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ISSN 0543-5846
METABK 52(1) 119-122 (2013)
UDC – UDK 621.81:621-772:338.58=111

OPTIMAL REPLACEMENT TIME ESTIMATION FOR MACHINES AND EQUIPMENT BASED ON COST FUNCTION

Received – Prispjelo: 2012-03-29
Accepted – Prihvaćeno: 2012-07-30
Preliminary Note – Prethodno priopćenje

The article deals with a multidisciplinary issue of estimating the optimal replacement time for the machines. Considered categories of machines, for which the optimization method is usable, are of the metallurgical and engineering production. Different models of cost function are considered (both with one and two variables). Parameters of the models were calculated through the least squares method. Models testing show that all are good enough, so for estimation of optimal replacement time is sufficient to use simpler models. In addition to the testing of models we developed the method (tested on selected simple model) which enable us in actual real time (with limited data set) to indicate the optimal replacement time. The indicated time moment is close enough to the optimal replacement time *t**.

Key words: machine replacement time, estimation, cost function

INTRODUCTION

The classic approach to determination of optimal replacement time in framework of renewal theory is based on probability. In the framework of replacement analysis [1-5] we can find different approaches as e.g. economic life models [6], productivity analysis [7] simulation model [8] profit maximization models [9]. Some authors incorporate different considerations: technological change [10-12], partially observed Markov process [13, 14], demand responsiveness [15], pricing policies [16], etc. to the analysis. Our approach is based on the idea, that „optimal maintenance policies can be obtained by minimizing the total expected cost...“ [17].

In our investigation we use real data set of commercial car operation, presented in Table 1. Presented input data received from an existing company contain information on cumulative costs and passed kilometers for each of 31 quarters (93 months) of the commercial car operation.

Table 1 Input data structure

Time *t* /quarters = 3 months	Distance s / km	Cumulative costs N / EURO
1	895	8 364,87
2	2 685	9 294,30
…	…	…
31	310 541	77 175,86

Remark: The cumulative costs cover: fuel consumption and consumption of other liquids needed for car operation (oil, distilled water, brake liquid, etc.), regular servicing (inspections, new oil, new tires, new bulbs, etc.), and unexpected repairs (servicing after a road accident, etc.).

Our approach to determination of optimal replacement time has not been developed from the mainstream renewal theory models. We started from a model [18-20] in the beginning which calculates optimal replacement time on the base of cost function in a form

$$N \approx A + Bt + Ct^{\delta} \tag{1}$$

where t is the time. Then for unit cost we get

$$\frac{N}{t} = \frac{A}{t} + B + Ct^{\delta-1}. \tag{2}$$

Optimal replacement time t^* at which unit costs are minimal is:

$$t^* = \sqrt[\delta]{\frac{A}{C(\delta-1)}} \tag{3}$$

The model is operating in a way based on the data set (sequence of periods of machine or equipment use with information on cumulative costs for each period (e.g. our data set in Table 1 (column 1 and 3)). Parameters A, B, C, δ of the model for N (resp. N/t) are determined by some fitting technique, and then the optimal replacement time t^* is calculated.

As the optimal replacement time we consider the time t^* calculated using data from longer period of time, e.g., in our case 31 quarters (about 8 years) of machine operation. But in practice, we would like to know earlier if it is suitable to replace a machine.

If we use the original model for calculation of optimal replacement time t^* successively at the end of each time period (starting, e.g. from the 5th quarter) the first estimates of t^* will be fundamentally different from t^* itself.

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In text below we present in detail an approach based on sequential improvement of optimal replacement time estimation.

In addition to presented successive estimation of replacement time we also studied some models for function *N/t* with more parameters and/or terms, and/or with incorporation of the second variable *s*. The reason is to study if more complicated models will have significantly better approximations of our real data set.

Models are compared by the value of the Sum of squared residuals (SSR):

$$SSR = \sum_{i=1}^{K}\left[f(t_i) - \frac{N_i}{t_i} \right]^2 \quad (4)$$

where *K* is the sample size, and *f*(*t*) is the corresponding model. For each considered model we have calculated the Mean SSR error (MSSRE):

$$MSSRE = \sqrt{\frac{SSR}{K}} \quad (5)$$

The model with minimal MSSRE value is considered as the most appropriate one for data approximation.

