the inverse compass setting s is displayed. This quantity corresponds to the precision of the measurement; the smaller the compass setting is, the more precise is the measurement. The vertical axis displays the logarithm of the length u. These log-log plots will always show how the logarithm of the total length $\log(u)$ changes with an increase in precision, i.e., with an increase of $\log(\frac{1}{s})$. In this diagram the points corresponding to the compass measurement of the coastline of Britain roughly fall on a straight line, i.e., the compass measurements are approximately in accordance with the power law:

$$u = c\left(\frac{1}{s}\right)^d, \qquad (1)$$

where the constants c and d can be fitted to the results of compass measurements and they characterize the growth law. The constant d corresponding to the slope of the fitted straight line in the log-log plot is the key to the fractal dimension of the underlying coastline. For former data for the coast of Britain one obtains $d \approx 0.3$. The points do not fall exactly on a straight line in the log-log diagram, i.e., deviations from the straight line are sizeable (PEITGEN, JUERGENS & SAUPE, 1992), revealing deviations from fractality and/or limited precision of the measurement. Therefore, the value of the constant d cannot be determined very accurately.

If we let the size of the compass setting go to smaller and smaller values on a geographic map, the power law would become invalid due to finite resolution of the map. In this case the measured length would tend to a certain limiting value, while according to the power law (1) it would go to infinity in the limit when the compass setting goes to zero. Thus, the power law (1) characterizes the complexity of the coast over some range of scales by expressing how quickly the coastal length increases if we measure with ever finer accuracy.

Eventually, such compass measurements do not make sense any more for sufficiently small compass setting, because one would run out of maps and would have to begin measuring the coast in reality and face all the problems of identifying the coastline (when to measure, at low or high tide, and so on). However, in any practical terms one must say that the coastline has no definite value for its length. The only meaningful thing we can say about the coastal length is that it behaves approximately like the power law (1) over a range of scales to be specified and that this behaviour will be characteristic of each particular coastline. Thus, the value of exponent in the power law (1) is likely to be different when we compare different coasts.

Using the map of island Cres on the scale 1 : 300,000 with the compass setting of 9 km, 4.5 km and 1.5 km, the values for coastal length of 177 km, 202 km and 249 km, respectively, were obtained (JOSIPOVIĆ, 1996).

Using the value of constant d, obtained by compass measurement, one defines the fractal dimension, denoted by D_c and referred to as compass fractal dimension:

$$D_c = 1 + d. \qquad (2)$$

2. Box-counting dimension - a form of fractal dimension

Now we consider the second and more reliable version of fractal dimension: the box-counting dimension. Description of fractals in terms of box-counting dimension has two advantages over compass dimension. First, it applies to any structure in the plane, no matter how complex, and can be readily adopted for structures in space. Second, it enables determination of fractal dimension with higher precision. Therefore, we have applied the box-counting dimension in our investigations of the coastline of island Cres.

The box-counting dimension is determined as follows. We put the geographic map onto a regular mesh with mesh size s, and simply count the number of grid boxes which contain some of the structure of the coastline being investigated. For each chosen mesh size s this gives a number of non-empty boxes denoted by $N(s)$.

We are changing s to progressively smaller sizes and for each mesh size count the corresponding number of non-empty boxes $N(s)$. Next we make a log-log diagram, displaying $\log_2[N(s)]$ in dependence on $\log_2(\frac{1}{s})$. We then fit a straight line to plotted points in a certain interval of mesh sizes and determine its slope which is denoted D_b. This number is referred to as the box-counting dimension, another special form of fractal dimension. Thus, the box-counting dimension is determined from the power law:

$$N(s)= c\left(\frac{1}{s}\right)^{D_b} \quad . \qquad (3)$$

In the case of mathematical selfsimilar fractals, the fractal dimension is determined by fitting the power law (3) in the limit of vanishing mesh size s:

