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USING ORTHONORMAL FUNCTIONS IN MODEL PREDICTIVE CONTROL

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Original scientific paper
This paper describes use of Legendre orthonormal functions for representation of the control trajectory in discrete model predictive control, precisely, above mentioned functions are used for efficient parametrization of the difference of control signal. When high demands, fast sampling or complicated dynamics are present in the design, classical approach is not computationally efficient and can lead to poorly numerically conditioned solutions as noted in [1]. Using orthonormal functions, moreover Legendre functions, described in this paper, number of parameters used for description of future control trajectory is reduced and the trajectory itself becomes smoother, with control signal being of smaller amplitude then in the Laguerre case.

Keywords: *model predictive control, orthonormal functions*

Korištenje ortonormalnih funkcija u prediktivnom upravljanju na osnovu modela

Izvorni znanstveni članak
U ovom se članku opisuje korištenje Legendreovih ortonormalnih funkcija za predstavljanje upravljačke putanje u modelu diskretnog prediktivnog upravljanja, točnije, spomenute funkcije se koriste za učinkovitu parametrizaciju razlike kontrolnog signala. Kada su velika potražnja, brzo uzorkovanje ili složena dinamika prisutni u projektu, klasični pristup nije računski učinkovit i može dovesti do slabih numerički uvjetovanih rješenja kako je spomenuto u [1]. Korištenjem ortonormalnih modela, naročito Legendre modela opisanog u ovom radu, smanjuje se broj parametara pri opisu buduće upravljačke putanje, a i sama putanja postaje jasnija, s manjom amplitudom upravljačkog signala nego u slučaju Laguerre.

Ključne riječi: *prediktivno upravljanje na osnovu modela, ortonormalne funkcije*

1 Introduction

In the last couple of years many approaches of model predictive control were proposed. They suggest using different model descriptions. They use hybrid fuzzy model [10, 12], Wiener-model based on PWL [11], hybrid MPC based on genetic algorithms [13], probabilistic neural-network [14].

The main technique used in the design of discrete model predictive controller as noted in [1] is based on modelling the control trajectory, control signal $u(k)$ or the difference of control signal $\Delta u(k)$ by forward shift operators. As noted in [1] the problem that arises when using aforementioned technique is a possible large number of forward shift operators used for the description of control trajectory if complicated dynamics of the process, fast sampling or high demands on closed-loop performance are present. Fast changes of control signal are also possible as there is no structural constraint on the future control signal.

When using discrete orthonormal functions to represent the control trajectory as in [1], the number of parameters used for description is in this way reduced, compared to the classical approach. Choosing the right scaling factor present in the orthonormal function itself allows the change of control signal to be managed. This paper will present the use of Legendre orthonormal functions in description of the difference of control signal $\Delta u(k)$ in discrete model predictive control. As will be seen later in the paper, the function itself is a special case of Generalized Orthonormal Basis Functions (GOBF) for a specific choice for poles of GOBF.

As noted in [1], there are two types of stability approach of model predictive controller systems. The first is the use of terminal constraints on the state variables which forces the terminal state variables to be zero, used in [1] with the Laguerre model. The same approach can be used with Legendre model.

Contribution of this paper is to extend modelling of the difference of control signal $\Delta u(k)$, using Legendre functions.

The paper consists of section 1 where Legendre functions are presented. Section 2 presents algorithm for model predictive controller design using Legendre functions. In section 3 results of simulation examples are presented. After the simulation section follows conclusions.

2 Legendre orthonormal functions

The following construction of discrete time bases from classical orthogonal polynomials in the case of Legendre polynomials which are useful in describing the solution of Laplace's equation in the sphere is defined in [3]. Legendre polynomials satisfy the Rodrigues's formula:

$$\mathrm{P}_n(x) = \frac{1}{2^n n!}\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left[(x^2-1)^n\right], \tag{1}$$

and they satisfy the orthogonality property

$$\int_{-1}^{1} \mathrm{P}_n(x)\mathrm{P}_m(x)\mathrm{d}x = \begin{cases} 2(2n+1)^{-1} & \text{for } n=m \\ 0 & \text{for } n \neq m \end{cases}. \tag{2}$$

Using the substitution $x = 2\mathrm{e}^{-\alpha t} - 1$, $\alpha > 0$ one obtains

$$\int_{0}^{\infty} \mathrm{P}_n(2\mathrm{e}^{-\alpha t}-1)\mathrm{P}_m(2\mathrm{e}^{-\alpha t}-1)\mathrm{e}^{-\alpha t}\mathrm{d}t = \begin{cases} \left[\alpha(2n+1)\right]^{-1} & \text{for } m=n \\ 0 & \text{for } m \neq n \end{cases}$$

so the Legendre functions $\mathrm{P}_n(t)$ defined by

$$\mathrm{P}_n(t)=\sqrt{\alpha(2n+1)}\mathrm{e}^{-\alpha t/2}\mathrm{P}_n\left(2\mathrm{e}^{-\alpha t}-1\right)$$

are $L_2([0,\infty))$ orthonormal.