DEVELOPMENT AND COMPARISON OF ALTERNATIVE MODELS

In this section we first present models which we have studied based on one variable – model (6), (7), and (8) for variable *t*, and models (9) and (10) for variable *s*. We have considered also two variable models (11), (12), (13), and (14).

For the model parameters marking unification reason let us consider first Selivanov model in the form:

$$\frac{N}{t} \approx c_1 + \frac{c_2}{t} + c_3 t^{c_4} \quad (6)$$

Beside the Selivanov model we have studied next model

$$\frac{N}{t} \approx c_1 + c_2 t + c_3 t^{c_4} \quad (7)$$

The next model have been more complicated one:

$$\frac{N}{t} \approx c_1 + \frac{c_2}{t^{c_3}} + c_4 t^{c_5} \quad (8)$$

which is a generalization of both models (6) and (7). For models (7) and (8) we got the following "optimal" formulae:

$$\frac{N}{t} \approx -1\,987{,}5 + 112{,}3t + 10\,246{,}8\, t^{-0{,}684}$$

$$\frac{N}{t} \approx -2\,273{,}3 + 4141{,}9 t^{0{,}947} + 10\,500{,}3 t^{-0{,}665}$$

The approximation (8) has smaller MSSRE (see Table 2) what is the result of additional parameter introduction.

In addition to models based on time variable *t*, we have considered next two models for distance variable *s*:

$$\frac{N}{t} \approx c_1 + c_2 s + c_3 s^{c_4} \quad (9)$$

$$\frac{N}{t} \approx c_1 + c_2 s + c_3 s^{c_4 + c_5 \ln s} \quad (10)$$

Next four two-variable models combine terms containing both variables *t* and *s*:

$$\frac{N}{t} \approx c_1 + c_2 t + c_3 t^{c_4} + c_5 s \quad (11)$$

$$\frac{N}{t} \approx c_1 + c_2 t + c_3 t^{c_4} + c_5 s + c_6 s^{c_7} \quad (12)$$

$$\frac{N}{t} \approx c_1 + c_2 t + c_3 t^{c_4} + c_5 s + c_6 t \cdot s, \quad (13)$$

$$\frac{N}{t} \approx c_1 + c_2 t + c_3 t^{c_4} + c_5 s + c_6 t \cdot s + c_7 s^{c_8} . \quad (14)$$

Parameters $c_1,\ldots,c_8$ for all considered models we have determined using least squares method, by minimizing SSR value. Corresponding MSSRE values are shown in Table 2.

Table 2 MSSRE errors for all models

Model	(6)	(7)	(8)	(9)
MSSRE	76,523	37,907	37,794	82,472
(10)	(11)	(12)	(13)	(14)
80,098	37,795	37,179	37,784	36,862

The residuals for all models are graphically presented in Figure 1. Dashed line corresponds to Selivanov model (5), dotted lines correspond to the *s*-variable models (9) and (10). We can see that residuals of these models are larger than residuals for other *t*-variable and two-variable models represented by solid lines which are all close to each other. So, models (7), (8), and (11)–(14) are according to the spread of residuals preferable.

From Table 2 it is evident that models (9) and (10) based only on the variable *s*, and also the Selivanov model (6) have larger MSSRE values than other models. Adding new parameter or successive new terms to the model (7), and combining *t* and *s* variables leads to