$$D_b = \lim_{s \to 0} \frac{\log[N(s)]}{\log(\frac{1}{s})} \qquad (4)$$

However, the fractals in nature, as for example the coastline, are characterized by statistical selfsimilarity down to level of a certain small size, below which the fractality disappears. Similar pattern can appear also in nonlinear dynamical systems treated computationally, and is referred to as truncated fractal (PAAR & PAVIN, 1998). Furthermore, when we use geographical maps on a certain scale, the finest mesh size which can be used is determined by the map scale. In previous calculations of overall fractal dimension of large sections of the coastlines, like the coastlines of Britain and Norway (PEITGEN, JUERGENS & SAUPE, 1992; SCHROEDER, 1991), a smaller number of large mesh sizes was used, with $s \approx 30$ km. From box occupations $N(s_1)$ and $N(s_2)$ for two mesh sizes s_1 and s_2, respectively, the fractal dimension was determined as (PEITGEN, JUERGENS & SAUPE, 1992):

$$D_b = \frac{\log[N(s_1)] - \log[N(s_2)]}{\log(\frac{1}{s_1}) - \log(\frac{1}{s_2})} \qquad (5)$$

The overall box-counting dimension has been previously determined for the coastlines of Britain, including Ireland ($D_b \approx 1.3$) (PEITGEN, JUERGENS & SAUPE, 1992) and of Norway (SCHROEDER, 1990) ($D_b \approx 1.5$).

In the classic example of the coastline of Britain, two underlying grids have been considered. Having normalized the width of the entire grid covering the geographic map on the scale 1 : 1,000,000 by 1 unit, the mesh sizes were taken 1/24 and 1/32, respectively. The box counting in these two cases yielded 194 and 283 non-empty boxes, respectively, that intersect the coastline in the corresponding grids (PEITGEN, JUERGENS & SAUPE, 1992). From these data, the box-counting dimension was calculated using Eq. (5), giving the slope of straight line that connects the corresponding two points in the log-log diagram:

$$D_b = \frac{\log 238 - \log 194}{\log 32 - \log 24} \approx \frac{2.45 - 2.29}{1.51 - 1.38} \approx 1.31 \, .$$

COMPUTATIONAL METHOD FOR DETERMINATION OF FRACTAL DIMENSION OF COASTLINE AND SENSITIVITY TO MESH SIZE

1. Computer program for box counting

To determine the box-counting dimension of the coastline using geographic map we have used our computational program which consists of four parts: 1) subroutine for digitalization of data from geographic map; 2) subroutine for reading digitalized data from disk ("reading subroutine"); 3) subroutine for data processing ("processing subroutine"); 4) subroutine for connecting the "reading" and "processing" subroutines.

We have used geographic map in the scale 1 : 100,000. The geographic map was scanned so that one pixel on the bitmap corresponds to 0.125 mm on the geographic map, i.e., one pixel corresponds to 12.5 m in nature. The C++ program was used for counting points on the geographic map and recording in bitmap graphical format. Each point in graphical format has two states: 0 (absence of any element of coastline) corresponds to white pixel, and 1 (presence of an element of coastline) corresponds to black pixel. Functions from *Mathematica* were used for creation of optimal tree of levels for counting and recording pixels (CRANDALL, 1994; EBERT, MUSGRAVE, PEACHEY, PERLIN & WORLEY, 1994; WOLFRAM, 1996). Here, the highest level in tree construction corresponds to the smallest mesh size s and the level gradually decreases with increasing mesh size. Structure of the tree is stored onto file on disk, and is being read in before the counting starts.

Data are being read sequentially (pixel by pixel) from disk. When the reading subroutine encounters a black pixel, this information is stored in the "processing" subroutine. "Processing subroutine" constructs the grid on the map. This subroutine counts black squares on the grid, while accepting the information on black pixels from map. Here, a square is black if no black pixel falls within it, and white if at least one black pixel is present. If a black pixel falls onto a previously white square, it turns it into black, increasing the total number of black squares by one, and transmits the information to all levels lying below it in the tree construction. On the other hand, if a black pixel falls onto a black square, it has no effect on box counting.