As shown in [3] Fourier transform is

$$\hat{p}_n(\omega)=\frac{\sqrt{\alpha(2n+1)}}{\mathrm{i}\omega+(n+1/2)\alpha}\prod_{k=0}^{n-1}\left(\frac{\mathrm{i}\omega-(k+1/2)\alpha}{\mathrm{i}\omega+(k+1/2)\alpha}\right). \tag{3}$$

Using Parseval's theorem one has that $\hat{p}_n(\omega)$ given by (3) is an orthonormal basis for $H_2(R)$. By using the bilinear transform defined by $\mathrm{i}\omega = (\mathrm{e}^{\mathrm{i}\varphi} - 1)(\mathrm{e}^{\mathrm{i}\varphi} + 1)^{-1} = M(\varphi)$ which maps the imaginary axis to T, one gets the Legendre orthonormal basis for $H_2(T)$:

$$\mathrm{B}_{n-1}(\mathrm{e}^{\mathrm{i}\omega})=\frac{\sqrt{1-\xi_n^2}}{\mathrm{e}^{\mathrm{i}\omega}-\xi_n}\prod_{k=0}^{n-1}\left(\frac{1-\xi_k\mathrm{e}^{\mathrm{i}\omega}}{\mathrm{e}^{\mathrm{i}\omega}-\xi_k}\right) \tag{4}$$

$$\xi_k=\frac{2-\alpha(2k+1)}{2+\alpha(2k+1)}. \tag{5}$$

The Legendre basis as stated in [3] can be simply obtained from the general GOBF construction

$$\mathrm{B}_n(z)=z^d\left(\frac{\sqrt{1-|\xi_n|^2}}{z-\xi_n}\right)\prod_{k=0}^{n-1}\left(\frac{1-\overline{\xi_k}z}{z-\xi_k}\right);\, d=0 \text{ or } 1, \tag{5a}$$

with the substitution for ξ_k given by (5).

As suggested in [3], no reference to these Legendre functions being used for system identification can be found. They even intuitively seem more useful than the popular Laguerre basis functions as they have a progression of the poles and would be a better choice at high frequencies. From the perspective of modelling of the control sequence in model predictive control with Legendre functions the same statement can be made.

3 Model predictive control and Legendre orthonormal functions

The following description is made based on the one presented in [1].

The state variable vector $x(k_i)$ provides the current plant information, where k_i, $k_i > 0$ is i^{th} sampling instant. In Model Predictive Control (MPC), the future control trajectory is denoted by

$$\Delta u(k_i), \Delta u(k_i+1), \ldots, \Delta u(k_i+N_C-1), \tag{6}$$

N_C being the control horizon dictating the number of parameters used to capture the future control trajectory. Having $x(k_i)$, the future state variables are predicted for

N_p number of samples, N_p being the prediction horizon. N_p is also the length of the optimization window. The future state variables are

$$x(k_i+1|k_i), x(k_i+2|k_i), \ldots, x(k_i+m|k_i), \ldots, x(k_i+Np|k_i),$$

where $x(k_i + m|k_i)$ is the predicted state variable at $k_i + m$ having current plant information $x(k_i)$. The control horizon N_C is chosen to be less than (or equal to) the prediction horizon N_p.

Receding horizon control principle takes only the first sample of the sequence of vector ΔU containing the controls, written in (6), and implements it, ignoring the rest of the sequence. In the next sample period, the more recent measurement is used to form the state vector for the calculation of the new sequence of control signal.

The z-transforms of the discrete-time Legendre networks are written in (4).