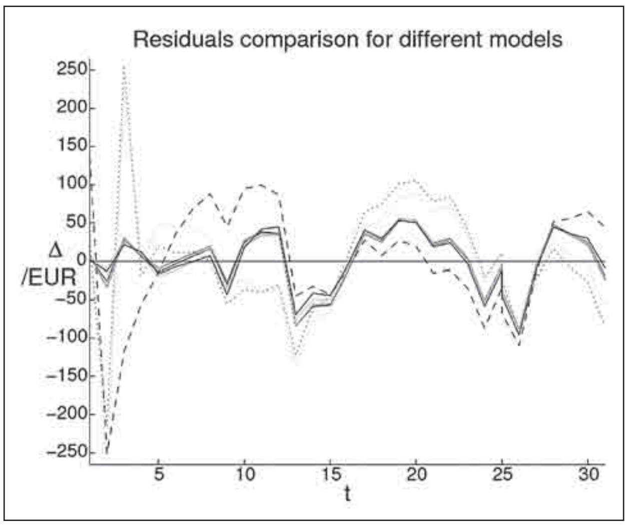

Figure 1 Residuals for all models

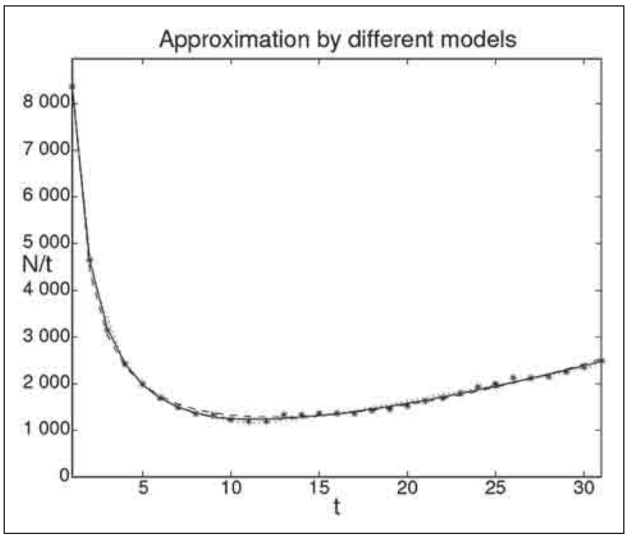

Figure 2 Approximation comparison

smaller MSSRE for models (8), and (11)–(14). The residuals for models (7), (8), and (11)–(14) are very similar, and practically equal, see Figure 1. Despite of different MSSRE values shown in Table 2, Figure 2 indicates that all approximation are good enough, hence all models are good descriptions of the behavior of unit costs from our data set. From this we can assume, that it is not necessary to use in practice more complicated models, respectively it is sufficient to use simple models (7) or (8).

OPTIMAL REPLACEMENT TIME ESTIMATION

We define the optimal replacement time t^* as the time at which the unit costs are minimal. After that moment, unit costs are increasing.

Based on the knowledge received in previous section we will further focus our optimal replacement time estimation to the models (7) and (8). For model (7) we come to the optimal replacement time t^* by the assumption that the derivative *N/t* at point t^* (resp. at the minimum of the cost function) should be equal to zero, so we get:

$$t_7^* = \left[-\frac{c_2}{c_3 \cdot c_4} \right]^{1/(c_4 - 1)} \tag{15}$$

In the same way we get optimal replacement time formula for model (8):

$$t_8^* = \left[\frac{c_2 \cdot c_3}{c_4 \cdot c_5} \right]^{1/(c_3 + c_5)} \tag{16}$$

If we count particular optimal replacement times for both models, we can see that difference between t_1^*=11,6388 and t_5^*=11,5869 is irrelevant, what is caused by fact, that both models are good approximations to the real cost data set, what we have found out in previous section. These optimal replacement times are but calculated on the base of models approximated with use of whole real data set (in our case 31 quarters) (Remark: Approximations presented in Figure 2 are all based on complete data for 31 periods). But the point is that at the time when we should decide about replacement of machine, we do not have whole data set (in our case 31 time periods). So our approach to enable practically usable replacement time estimation also with cost data set from limited number of time periods of machine operation consists in an idea that it is possible to use actual data which arrive at the end of each time period, and successive improve the estimation.