2. Grid-translation averaging method

In order to diminish fluctuations in box populations due to a particular placement of grid on the map, we have introduced an averaging method with a chain of grid translations. For each mesh size, m computations were performed for m uniformly shifted grids: in the j-th step of translation the grid on map was shifted by j pixels upwards and by j pixels to the left. The number of black boxes in the j-th step of translation of the grid with mesh size s_i is denoted by $n_j(s_i)$. Then, the average value of black boxes for the grid with mesh size s_i is:

$$n(s_i) = \frac{1}{m} \sum_{j=1}^{m} [n_j(s_i)] \; . \tag{6}$$

Using these average occupation numbers, the corresponding log-log diagram, displaying $\log[N(s)]$ in dependence on $\log\left(\frac{1}{s}\right)$ was constructed.

3. Selfsimilar section of log-log diagram and operational definition of fractal dimension of the coastline

In the computation of fractal dimension from the log-log diagram we adopt the following procedure. The log-log diagram is constructed for the range of mesh sizes from $s_{min} = 10$ pixels to the maximum value s_{max} which is equal to about a quarter of the size of geographic map of coastline. In our computation for Cres we take $s_{max} = 512$ pixels (the width of Cres is 1696 pixels).

In the interval $[\log(\frac{1}{s_{max}}), \log(\frac{1}{s_{min}})]$ on the horizontal axis, denoted as I_{tot}, we have chosen m = 32 uniformly distributed test points. Around each test point we form a small subinterval of the width equal to 1/16 of the width of the interval I_{tot}. In the subinterval around each test point we fit a straight line to data points and determine its slope. Such a slope associated with the *k*-th test point is denoted by D_k, and referred to as the *k*-th fractional slope. Thus we obtain a set of fractional slopes $D_1, D_2, D_3, \ldots D_m$. Starting from D_1 (associated with the first test point on the l.h.s. of the log-log diagram), we compare the consecutive values of fractional slopes. If D_1 differs from D_2 and/or D_3 by more than 5%, it is discarded. The same procedure is repeated for D_2 in comparison to D_3 and D_4, and so on, until we come to the *l*-th test point with a slope D_l, which differs from the next two slopes D_{l+1} and D_{l+2} by less than 5%. The value of mesh size at the beginning of the *l*-th interval is denoted s_l and referred to as the critical mesh size of the corresponding log-log diagram.

Then we divide the total interval I_{tot} on the horizontal axis of the log-log diagram into two parts:

1) The section of the log-log diagram from the initial point $[\log(\frac{1}{s_{max}})]$ to the critical mesh size $[\log(\frac{1}{s_l})]$. This section of the log-log diagram will be referred to as non-selfsimilar.

2) The section of the log-log diagram from the critical mesh size $[\log(\frac{1}{s_l})]$ to the final point $[\log(\frac{1}{s_{min}})]$. This section of the log-log diagram will be referred to as selfsimilar.

We fit a straight line to data points in the selfsimilar section of the log-log diagram. Then we define the slope of this straight line as fractal dimension D_b of the coastline.

Quite generally, for each form in the plane, of a fractal (for example, Koch curve, Sierpinski gasket, geographical map of a coastline, etc.) or nonfractal type (for example, a circle), the log-log diagram is divided into non-selfsimilar and selfsimilar sections.