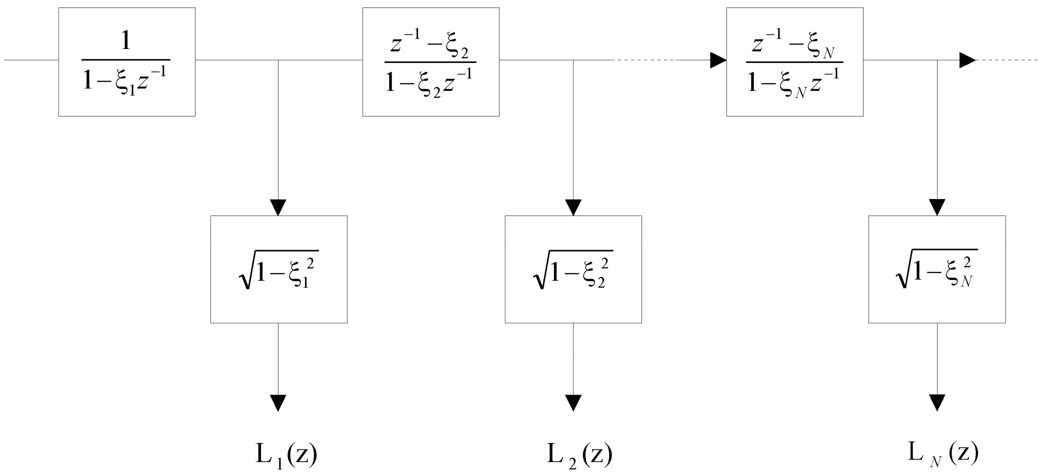

Figure 1 Legendre network

$l_i(k)$ denotes the inverse z-transform of $\mathrm{B}_i(z, \xi_1, ..., \xi_i)$. The set of discrete-time Legendre functions is expressed in a vector form as

$$\mathrm{L}(k)=\left[l_1(k)\ l_2(k) \ldots l_N(k)\right]^{\mathrm{T}}. \tag{7}$$

The set of discrete Legendre functions satisfies the following difference equation:

$$\mathrm{L}(k+1)=\Omega\mathrm{L}(k), \tag{7a}$$

similar to the Laguerre case in [1].

By analyzing Legendre network given in Fig. 1 or by analyzing construction given by (4) and using the following z transform properties given in Tab. 1.

Table 1 Some properties of z transform

$X(z)$	$x(k)$
$X(z)\,H(z)$	$\sum_{m=0}^{\infty} x(m)h(k-m)$
$\lim_{z\to 1} X(z)$	$x(0)$

One obtains

$$\Omega=\begin{bmatrix} \xi_1 & 0 & 0 & \dots & 0 \\ 0 & \xi_2 & 0 & \dots & 0 \\ 0 & 0 & \xi_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & \xi_n \end{bmatrix} \tag{7b}$$

$$\mathrm{L}(0)=\begin{bmatrix} \sqrt{\beta_1} \\ -\xi_1\sqrt{\beta_2} \\ \xi_1\xi_2\sqrt{\beta_3} \\ -\xi_1\xi_2\xi_3\sqrt{\beta_4} \\ \xi_1\xi_2\xi_3\xi_4\sqrt{\beta_5} \\ \vdots \end{bmatrix} \tag{7c}$$

β_i being $\beta_i = 1-\xi_i^2$.

At time k_i, the control trajectory $\Delta u(k_i)$, $\Delta u(k_i+1)$, ..., $\Delta u(k_i+k)$, is regarded as the impulse response of a stable dynamic system. A set of Legendre functions, $l_1(k)$, $l_2(k)$, ..., $l_N(k)$ is used to capture the dynamic response with a set of Legendre coefficients that are to be determined from the design process. Based on this,

$$\Delta u(k_i+k)=\sum_{j=1}^{N} c_j(k_i)l_j(k), \tag{8}$$

where k_i is the initial time of the moving horizon window and k is the future sampling instant, N is the number of terms used in the expansion, c_j, j = 1, 2, ..., N, are the coefficients being functions of the initial time of the moving horizon window, k_i. In this way, the control horizon N_C from the earlier (classical) approach has vanished, as thoroughly explained in [1]. Now, the number of terms N along with the parameters ξ_1, ξ_1, ..., ξ_N is used to capture the trajectory.

Eq. (8) can also be expressed in a vector form:

$$\Delta u(k_i+k)=[\mathrm{L}(k)]^{\mathrm{T}}\eta. \tag{9}$$

Where η has N Legendre coefficients:

$$\eta=[c_1 \quad c_2 \quad \dots \quad c_N]^{\mathrm{T}}. \tag{10}$$

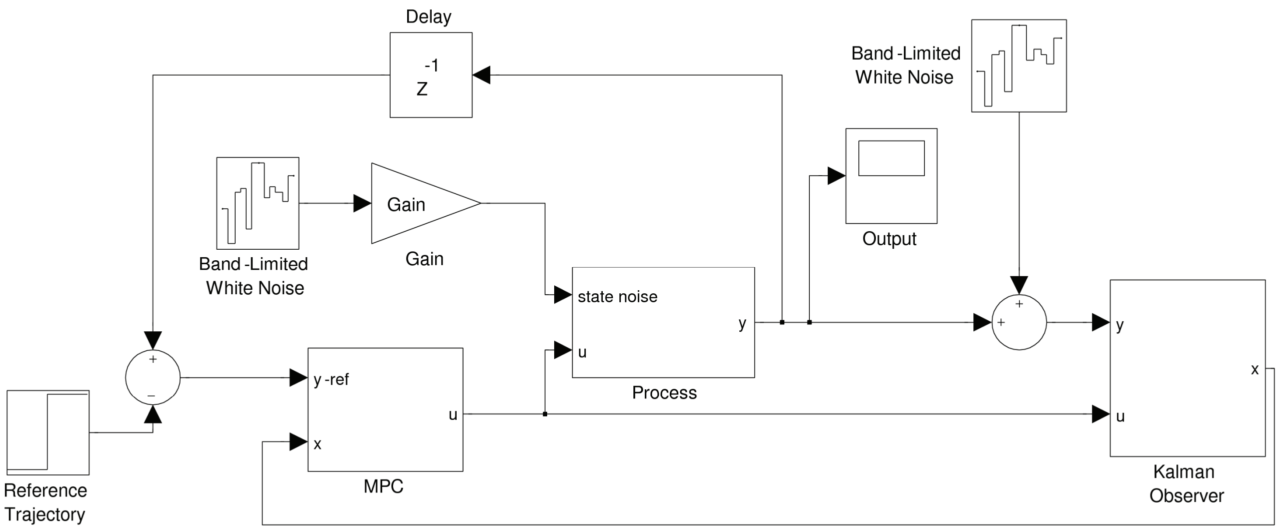

Figure 2 Block scheme used for simulation of processes

So, the coefficient vector η is optimized and computed in the design.

As thoroughly explained in [1], an alternative formulation of the cost function is used and formulated with a link to discrete-time linear quadratic regulators (DLQR). The task is finding the coefficient vector η to minimize the cost function:

$$J=\sum_{m=1}^{N_p} x(k_i+m|k_i)^{\mathrm{T}}Qx(k_i+m|k_i)+\eta^{\mathrm{T}}R_L\eta \tag{11}$$

$Q\geq 0$ and $R_L>0$ being weighting matrices.

Having optimal parameter vector η, the receding horizon control law is realized as

$$\Delta u(k_i)=[\mathrm{L}(0)]^{\mathrm{T}}\eta. \tag{11a}$$

Design parameter N is the number of terms used in capturing the control signal, as in the Laguerre case. As stated in [1] when hard constraints are involved, a quadratic programming procedure is typically used in the optimization algorithm.

Parameter ξ_i defined in (5) is like parameter a in the Laguerre case, except that it has progression.

Stability of the closed loop system is based on the property of model predictive control that it can be guaranteed under certain circumstances. An approach that uses terminal constraints on the state variables, which forces the terminal state variables to be zero, is thoroughly explained in [1] and can be used in the Legendre case.

4 Simulation

Simulation has been done for two processes of different dynamics.

4.1 First process

Simulation has been done for a mechanical highly oscillatory process with non-minimum phase also used in [1] and [2]. The process is given by the transfer function:

$$G_1(z)=\frac{-5{,}7980\cdot z^3+19{,}5128\cdot z^2-21{,}6452\cdot z+7{,}9547}{z^4-3{,}0228\cdot z^3+3{,}8630\cdot z^2-2{,}6426\cdot z+0{,}8084}, \quad (12)$$

having sampling time $T_s = 0{,}2$ s.

Open loop step response of aforementioned process is given in Fig. 3.

Model predictive control has been used with parameterization using Laguerre and Legendre orthonormal functions. In both cases, number of functions used for modelling the control sequence is $N = 10$, and prediction horizon is $N_p = 100$. Weighting matrices R and Q are in both cases the same, being $R = 0{,}1$ and

$$Q=\begin{bmatrix}0 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 1\end{bmatrix}. \quad (12a)$$

As can be seen in Fig. 2 state x of the process has been obtained using Kalman observer.

Fig. 4 shows output of the process for three cases, using Laguerre and Legendre functions and using PID controller tuned with Ziegler-Nichols (ZN) recommendations. In case when Laguerre functions are used $\xi = 0{,}95$ and in the case when Legendre functions are used, progression of the pole is given in (5) with $\xi_1 = 0{,}95$. In the classical MPC case with forward shift operators, performance is worse than in case when using Laguerre functions which is shown in [1]. After making numerous simulation experiments, where pole values are greater than $\xi = 0{,}95$ in Laguerre case, better results are obtained using Legendre functions as control signal is of smaller amplitude and the difference of control signal Δu(k) is also smoother. Output of the process is much slower in the case when PID controller is used, tuned with Ziegler-Nichols recommendations and has bigger amplitude of control signal.

Fig. 5 shows control signals in PID, Laguerre and Legendre cases. As can be noted, difference of control signal is also much smoother in Legendre case than in Laguerre case.

As can be seen from Fig. 6 when using Legendre functions, non-minimum phase of the process has smaller effect on output than in case when using Laguerre functions. For PID case it has smaller effect than in the other two cases, but output is notably slower than in other two cases.

