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Pregledni znanstveni članak

An Analysis of the Impact of Errors Occurring in the Auxiliary Parameters while Determining Geometric Corrections of Distance

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ABSTRACT. Distance measurement results are hampered by systematic and accidental errors. The influence of systematic errors is reduced or eliminated mostly through the entering of appropriate corrections or through the use of a particular measurement method. A large number of different sources of corrections may be divided into three groups, namely into meteorological, geometric and projection corrections. In this paper, exclusively geometric corrections are explored. The determination of geometric corrections and the elimination of their influence upon measurement results demands knowledge or measurement of various auxiliary parameters. Auxiliary parameters are also hampered by accidental errors, which under the law of propagation of errors affect the precision of the distance measured adjusted for geometric corrections. These are all generally known facts, but due to reasons unknown to the authors, the problematics exposed in this paper have not yet been treated in literature. This paper presents an analysis of the impact of auxiliary parameters on the precision of measured distances that have been adjusted for geometric corrections. Knowing the precision of a distance adjusted in this manner is significant, as the previously listed effects are present in the daily implementation of surveying tasks in the field of precise geodetic measurements and in the calibration of electronic instruments used to measure distances. This paper also presents a detailed example of calculating the impact of geodetic sources and their standard deviations on the precision of an electronically measured distance. In addition, this paper can also serve as a detailed general instruction manual for everyday professional application during precision distance measurements.

Keywords: electronic distance measurement, geometric corrections, standard deviation.

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1. Introduction

Usually, slope distances are measured on the survey site. For further calculations, it is necessary to reduce slope distances to the horizon, respectively to the projection plane (Džapo and Zrinjski 2004, Zrinjski 2010, Zrinjski and Džapo 2010). Results of distance measurements are encumbered by different systematic and accidental errors. Therefore, it is necessary to eliminate different impacts from the results of distance measurements by the operating method or by entering appropriate corrections (Schofield and Breach 2007). With respect to the character of the impacts existing within the results of slope distances measurements, the corrections can be divided into three groups (Rüeger 1996):

- meteorogical corrections,
- geometric corrections and
- projection corrections.

Firstly, measured distances are to be corrected based on meteorogical impacts. The electromagnetic waves propagate through the air, which is heterogeneous, and the speed of wave propagation depends on certain meteorogical characteristics of the atmosphere, such as: air temperature, air humidity and air pressure.

Geometric corrections are the result of the imperfection of instruments and measuring equipment, spatial distortion of the path of electromagnetic waves as well as the position of the points in space, which includes the distance to be measured, in relation to the instrument itself and the reflector (Hashemi et al. 1994).

The projection corrections are to be entered because of the aforementioned need to determine the horizontal projection of the distance to be measured and its representation within the projection plane.

In order to determine the appropriate corrections, other auxiliary parameters have to be measured on the survey site apart from the distance itself, such as for example: air temperature, difference in altitude, height of the instrument and the reflector (Zrinjski 2010, Zrinjski and Džapo 2010), etc. Simultaneously, the auxiliary parameters are, just like the distance itself, encumbered by accidental errors. Pursuant to the Law on the Propagation of Errors, the precision of the measurement of auxiliary parameters affects the precision of the corrected distance. Understanding the total standard deviation of the corrected distance is very important in because not only the precision of the other parameters, but also further field work depend on the precision of the corrected distance.

2. Geometric Corrections of the Distance

Geometric corrections of the measured distances are as follows (Zrinjski 2010):

- correction of the distance by an additive constant,
- correction of the distance by a multiplication constant,
- distance correction due to refraction,
- distance reduction.

While performing usual engineering tasks, distances are not to be corrected on account of the impact of refraction. The reason for this is that such corrections have a minor influence, i.e., they are smaller than the precision of the distance measurement itself (Saastamoinen 1964). Specifically, the value k = 0,13 has been adopted both in Slovenia and in Serbia as the refraction coefficient for light waves and only distances larger than 39 km are corrected for the refraction, which is in the millimetre size range (Mrkić 1991).

The necessary calculations are shown using as an example a distance measured from the point Šoštanj (The Republic of Slovenia).