The fractional slope in non-selfsimilar section varies in size with mesh size and its value is generally higher than in the selfsimilar section of log-log diagram. The fractional slope in non-selfsimilar section may exhibit irregular oscillations with mesh size. The origins of this behaviour are deviations from selfsimilarity and two mathematical effects:

First, there is an effect due to small number of non-empty boxes. When the object becomes comparable to the size of boxes, the boxes containing different sides of the object start to overlap, which increases the fractional slope. On the other hand, for small number of non-empty boxes, further smaller increases in the mesh size may have no effect on box populations; in this range of mesh sizes the fractional slope decreases with increasing mesh size until the value of mesh size is reached where the number of non-empty boxes decreases by one. At this value of mesh size, the fractional slope will have an increase. Such changes may be repeated until the mesh size s_{max} is reached. (The largest mesh size was chosen to be smaller than the size of the object. Namely, for a box larger than the object itself, the corresponding fractional slope would be equal to zero.)

Second, for large mesh sizes the grid translation averaging method becomes less effective, because in the sequence of m successive shifts of the grid all positions within each box are not accounted for on equal footing.

4. Computation of box-counting dimension of the island Cres

In the computation of fractal dimension of the island Cres we have used the geographical map in scale 1 : 100,000 as the source of data to be digitalized. The mesh size s was expressed in pixels. The size of map of Cres was 1696x5052 pixels. The log-log diagram was computed in the interval between $s_{min} = 10$ pixels and $s_{max} = 512$ pixels. In the grid translation averaging method we employed for each mesh size a set of $m = 200$ grids, shifted consecutively by one pixel with respect to each other. This method has led to sizeable reduction of fluctuations in the change of box occupations with mesh size.

In Fig. 1 we present a diagram of fractional slopes computed for the island Cres. It is seen that the critical mesh size is $\log_2(1/s) \approx -7.6$.

The selfsimilar section of the log-log diagram for the island Cres is displayed in Fig. 2. Interval of binary logarithms of mesh sizes (in pixels) included in the calculation goes from $\log_2(1/s) = -7.7$, i.e., $s = 200$ pixels (which corresponds to the real length of 2.5 km) to $\log_2(1/s) = -3.3$, i.e., $s = 10$ pixels (which corresponds to the real length of 0.125 km). As seen in figure these data points lie to a very good approximation on a straight line. The fit gave the slope of this straight line, i.e., the corresponding overall box-counting dimension for the coastline of the island Cres:

$$D_b\,(\text{Cres}) = 1.118 \pm 0.001. \qquad (7)$$

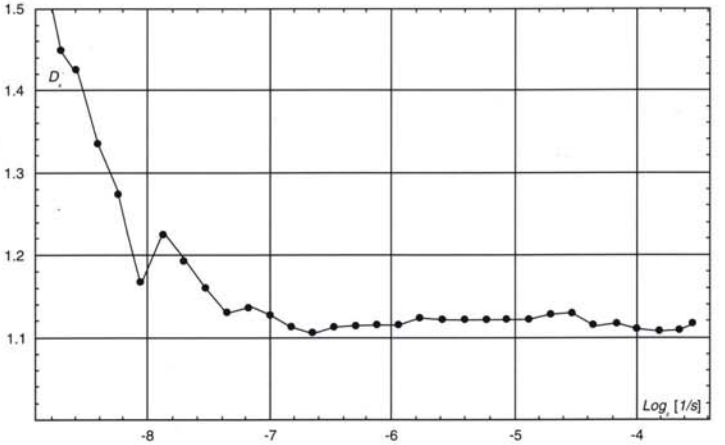

Fig. 1. Diagram of fractional slopes computed for the island Cres
The size of the geographic map used as the source of data for digitalization is 1696x5052 pixels.The interval of mesh sizes on the horizontal axis is between $\log_2(1/s) = -9$, i.e., $s = 512$ pixels (which corresponds to the real length of 6.5 km) and $\log_2(1/s) = -3.3$, i.e., $s = 10$ pixels (which corresponds to the real length of 0.125 km^2. It is seen that the critical mesh size is $\log_2(1/s) \approx -7.6$. For description see the text.