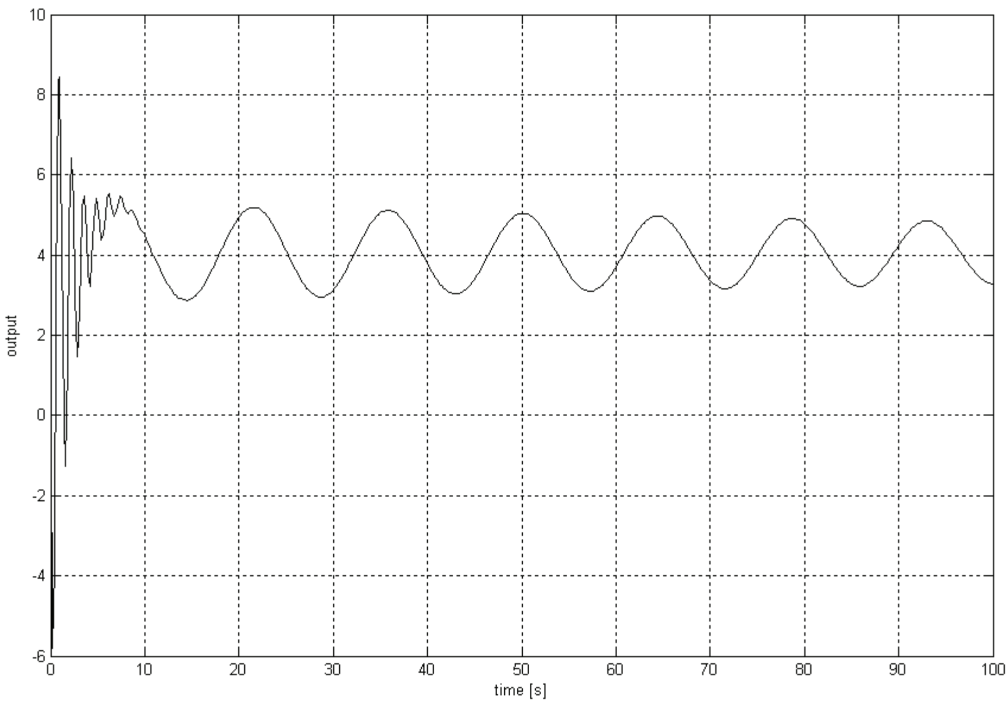

Figure 3 Open loop step response of the process given in (12)

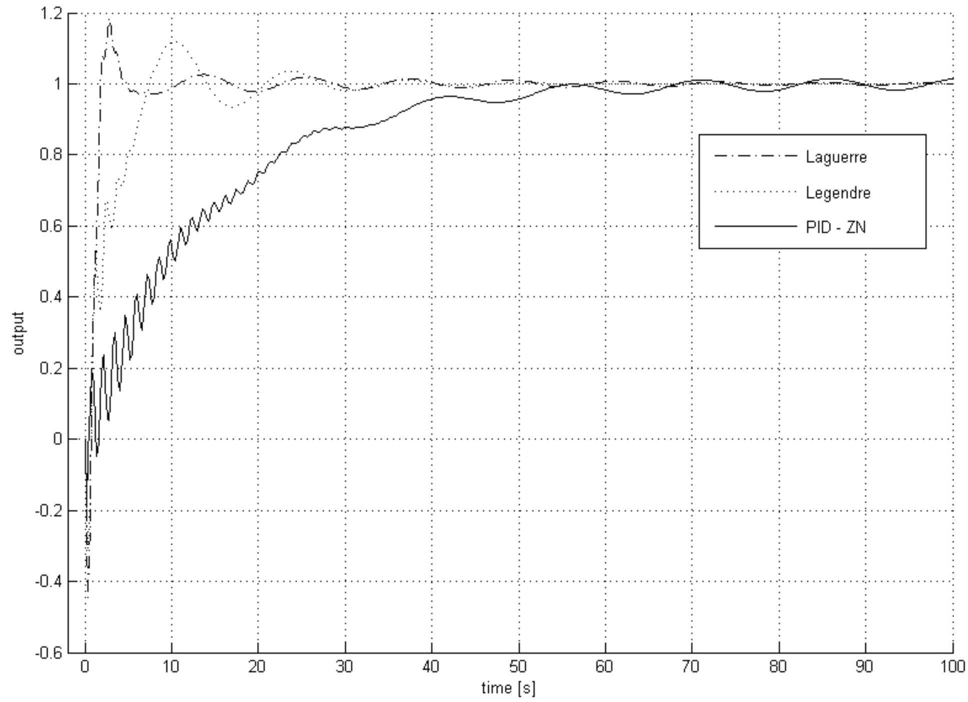

Figure 4 Output of the process given in (12) for the three cases: solid – using PID controller tuned with Ziegler-Nichols recommendations; dashed - using Laguerre functions; dotted – using Legendre functions