Data relevant for calculation are (Kogoj 2005):

- total station: Leica TDM5000
- standard deviation of the distance measurement: 2 mm + 2 ppm
- standard deviation of the angle measurement: 1"
- additive constant: –0.8 mm
- standard deviation of the additive constant: 0.2 mm
- measured distance: 1281.0078 m
- height of the total station: 0.234 m
- height of the reflector: 1.390 m
- slope distance reduced by meteorogical corrections: 1281.0055 m
- standard deviation of the distance reduced by meteorogical corrections: 4.6 mm
- zenith distance: 86°55'48.85"
- elevation of the station: 450.990 m.

As support for this example, all computations within this paper were performed in the program package Microsoft Office Excel using the corresponding functions, which were specifically implemented into the program (URL 1).

3. Determination of the Distance Correction for the Additive Constant

The additive constant is the horizontal distance between the centre of the emission of electromagnetic waves and the point at which the measurement is performed (Dzierzega and Scherrer 2003). This correction includes all geometric, electronic and linear eccentricities of both the total station and the reflector, as well as errors that occurred because of the difference in the speed of wave propagation through the electro-optical system of the instrument and reflector, and through the air.

The eccentricity of the total station K_I is the result of the path geometry of the reference signal and the electronic delay within the total station. It occurs because the centre of the measurement is not positioned on the vertical axis of total station. The eccentricity of the reflector K_R is the result of the path geometry of the measuring signal through the reflector prisms. This difference occurs because the wave reflection plane is not on the same level as vertical axis of the reflector.

The additive constant represents the algebraic sum of the additive constant of the total station K_I and additive constant of the reflector K_R:

$$K_a = K_I + K_R. \quad (1)$$

where: K_I = –0.8 mm and K_R = 0. Thus:

$$K_a = -0.8 \text{ mm}.$$

Due to the additive constant, the distance measured is shorter or longer than the actual value. The value of the additive constant is entered into the total station and the distance is automatically reduced or it is taken into account when further computations are performed. The distance corrected for the additive constant S_{ra} amounts to:

$$S_{ra} = S_{met} + K_a \tag{2}$$

$$S_{ra} = 1281.0047 \text{ m}.$$

3.1. Measurement Errors in the Input Values of the Distance Corrected for the Additive Constant

The precision of the calculation of distance corrected for the additive constant is affected by:

- the standard deviation of the distance reduced by the meteorogical corrections $\sigma_{S_{met}}$,
- the standard deviation of the additive constant σ_{K_a}.

The impact of the distance reduced by the meteorogical correction S_{met} on the distance corrected for additive constant S_{ra} is obtained when the partial derivative upon the variable S_{met} is derived from Equation (2) and then multiplied by the standard deviation of the distance reduced by meteorogical correction $\sigma_{S_{met}}$.

Standard deviation of the distance reduced by meteorogical correction was computed previously and amounts to $\sigma_{S_{met}}$ = 4.6 mm.

$$I_{S_{met}} = \frac{\partial S_{ra}}{\partial S_{met}} \cdot \sigma_{S_{met}} = \sigma_{S_{met}} \tag{3}$$

$$I_{S_{met}} = 4.6 \text{ mm}$$

The impact of the error of the additive constant K_a on the distance corrected for the additive constant S_{ra} is obtained when the partial derivative upon the variable K_a is derived from Equation (2) and then multiplied by the standard deviation of the additive constant σ_{K_a}.

The standard deviation of the additive constant was determined by the manufacturer of the total station and reflector and amounts to σ_{K_a} = 0.2 mm.

$$I_{K_a} = \frac{\partial S_{ra}}{\partial K_a} \cdot \sigma_{K_a} = \sigma_{K_a} \tag{4}$$

$$I_{K_a} = 0.2 \text{ mm}$$

3.2. Standard Deviation of the Distance Corrected for Additive Constant

The standard deviation of the distance corrected for the additive constant is derived from the square root of the sum of squares of the impact:

- the distance reduced by meteorogical correction $I_{S_{met}}$ determined according to the equation (3),
- additive constant I_{K_a} determined according to the equation (4).

$$\sigma_{S_{ra}}^2 = I_{S_{met}}^2 + I_{K_a}^2 = \sigma_{S_{met}}^2 + \sigma_{K_a}^2 \tag{5}$$

$$\sigma_{S_{ra}} = 4.6 \text{ mm}$$

4. Determination of the Distance Correction Based on the Multiplication Constant

The multiplication constant q is the result of a change in the oscillation frequency of quartz within the total station, which is a cause for distances different from actual values being obtained (Dzierzega and Scherrer 2003). One of the methods to determine the multiplication constant is to measure a particular, conditionally taken as accurate, distance using the total station the multiplication constant of which is to be determined.