Sl. 1. Dijagram frakcijskih nagiba izračunatih za obalnu crtu otoka Cresa
Veličina geografske karte korištene kao izvor podataka za digitalizaciju je 1696x5052 piksela. Interval duljina kutije na vodoravnoj osi ide od $\log_2(1/s) = -9$, t.j. $s = 512$ piksela (što odgovara stvarnoj duljini od 6,5 km) do $\log_2(1/s) = -3,3$, t.j. $s = 10$ piksela (što odgovara stvarnoj duljini od 0,125 km). Sa slike se vidi da je kritična duljina kutije $\log_2(1/s) \approx -7,6$. Za opis vidjeti tekst.

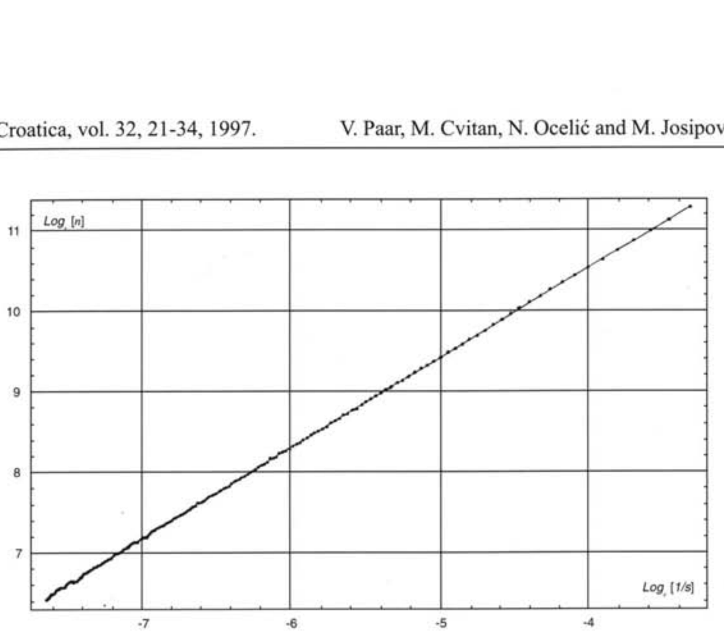

Fig. 2. Selfsimilar section of the log-log diagram computed for the coastline of the island Cres
Interval of binary logarithms of mesh sizes (in pixels) is between $\log_2(1/s) = -7.7$, i.e., $s = 200$ pixels (which corresponds to the real length of 2.5 km) and $\log_2(1/s) = -3.3$, i.e., $s = 10$ pixels (which corresponds to the real length of 0.125 km). Thus, for the finest mesh size the island Cres is covered by a grid of 170x505 boxes, each of the size 10x10 pixels. Solid straight line displays a fit to data points. The corresponding box-counting dimension for the coastline of island Cres is: $D_b = 1.118 \pm 0.001$.
Sl. 2. Samoslični dio log-log dijagrama izračunat za obalnu crtu otoka Cresa
Interval binarnih logaritama duljine kutije (u pikselima) je između $\log_2(1/s) = -7{,}7$, tj. $s = 200$ piksela (što odgovara stvarnoj duljini od 2,5 km) i $\log_2(1/s) = -3{,}3$, tj. $s = 10$ piksela (što odgovara stvarnoj duljini od 0,125 km). Na taj način, za najfiniju duljinu kutije otok Cres je prekriven rešetkom od 170x505 kutija, svaka veličine 10x10 piksela. Pravac predstavlja fit na točke podataka. Odgovarajuća fraktalna dimenzija metode brojanja kutija za otok Cres je: $D_b = 1{,}118 \pm 0{,}001$.

Here, the statistical error is obtained by linear regression method. In the following considerations we use the rounded off value of 1.12.