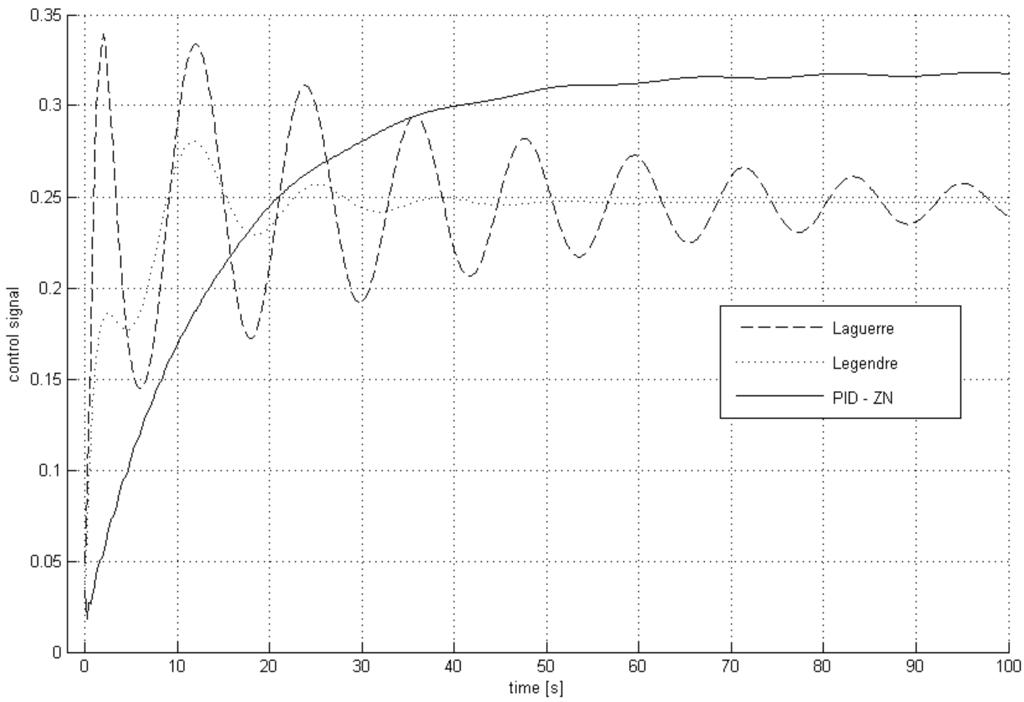

Figure 5 Control signal of the process given in (12) for the three cases: solid – using PID controller tuned with Ziegler-Nichols recommendations; dashed - using Laguerre functions; dotted – using Legendre functions

4.2 Second process

Simulation has been done for a third order process with dead time given by the transfer function:

$$G_2(s) = \frac{K\mathrm{e}^{-Ds}}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)}, \tag{13}$$

being $K = 1$, $D = 15$, $T_1 = 10$, $T_2 = 20$, $T_3 = 30$.

Sampling time used is $T_s = 1$ s.

Fig. 7 shows open loop step response of the process given in (13).

Model predictive control has been used with parameterization using Laguerre and Legendre orthonormal functions. In both cases, the number of functions used for modelling the control sequence is $N = 5$, and prediction horizon is $N_p = 200$. Weighting matrices R and Q are in both cases the same and are as in section 3.1. The same block scheme for simulation is used as in section 3.1 and is given in Fig. 2.