The distance corrected for the multiplication constant is calculated by using the equation:

$$S_r = q \cdot S_{ra} \tag{6}$$

$$S_r = 1281.0047 \text{ m.}$$

The multiplication constant for the total station used in the example amounts to $q = 1$.

4.1. Errors in the Measurement of the Input Values of Distance Corrected for the Multiplication Constant

The precision of the calculation of distance corrected for the multiplication constant is affected by:

- the standard deviation of the multiplication constant σ_q,
- the standard deviation of the distance corrected for the additive constant $\sigma_{S_{ra}}$.

The impact that the multiplication constant q has on the distance S_r is obtained when the partial derivative upon variable q is derived from Equation (6) and then multiplied by standard deviation σ_q.

The standard deviation of the multiplication constant amounts to $\sigma_q = 0.002$ mm/m.

$$I_q = \frac{\partial S_r}{\partial q} \cdot \sigma_q = \sigma_q \cdot S_{ra} \tag{7}$$

$$I_q = 2.6 \text{ mm}$$

The impact that the distance corrected for the additive constant S_{ra} has on the distance corrected for the multiplication constant S_r is obtained when the partial derivative upon the variable S_{ra} is derived from Equation (6) and then multiplied by the standard deviation of the distance corrected for the additive constant $\sigma_{S_{ra}}$.

The standard deviation of the distance corrected for the additive constant (5) is $\sigma_{S_{ra}} = 4.6$ mm.

$$I_{S_{ra}} = \frac{\partial S_r}{\partial S_{ra}} \cdot \sigma_{S_{ra}} = \sigma_{S_{ra}} \cdot q \tag{8}$$

$$I_{S_{ra}} = 4.6 \text{ mm}$$

4.2. Standard Deviation of the Distance Corrected for the Multiplication Constant

Standard deviation of the distance corrected for the multiplication constant is derived from the square root of the sum of squares of the impact:

- the multiplication constant I_q determined according to Equation (7),
- the distance corrected for the additive constant $I_{S_{ra}}$ determined according to Equation (8).

$$\sigma_{S_r}^2 = I_q^2 + I_{S_{ra}}^2 \tag{9}$$

$$\sigma_{S_r} = 5.3 \text{ mm}$$

5. Determination of the Distance between the Centres of Reference Points

It is necessary to convert the slope distance S_r, preliminary corrected for the meteorogical correction and the additive and multiplication constant, to the distance S_k between centres of stabilized endpoints of the distance measured (17). These calculations are to be performed using one of two following methods:

- based on the measured zenith distance,
- based on the known elevations of endpoints on a line segment.

The equation used to calculate the distance S_k also contains the radius of the Earth's curvature R at a reference point (station). First, the Earth's radius at the reference point Šoštanj has to be calculated, as well as the impact of the error of the radius on the precision of the distance measured.

6. Determination of the Earth's Radius at the Reference Point

The Earth's datum level is defined by the reference ellipsoid of revolution for the specific region. Based on the given ellipsoid, the mean radius of the Earth's curvature R is calculated. The magnitude of the radius at some particular point depends on the semi-major and semi-minor axis of the ellipsoid of revolution as well as on the ellipsoidal width of the reference point according to the equation:

$$R = \sqrt{\frac{a^4}{b^2\left[1+\frac{(a^2-b^2)\cos^2\varphi}{b^2}\right]^2}} \tag{10}$$

where:

a = 6378137.000 m – semi-major axis of the ellipsoid of revolution,

b = 6356752.314 m – semi-minor axis of the ellipsoid of revolution,

φ = 46°20' – ellipsoidal width of the reference point.

The semi-major and semi-minor axes refer to the Geodetic Reference System 1980 – GRS80 ellipsoid.

In this example, the Earth's radius at the reference point Šoštanj is:

$$R = 6379097.775 \text{ m}.$$

6.1. Errors of the Input Values Affecting the Radius R

The precision of calculation of the Earth's radius is affected by:

- the standard deviation of the ellipsoidal width at the chosen point σ_φ,
- the standard deviation of the semi-major axis of the ellipsoid of revolution σ_a,
- the standard deviation of the semi-minor axis of the ellipsoid of revolution σ_b.