In the next step, the map of Cres was divided into three parts: the northern part, the central part and the southern part (Fig. 3, l.h.s.). For each part of Cres, the fractal dimension was determined separately, following the same procedure as before. The results were:

$$D_b \text{ (Northern Cres)} = 1.03, \quad (8)$$

$$D_b \text{ (Central Cres)} = 1.08, \quad (9)$$

$$D_b \text{ (Southern Cres)} = 1.17. \quad (10)$$

We see that the coastline of the island Cres can be considered as a combination of three statistical fractals, corresponding to three parts of Cres with partial fractal dimensions (8)-(10). As seen, in the case of present geographic statistical fractal, the fractal dimension of the whole (total coastline of Cres) is smaller than for its most fractal segment (Southern Cres).

Dividing further the map of island Cres into 2x6 = 12 parts, we obtain the partial fractal dimensions given in Fig. 3 (r.h.s.). In that case, the part of Cres in the lower right corner has the largest fractal dimension (1.24), close to the fractal dimension of the Koch curve, which is a prototype of mathematical selfsimilar fractal (OTT, 1993). Dividing this part even further into four parts, its lowest right part, containing the most developed part of the coastline of Cres, has the fractal dimension of 1.29, which is above the value of fractal dimension of the Koch curve.

5. Dependence of measured value of the length of coastline on the precision (compass setting)

Let us now address the question of the total length of coastline of Cres measured at different levels of precision. We have

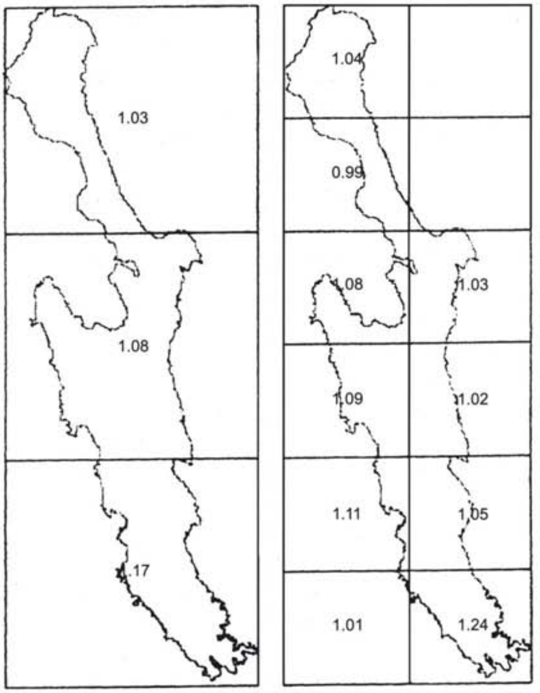

Fig. 3. Fractal dimension computed for parts of island Cres
Left map: Cres is divided into three parts: northern, central and southern. Right map: Cres is divided into twelve parts. Within each part of the island the corresponding calculated fractal dimension is given.

Sl. 3. Fraktalna dimenzija izračunata za dijelove otoka Cresa Lijeva karta: Cres je podijeljen na tri dijela: sjeverni, srednji i južni. Desna karta: Cres je podijeljen na 12 dijelova. Unutar svakog dijela otoka navedena je odgovarajuća izračunata fraktalna dimenzija.

determined that the overall fractal dimension of Cres in the range of mesh size between 1.5 km and 0.125 km is $D_b \approx 1.12$. Therefrom we can obtain an estimate for the ratio of lengths of the coastline of Cres obtained by measurement at two different precisions (compass settings). Let us denote by $L(s_1)$ the value obtained for the coastal length using compass setting s_1, and by $L(s_2)$ the value obtained using compass setting s_2. Then the ratio of these two values obtained for the length of coast is given by approximate expression:

$$\frac{L(s_1)}{L(s_2)} = \left(\frac{s_2}{s_1}\right)^{Db-1} \qquad (11)$$

This approximate expression is obtained under two assumptions. First, it is assumed that the precisions s_1 and s_2, corresponding to mesh sizes, lie in selfsimilar section of the log-log diagram for this coast. Second, it is assumed that the length of a segment of coastline lying in each non-empty box is equal to the length of box (mesh size).