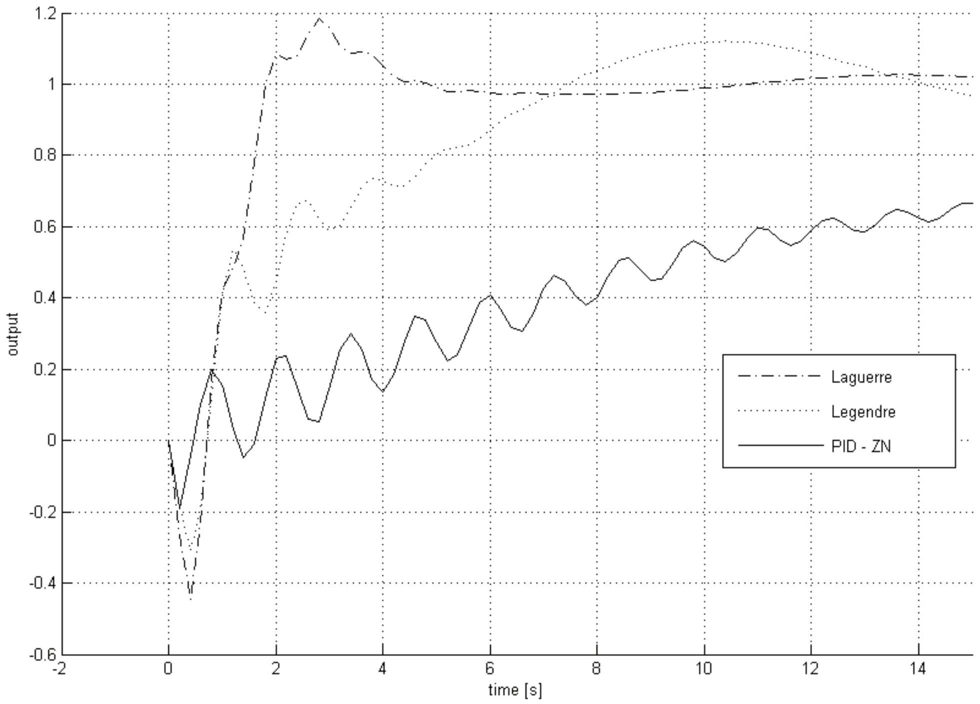

Figure 6 Output of the process given in (12) for the three cases: solid – using PID controller tuned with Ziegler-Nichols recommendations; dashed – using Laguerre functions; dotted – using Legendre functions

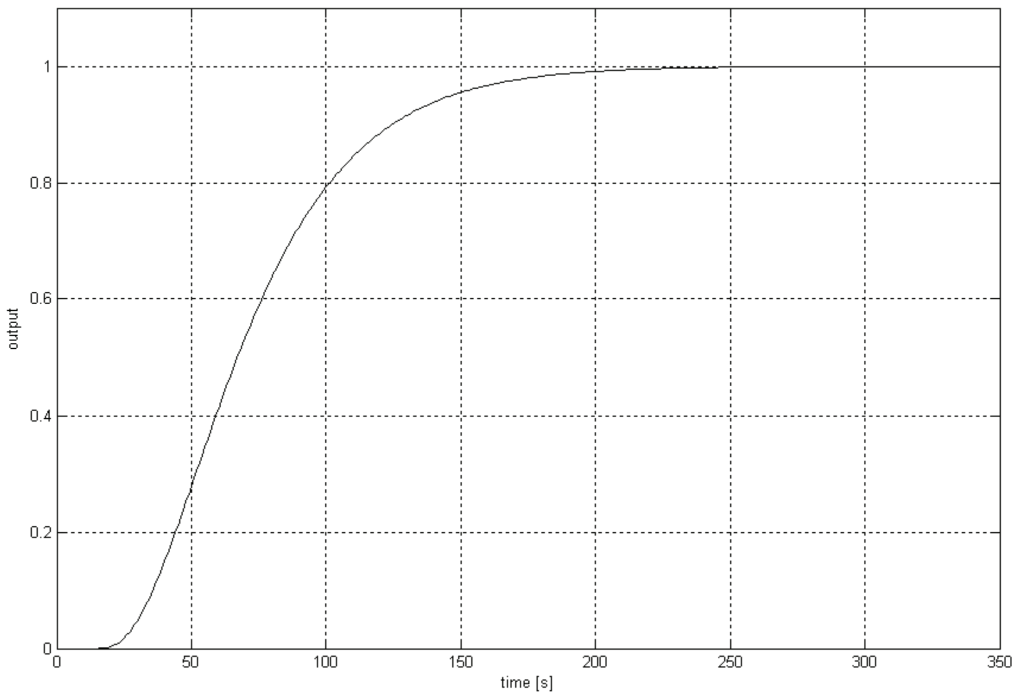

Figure 7 Open loop step response of the process given in (13)

Dynamics of the process described in (13) is much simpler than in the process described in (12).

In the case when Laguerre functions are used $\xi = 0{,}98$ and in the case when Legendre functions are used, progression of the pole is given in (5) with $\xi_1 = 0{,}9$. From Fig. 8 and Fig. 9 one can note that the output of the process when using classical approach (FIR) that equals Laguerre one with the choice of all poles being zeros [1] is faster than in the cases when Laguerre and Legendre functions are used. The cost is in several times bigger amplitude of control signal. When comparing cases in which Laguerre and Legendre functions are used, one can note that with the chosen pole locations, for almost the same speed and slightly smaller overshoot, in the case when Legendre functions are used, amplitude of control signal is also slightly smaller in the case when Legendre functions are used.