The ellipsoidal width of Slovenia is one degree; hence, the same radius can be used throughout the country, considering the fact that its value does not affect significantly the calculation of distances. The impact of the ellipsoidal width φ on the radius R is obtained when the partial derivative upon the variable φ is derived from Equation (10) and then multiplied by the standard deviation value of the ellipsoidal width σ_φ.

The standard deviation value of the ellipsoidal width of the chosen point is σ_φ = 30'.

$$I_\varphi = \frac{\partial R}{\partial \varphi}\cdot\sigma_\varphi = \frac{\sigma_\varphi(a^2-b^2)\cdot\sin 2\varphi\sqrt{\frac{a^4b^2}{[b^2+(a^2-b^2)\cdot\cos^2\varphi]^2}}}{b^2+(a^2-b^2)\cdot\cos^2\varphi} \tag{11}$$

$$I_\varphi = 373.5686 \text{ m}$$

The impact that the semi-major axis of the ellipsoid of revolution a has on the radius R is obtained when the partial derivative upon the variable a is derived from Equation (10) and then multiplied by the standard deviation of the semi-major axis of the ellipsoid of revolution σ_a.

The standard deviation of the semi-major axis of the ellipsoid of revolution is $\sigma_a = 0.5$ mm.

$$I_a = \frac{\partial R}{\partial a} \cdot \sigma_a = \frac{\sigma_a \cdot 2a^3 \cdot b^4 \cdot \sin^2 \varphi}{[b^2 + (a^2 - b^2) \cdot \cos^2 \varphi]^3 \sqrt{\dfrac{a^4 b^2}{[b^2 + (a^2 - b^2) \cdot \cos^2 \varphi]^2}}} \tag{12}$$

$$I_a = 0.5 \text{ mm}$$

The impact that the semi-minor axis of the ellipsoid of revolution b has on the radius R is obtained when the partial derivative upon the variable b is derived from Equation (10) and then multiplied by the standard deviation of the semi-minor axis of the ellipsoid of revolution σ_b.

The standard deviation of the semi-minor axis of the ellipsoid of revolution is $\sigma_b = 0.5$ mm.

$$I_b = \frac{\partial R}{\partial b} \cdot \sigma_b = \frac{\sigma_b \cdot [b^2 + (a^2 - b^2) \cdot \cos^2 \varphi] \sqrt{\dfrac{a^4 b^2}{[b^2 + (a^2 - b^2) \cdot \cos^2 \varphi]^2}}}{-b\,[b^2 + (a^2 - b^2) \cdot \cos^2 \varphi]} \tag{13}$$

$$I_b = 2.2 \cdot 10^{-2} \text{ mm}$$

6.2. Standard Deviation of the Earth's Radius in Reference Point

The standard deviation of the Earth's radius R at the reference point is derived from the square root of the sum of squares of the impact:

- the ellipsoidal width I_φ determined according to Equation (11),
- the semi-major axis of the ellipsoid of revolution I_a determined according to Equation (12),
- the semi-minor axis of the ellipsoid of revolution I_b determined according to Equation (13).

$$\sigma_R^2 = I_\varphi^2 + I_a^2 + I_b^2 \tag{14}$$

$$\sigma_R = 373.5686 \text{ m}$$

7. Calculation of the Distance S_k Based on the Measured Zenith Distance

The calculation of the slope distance S_k is necessary because of the different heights of the total station and reflector, i.e., because of their different vertical distance from the ground. The measured distance corrected for the additive and multiplication constant must be converted into the slope distance existing between the very centres of the stabilized reference points. Therefore this type of distance may be called the stone-stone distance.