Let us now illustrate the application of formula (11) in the case of island Cres. For example, Eq. (11) means that the ratio of values for the length of coastline of Cres obtained by measurements with precisions (compass settings) $s_1 = 0.125$ km and $s_2 = 1.5$ km is:

$$\frac{L(0.125\,km)}{L(1.5\,km)} = \left(\frac{1.5\text{ km}}{0.125\text{ km}}\right)^{1.12-1} = \left(\frac{1.5}{0.125}\right)^{0.12} = 1.35$$

Let us now assume that the same value of the overall fractal dimension of 1.12, obtained in the range of mesh sizes between 1.5 km and 0.125 km, can be extrapolated to lower values of the mesh size. Then, we can obtain estimates for the total length of Cres measured at very high precisions, for example at the precision of 1 m. For example, the calculated ratio of the values for the length of Cres at the precisions (compass settings) of 1 m and 1 km will be:

$$\frac{L(0.001\text{ km})}{L(1\text{ km})} = (1\ 000)^{0.12} = 2.29$$

Thus, it is predicted that the value of the coastline of Cres measured at the precision (compass setting) of 1 m will be 2.29 times larger than the value measured at the precision of 1 km. This means that, under previous assumptions, the total length of the coastline of Cres measured at the precision of 1 m would be more than 500 km.

CONCLUSION

In this paper we have investigated the sensitivity of calculated box counting fractal dimension of the coastal line on the interval of mesh sizes used in the computation. Furthermore, we investigated the partial box-counting dimensions of segments of the coastline. In order to perform these computations, we developed a set of computer codes involving subroutines for digitalization of data from geographic map, subroutine for reading digitalized data, subroutine for data processing and subroutine for connecting the "reading" and "processing" subroutines.

As a case study, we have performed computations for the island Cres, which has a size of about 20 km ´ 60 km. The main results are as follows.

The log-log diagram displaying the number of non-empty boxes $N(s)$ versus inverse mesh size $1/s$ can be separated into two sections: non-selfsimilar and selfsimilar. The non-selfsimilar section, lying at the side of large mesh sizes, is characterized by large fluctuations in fractional slopes of the log-log graph, which increase with increasing mesh size. On the other hand, in the selfsimilar section, which lies below a certain critical value of mesh size all the way down to the limit given by scale of geographic map, data points on the log-log graph lie to a very good approximation on a straight line. The slope of this straight line is associated with overall fractal dimension of the coastline. In our case study for the island Cres the interval of mesh size in the selfsimilar section goes from the critical value $s = 2.5$ km to the limiting value $s = 0.125$ km which is determined by the map scale.

It should be pointed out, that in any determination of fractal dimension one should be careful to check whether the mesh sizes being used lie in selfsimilar section of the log-log diagram. Otherwise, it may happen that the slope of log-log graph is, in fact, determined in non-selfsimilar section of the log-log graph. As shown in this paper, the result of calculations in such cases would present fractional slope, which may sizeably exceed the fractal dimension, leading to misleading conclusion.

Separating the coastline of the island Cres into parts, and determining the fractal dimension of each part from the slope of log-log graph in the selfsimilar region as before, partial fractal dimensions may differ sizeably between different parts of the coastline.

It would be interesting to investigate how the same selfsimilar section of log-log diagram extrapolates towards lower values of mesh size and whether a truncation appears at the low side of mesh sizes. At this low mesh size limit the effects of tides and of technical interventions in changing the coastline should be taken into account. On the other hand, such systematic investigations should be extended to larger sections of the coastline, for example to the total Adriatic coastline. Furthermore, an interesting problem is related to correlations between fractality of the coastline at different scales, i.e., between the fractal dimension of the coastline and its constituent parts at successively increasing scales.

It would be interesting to interpret these results in light of general theoretical investigations of how the fractalities on the earth˝s relief can be created by water erosion, with water as a medium which transmits the information about the surrounding structures to the local growth point, being a crucial factor for local growth characterized by long-range correlations (INAOKA & TAKAYASU, 1993) and by tectonic movements which are also characterized by fractality (HIRABAYASI, ITO, & YOSHII, 1992).