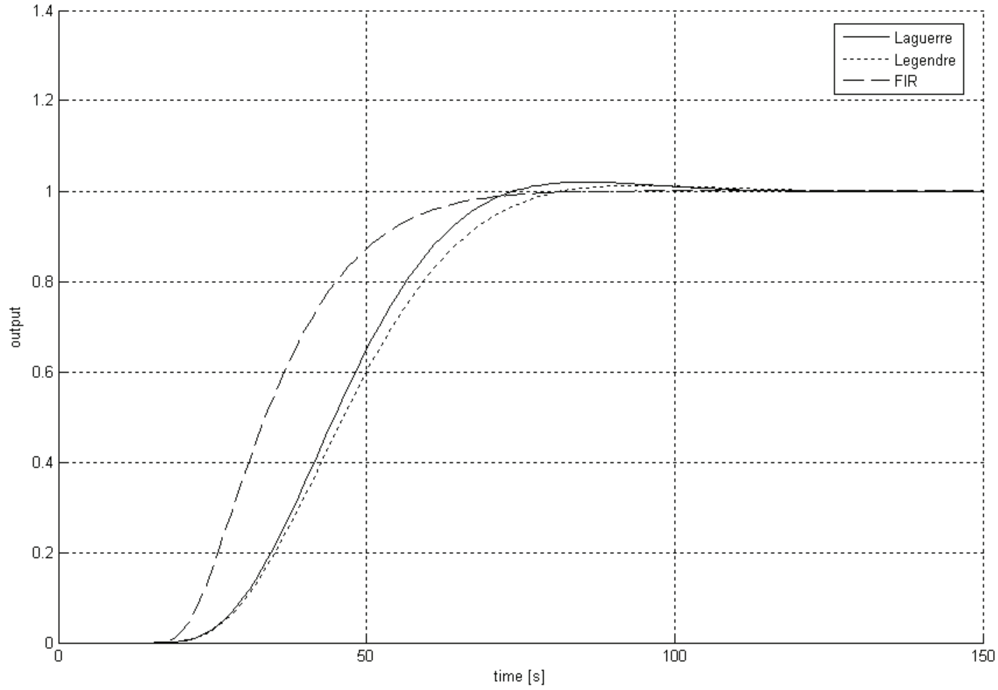

Figure 8 Output of the process given in (13) for the three cases: solid – using Laguerre functions; dotted – using Legendre functions; dashed – using classical approach (FIR) when poles equal zero when using Laguerre functions

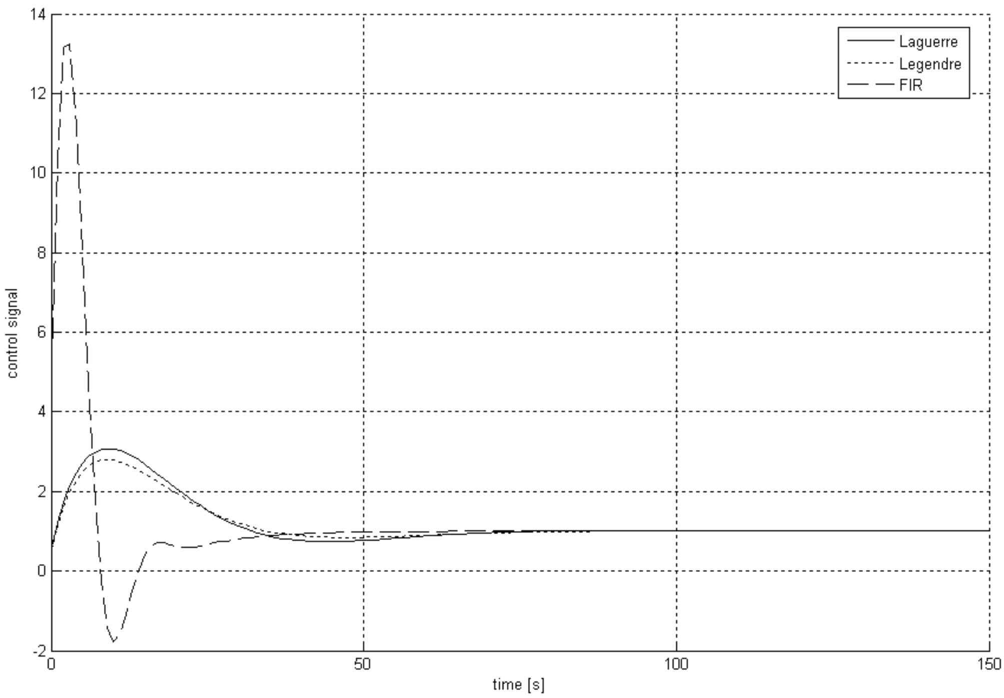

Figure 9 Control signal of the process given in (13) for the three cases: solid – using Laguerre functions; dotted – using Legendre functions; dashed – using classical approach (FIR) when poles equal zero when using Laguerre functions

5 Conclusion

As can be seen from results presented, the proposed use of Legendre functions in model predictive control is a much better choice for systems with complicated dynamics in which amplitude of control signal needs to be lowered as much as possible, for almost no cost in speed and overshoot of output signal or in computational complexity compared to the case where Laguerre functions are used.

Also, non-minimum phase of the process has smaller impact to output as can be seen from Fig 6. Difference of control signal $\Delta u(k)$ is also smoother when using Legendre functions than when using the Laguerre ones.

It can be noted that in the case when Legendre functions are used, one has better ratio between speed of the process and its overshoot on one side and amplitude of the control signal and its difference on the other side, especially in case of complicated process dynamics.

Further extensions may go in the direction of investigating other possible pole locations for the general GOBF constructions and their effect on the output and control signals in cases when the same number of functions N is used.

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