The slope distance S_p at the height the total station above sea level is given by:

$$S_p = S_r - (l - i) \cdot \cos z_r + \frac{(l - i) \cdot \sin z_r}{2S_r} \tag{15}$$

$$S_p = 1280.9432 \text{ m.}$$

It is necessary to convert the distance S_p at the altitude of total station into the reference point elevation:

$$S_k = S_p - \frac{i \cdot S_p}{R + H_i + i}. \tag{16}$$

By replacing Equation (15) with Equation (16) and after re-arrangement, the equation for the slope distance at the elevation of the reference point is obtained:

$$S_k = \frac{(R + H_i)[2S_r^2 + 2S_r(i - l)\cos z_r + (l - i)\sin z_r]}{2S_r(R + H_i + i)} \tag{17}$$

$$S_k = 1280.9432 \text{ m.}$$

7.1. Errors in the Input Values Affecting the Slope Distance S_k Determined by Means of Zenith Distance

The precision of the calculation of the slope distance S_k determined by means of zenith distance is affected by:

- the standard deviation of the distance corrected for the multiplication constant σ_{S_r},
- the standard deviation of the altitude of the total station σ_i,
- the standard deviation of the altitude of the reflector σ_l,
- the standard deviation of the zenith distance σ_{z_r},
- the standard deviation of the elevation of a station σ_{H_i},
- the standard deviation of the Earth's radius σ_R.

The impact that the distance corrected for the multiplying constant S_r has on the slope distance S_k calculated based the zenith distance is obtained when the partial derivative upon the variable S_r is derived from Equation (17) and then multiplied by the standard deviation of the distance corrected for the multiplication constant σ_{S_r}.

The standard deviation of the distance corrected for the multiplication constant (9) is $\sigma_{S_r} = 5.3$ mm.

$$I_{S_r} = \frac{\partial S_k}{\partial S_r} \cdot \sigma_{S_r} = \frac{\sigma_{S_r}(R + H_i)[2S_r^2 + (i - l)\sin z_r]}{2S_r^2(R + H_i + i)} \tag{18}$$

$$I_{S_r} = 5.3 \text{ mm}$$

The impact that the altitude of the total station i has on the slope distance S_k calculated on the basis of the zenith distance is obtained when the partial derivative upon the variable i is derived from Equation (17) and then multiplied by the standard deviation of the altitude of the total station σ_i.

The standard deviation of the altitude of the total station is the result of the error in the measurement of the altitude and it amounts to $\sigma_i = 1$mm.

$$I_i = \frac{\partial S_k}{\partial i} \cdot \sigma_i = \sigma_i \cdot \frac{(R+H_i)[(R+H_i+l)(2S_r \cos z_r - \sin z_r) - 2S_r^2]}{2S_r(R+H_i+i)^2} \tag{19}$$

$$I_i = 5.3 \cdot 10^{-2} \text{ mm}$$

The impact that the altitude of the reflector l has on the slope distance S_k calculated based on the zenith distance is obtained when the partial derivative upon the variable l is derived from Equation (17) and then multiplied by the standard deviation of the altitude of the reflector σ_l.

The standard deviation of the altitude of the reflector is the result of the error in the measurement of the altitude and amounts to $\sigma_l = 1$ mm.

$$I_l = \frac{\partial S_k}{\partial l} \cdot \sigma_l = \frac{\sigma_l (R+H_i)(\sin z_r - 2S_r \cos z_r)}{2S_r(R+H_i+i)} \tag{20}$$

$$I_l = 5.3 \cdot 10^{-2} \text{ mm}$$

The impact that the zenith distance z_r has on the slope distance S_k calculated based on the zenith distance is obtained when the partial derivative upon the variable z_r is derived from Equation (17) and then multiplied by the standard deviation of the zenith distance σ_{z_r}.

The standard deviation of the zenith distance is determined based on an a-posteriori evaluation of the precision of the measurements performed and it amounts to $\sigma_{z_r} = 5''$.

$$I_{z_r} = \frac{\partial S_k}{\partial z_r} \cdot \sigma_{z_r} = -\frac{\sigma_{z_r}(i-l)(R+H_i)(2S_r \cdot \sin z_r + \cos z_r)}{2S_r(R+H_i+i)} \tag{21}$$

$$I_{z_r} = 2.8 \cdot 10^{-2} \text{ mm}$$

The impact that the elevation of the station H_i has on the slope distance S_k calculated based on the zenith distance is obtained when the partial derivative upon the variable H_i is derived from Equation (17) and then multiplied by the standard deviation of the altitude of the station σ_{H_i}.