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Sažetak

FRAKTALNA DIMENZIJA OBALNE CRTE HRVATSKOG OTOKA CRESA

VLADIMIR PAAR, MARO CVITAN, NENAD OCELIĆ i MIROSLAV JOSIPOVIĆ

U ovom radu istražuje se osjetljivost fraktalne dimenzije obalne crte izračunate metodom kutija o intervalu duljina kutije. Nadalje, istražuju se parcijalne fraktalne dimenzije za pojedine dijelove obalne crte. Za te račune izrađen je skup kompjutorskih programa koji uključuje potprograme za digitalizaciju podataka iz geografske karte, za čitanje podataka s diska, za obradu podataka te za povezivanje potprograma za čitanje i obradu.

Ova kompjutorska metoda je primijenjena na slučaj obalne crte otoka Cresa. Glavni rezultati ovih istraživanja su sljedeći.

Log-log dijagram koji prikazuje broj nepraznih kutija $N(s)$ u ovisnosti o recipročnoj vrijednosti duljine kutije razlaže se na dva dijela: nesamoslični i samoslični. Nesamoslični dio, koji leži na strani velikih duljina kutije, karakteriziran je velikim fluktuacijama frakcijskog nagiba log-log grafa, koji rastu s duljinom kutije. S druge strane, u samosličnom dijelu log-log dijagrama koji leži ispod određene kritične vrijednosti duljine kutije i proteže se prema manjim vrijednostima sve do donje granice određene mjerilom geografske karte, točke log-log grafa leže vrlo približno na pravcu. Nagib tog pravca pridružuje se ukupnoj fraktalnoj dimenziji obalne crte. U našem računu za otok Cres interval duljine kutije proteže se od kritične vrijednosti $s = 2{,}5$ km prema manjim vrijednostima sve do donje granice 0,125 km, koja je određena skalom geografske karte.

Nadalje, obalna crta otoka Cresa podijeljena je na dijelove i fraktalna dimenzija svakog dijela određena je kao nagib log-log grafa u samosličnom segmentu. Ovako dobivene parcijalne fraktalne dimenzije mogu se međusobno znatno razlikovati za različite dijelove obale.

Zanimljivo pitanje za daljnja istraživanja je kako se samoslični segment log-log grafa ekstrapolira prema još manjim vrijednostima duljine kutije i pojavljuje li se rezanje fraktalne strukture na strani malih duljina kutije. U istraživanju ovisnosti log-log dijagrama na toj vrlo preciznoj skali trebalo bi također uzeti u obzir ovisnost o razini plime i oseke (trebalo bi razmatrati obalnu crtu za neku određenu visinu morske razine) kao i tehničke intervencije u obalnu crtu koje mijenjaju njezinu prirodnu fraktalnost. S druge strane, sistematska istraživanja bi trebalo proširiti prema većim segmentima obalne crte, na primjer na ukupnu obalnu crtu Jadranskog mora. Nadalje, zanimljiv je problem korelacija između fraktalnosti obalne crte na različitoj skali, kao na primjer između ukupne fraktalne dimenzije obalne crte i fraktalnih dimenzija njezinih sastavnih dijelova na uzastopno rastućoj ljestvici.

Također bi bilo zanimljivo da se ovi rezultati interpretiraju u svijetlu općih teorijskih istraživanja kako fraktalnost reljefa nastaje djelovanjem erozije, pri čemu voda djeluje kao medij za prijenos informacije o okolnoj strukturi na točku rasta kao ključni čimbenik lokalnog rasta, kojeg karakteriziraju korelacije dugoga dosega (INAOKA & TAKAYASU, 1993) i tektonskih pomicanja koja su također karakterizirana fraktalnošću (HIRABAYASI, ITO, & YOSHII, 1992).

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