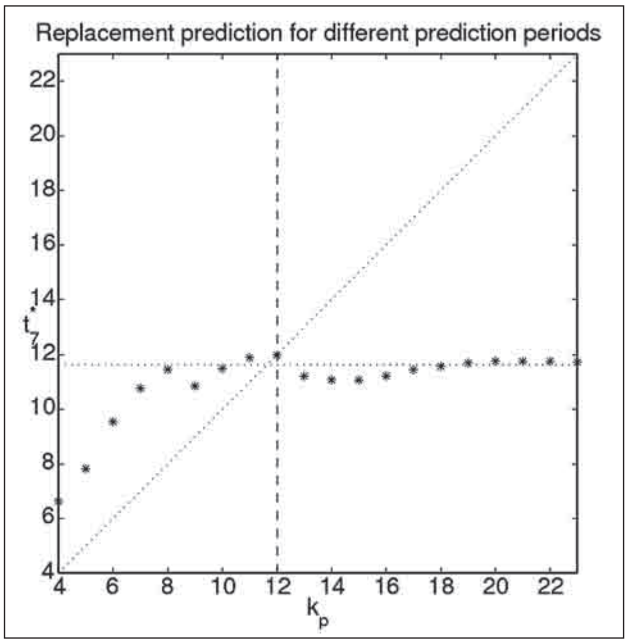

Figure 3 Replacement time estimation

On the Figure 3 we can see a sequence of estimates of “optimal” replacement times t_7^* (it means t^* calculated by using model (7) and corresponding formula for optimal replacement time (15)) for different number of time periods from beginning (k_p) (in our case number of quarters from the beginning of car purchase). For each value k_p we use corresponding partial data set for the calculation of parameters for model (7) and consequently for the replacement time t_7^* calculation. The dotted horizontal line indicates the value of the optimal replacement time calculated for the whole data set (t_7^*=11,6388).

How we could use described behavior of successive t_1^* estimations on Figure 3 on prediction of the optimal replacement time in real time? This behavior bring us to the idea, that we can make the replacement decision at the moment k_p, when the corresponding t_7^* value get below the bisectrix $t_7^* = k_p$ (see Figure 3 or in more detail Figure 4). After that value k_p we get the next estimations of the replacement time in the past ($t_1^* < k_p$), indicating that we have passed the right replacement time. So, it is an appropriate moment for replacement. Hence in our real data case the first value k_p for which $t_1^* < k_p$ is k_p=12 (see Figure 4) which is for practical use close enough to the optimal replacement time t_1^*=11.6388 calculated from the whole real data set.

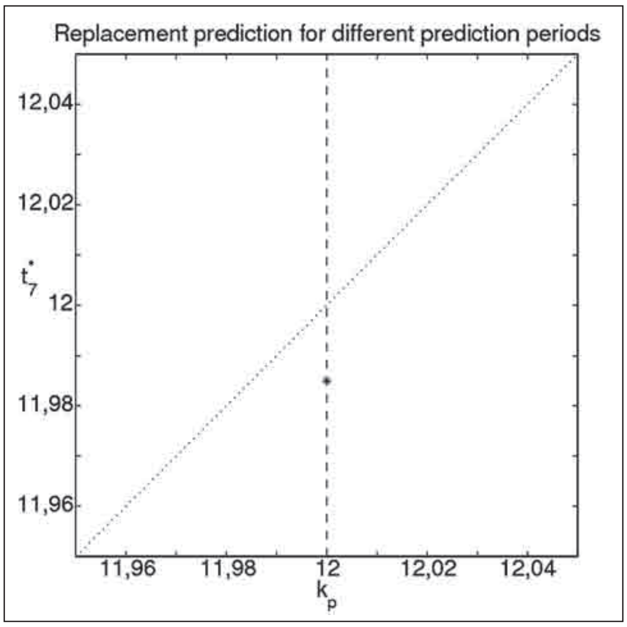

Figure 4 Replacement time at point $k_p = 12$

CONCLUSION

We can conclude that for determination of optimal replacement time t^* defined as the time at which the unit costs are minimal we can use presented method using behavior of successive t^* estimations. It is indicating when estimated replacement time is for the first time smaller than actual real time ($t^*<k_p$), what means that we have just passed the right replacement time. So the method enable us to decide to replace a machine at the real time moment which is close enough to the optimal replacement time t^* calculated from whole data set (it means the data set for long operation of the machine).

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Acknowledgement:

This contribution has been supported by research grants: KEGA 023TUKE-4/2012 – “Creation of complex education–teaching material for subject The production technology with using a combination of classical and modern information technology and e-learning” and VEGA 1/0102/11 Methods and techniques of experimental modeling of production and nonproductive processes in enterprises.

Note: The responsible for English language is Peter Kovkol from State Language School, Košice.