The standard deviation of the elevation of the station is the result of the levelling error and it amounts to $\sigma_{H_i} = 20$ mm.

$$I_{H_i} = \frac{\partial S_k}{\partial H_i} \cdot \sigma_{H_i} = \frac{\sigma_{H_i} \cdot i[2S_r^2 + 2S_r(i-l)\cos z_r + (l-i)\sin z_r]}{2S_r(R+H_i+i)^2} \tag{22}$$

$$I_{H_i} = 1.5 \cdot 10^{-10} \text{ mm}$$

The impact that the Earth's radius R has on the slope distance S_k calculated based on the zenith distance is obtained when the partial derivative upon the variable R is derived from Equation (17) and then multiplied by the standard deviation of the Earth's radius σ_R.

The standard deviation of the Earth's radius (14) is $\sigma_R = 373.5686$ m.

$$I_R = \frac{\partial S_k}{\partial R} \cdot \sigma_R = \frac{\sigma_R \cdot i[2S_r^2 + 2S_r(i-l)\cos z_r + (l-i)\sin z_r]}{2S_r(R+H_i+i)^2} \tag{23}$$

$$I_R = 2.8 \cdot 10^{-6} \text{ mm}$$

7.2. Standard Deviation of the Slope Distance S_k

The standard deviation of the slope distance S_k calculated based on the zenith distance (17) is derived from the square root of the sum of the squares of the impact of:

- the distance corrected for the multiplication constant I_{S_r} determined according to Equation (18),
- the altitude of the total station I_i determined according to Equation (19),
- the altitude of the reflector I_l determined according to Equation (20),
- the zenith distance I_{z_r} determined according to Equation (21),
- the elevation of the station I_{H_i} determined according to Equation (22),
- the impact of the Earth's radius I_R determined according to Equation (23).

$$\sigma_{S_k}^2 = I_{S_r}^2 + I_i^2 + I_l^2 + I_{H_i}^2 + I_{z_r}^2 + I_R^2 \tag{24}$$

$$\sigma_{S_k} = 5.3 \text{ mm}$$

7.3. Determination of the Point Elevation of the Reflector

The point elevation of the reflector is to be determined according to the following equation:

$$H_l = \sqrt{(R+H_l)^2} - R \tag{25}$$

whereby:

$$(R+H_l)^2 = (R+H_i)^2 + S_k^2 + 2S_k(R+H_i)\cos z_r, \tag{26}$$

hence, after the replacement, the following equation is obtained:

$$H_l = \sqrt{(R+H_i)^2 + S_k^2 + 2S_k(R+H_i)\cos z_r} - R. \tag{27}$$

The point elevation of the reflector H_l amounts to:

$$H_l = 519.715 \text{ m}.$$

7.4. Error in the Measurement of the Input Values on the Determination of the Point Elevation of the Reflector H_l

The precision of the determination of the point elevation of the reflector H_l is affected by:

- the standard deviation of the elevation of the station σ_{H_i},
- the standard deviation of the slope distance σ_{S_k},
- the standard deviation of the Earth's radius σ_R,
- the standard deviation of the zenith distance σ_{z_r}.

The impact that the elevation of the station H_i has on the point elevation of the reflector H_l is obtained when the partial derivative upon the variable H_i is derived from Equation (27) and then multiplied by the standard deviation of the altitude of the station σ_{H_i}.

The standard deviation of the elevation of the station is the result of the levelling error and amounts to $\sigma_{H_i} = 20$ mm.

$$I_{H_i} = \frac{\partial H_l}{\partial H_i} \cdot \sigma_{H_i} = \frac{\sigma_{H_i}(R + H_i + S_k \cdot \cos z_r)}{\sqrt{(R+H_i)^2 + S_k^2 + 2S_k(R+H_i) \cdot \cos z_r}} \tag{28}$$

$$I_{H_i} = 20.0 \text{ mm}$$

The impact that the slope distance S_k has on the point elevation of the reflector H_l is obtained when the partial derivative upon the variable S_k is derived from Equation (27) and them multiplied by the standard deviation of the slope distance σ_{S_k}.

The standard deviation of the slope distance (24) is $\sigma_{S_k} = 5.3$ mm.

$$I_{S_k} = \frac{\partial H_l}{\partial S_k} \cdot \sigma_{S_k} = \frac{\sigma_{S_k}[S_k + (R+H_i) \cdot \cos z_r]}{\sqrt{(R+H_i)^2 + S_k^2 + 2S_k(R+H_i) \cdot \cos z_r}} \tag{29}$$

$$I_{S_k} = 0.3 \text{ mm}$$

The impact that the Earth's radius R has on the point elevation of the reflector H_l is obtained when the partial derivative upon the variable R is derived from Equation (27) and then multiplied by the standard deviation of the Earth's radius σ_R.

The standard deviation of the Earth's radius (14) is $\sigma_R = 373.5686$ m.

$$I_R = \frac{\partial H_l}{\partial R} \cdot \sigma_R = \sigma_R \left[\frac{R + H_i + S_k \cdot \cos z_r}{\sqrt{(R + H_i)^2 + S_k^2 + 2S_k \cdot (R + H_i) \cdot \cos z_r}} - 1 \right] \tag{30}$$

$$I_R = 7.5 \cdot 10^{-3} \text{ mm}$$

The impact that the zenith distance z_r has on the point elevation of the reflector H_l is obtained when the partial derivative upon the variable z_r is derived from Equation (27) and then multiplied by the standard deviation of the zenith distance σ_{z_r}.

The standard deviation of the zenith distance is determined based on the a-posteriori evaluation of the precision of the measurements performed and amounts to $\sigma_{z_r} = 5''$.

$$I_{z_r} = \frac{\partial H_l}{\partial z_r} \cdot \sigma_{z_r} = -\frac{\sigma_{z_r} (R + H_i) \cdot S_k \cdot \sin z_r}{\sqrt{(R + H_i)^2 + S_k^2 + 2S_k (R + H_i) \cdot \cos z_r}} \tag{31}$$

$$I_{z_r} = 31.0 \text{ mm}$$

7.5. Standard Deviation of the Point Elevation of the Reflector

The standard deviation of the point elevation of the reflector H_l determined by Equation (27) is derived from the square root of the sum of the squares of the impact:

- of the elevation of the station I_{H_i} determined according to Equation (28),
- of the slope distance I_{S_k} determined according to Equation (29),
- of the Earth's radius I_R determined according to Equation (30),
- of the zenith distance I_{z_r} determined according to Equation (31).

$$\sigma_{H_l}^2 = I_{H_i}^2 + I_{S_k}^2 + I_R^2 + I_{z_r}^2 \tag{32}$$

$$\sigma_{H_l} = 36.9 \text{ mm}$$

8. Calculation of the Distance S_k Based on the Known Elevation Between the Endpoints on the Line Segment

The reduction is performed based on the known elevation between the endpoints on the line segment to be measured. The elevation between points is obtained by means of geometric or trigonometric levelling. The method of trigonometric levelling is applied on steep and overgrown terrains, because in these cases, it is simpler to obtain differences in altitude using this method than by using the geometric levelling method, which is applied on an even and smooth terrain.

The slope distance is to be calculated because the total station and the reflector are positioned at different altitudes, respectively at the different vertical distances from the ground. It is therefore necessary to convert the measured distance into the slope distance between the centres of the stabilized reference points. Such a distance is, therefore, called the stone-stone distance.

This method of calculation implies that the elevation between the endpoints is known, however, it is necessary to calculate the difference in altitude between the total station and the reflector according to the equation:

$$\Delta S = \frac{(i-l)(H_l - H_i)}{S_r} - \frac{(i-l)^2}{2S_r} - \frac{(i+l)}{2R} \cdot S_r \tag{33}$$

$$\Delta S = -0.0627 \text{ m.}$$

The slope distance is to be calculated according to the equation:

$$S_k = S_r + \Delta S \tag{34}$$

$$S_k = 1280.9420 \text{ m.}$$

8.1. Errors of the Input Values Affecting the Slope Distance S_k Determined based on the Known Elevation Between the Endpoints

The precision of the slope distance S_k determined based on the known elevation between the endpoints is affected by:

- the standard deviation of the distance corrected for the multiplication constant σ_{S_r},
- the standard deviation of the altitude of the total station σ_i,
- the standard deviation of the altitude of the reflector σ_l,
- the standard deviation of the elevation of the station σ_{H_i},
- the standard deviation of the point elevation of the reflector σ_{H_l},
- the standard deviation of the Earth's radius σ_R.

The impact that the distance corrected for the additive constant S_r has on the slope distance S_k calculated based on the known elevation between the endpoints is obtained when the partial derivative upon the variable S_r is derived from Equation (34) and then multiplied by the standard deviation of the distance corrected for the multiplication constant σ_{S_r}.

The standard deviation of the distance corrected for the multiplication constant (9) is $\sigma_{S_r} = 5.3$ mm.

$$I_{S_r} = \frac{\partial S_k}{\partial S_r} \cdot \sigma_{S_r} = \frac{\sigma_{S_r}[(i-l)(2H_i - 2H_l + i - l) \cdot R + S_r^2(2R - i - l)]}{2S_r^2 R} \tag{35}$$

$$I_{S_r} = 5.3 \text{ mm}$$

The impact that the altitude of the total station i has on the slope distance S_k calculated based on the known elevation between the endpoints is obtained when the partial derivative upon the variable i is derived from Equation (34) and then multiplied by the standard deviation of the altitude of the total station σ_i.

The standard deviation of the altitude of the total station is the result of the error in the measurement of the altitude and amounts to $\sigma_i = 1$ mm.

$$I_i = \frac{\partial S_k}{\partial i} \cdot \sigma_i = -\frac{\sigma_i [S_r^2 + 2R(H_i - H_l + i - l)]}{2S_r R} \tag{36}$$

$$I_i = 5.5 \cdot 10^{-2} \text{ mm}$$

The impact that the altitude of the reflector l has on the slope distance S_k calculated based on the known elevation between the endpoints is obtained when the partial derivative upon the variable l is derived from Equation (34) and then multiplied by the standard deviation of the altitude of the reflector σ_l.

The standard deviation of the altitude of the reflector is the result of the error in the measurement of the altitude and amounts to $\sigma_l = 1$ mm.

$$I_l = \frac{\partial S_k}{\partial l} \cdot \sigma_l = \frac{\sigma_l [2R(H_i - H_l + i - l) - S_r^2]}{2S_r R} \tag{37}$$

$$I_l = 5.5 \cdot 10^{-2} \text{ mm}$$

The impact that the elevation of the station H_i has on the slope distance S_k calculated based on the known elevation between the endpoints is obtained when the partial derivative upon the variable H_i is derived from Equation (34) and then multiplied by the standard deviation of the elevation of the station σ_{H_i}.

The standard deviation of the elevation of the station is the result of the levelling error and amounts to $\sigma_{H_i} = 20$ mm.

$$I_{H_i} = \frac{\partial S_k}{\partial H_i} \cdot \sigma_{H_i} = \frac{\sigma_{H_i}(l - i)}{S_r} \tag{38}$$

$$I_{H_i} = 1.8 \cdot 10^{-2} \text{ mm}$$

The impact that the point elevation of the reflector H_l has on the slope distance S_k calculated based on the known elevation between the endpoints is obtained when the partial derivative upon the variable H_l is derived from Equation (34) and then multiplied by the standard deviation of the point elevation of the reflector σ_{H_l}.

The standard deviation of the point elevation of the reflector (32) is $\sigma_{H_l} = 36.9$ mm.

$$I_{H_l} = \frac{\partial S_k}{\partial H_l} \cdot \sigma_{H_l} = \frac{\sigma_{H_l}(i - l)}{S_r} \tag{39}$$

$$I_{H_l} = 3.3 \cdot 10^{-2} \text{ mm}$$

The impact that the Earth's radius R at the reference point has on the slope distance S_k calculated based on the known elevation between the endpoints is obtained when the partial derivative upon the variable R is derived from Equation (34) and then multiplied by the standard deviation of the Earth's radius σ_R.

The standard deviation of the Earth's radius (14) is $\sigma_R = 373.5686$ m.

$$I_R = \frac{\partial S_k}{\partial R} \cdot \sigma_R = \frac{\sigma_R(i+l) \cdot S_r}{2R^2} \tag{40}$$

$$I_R = 9.5 \cdot 10^{-6} \text{ mm}$$

8.2. Standard Deviation of the Slope Distance Determined Based on the Known Elevation between the Endpoints

The standard deviation of the slope distance S_k determined based on the known elevation between the endpoints (34) is derived from the square root of the sum of the squares of the impact:

- of the distance corrected for the additive constant I_{S_r} determined according to Equation (35),
- of the altitude of the total station I_i determined according to Equation (36),
- of the altitude of the reflector I_l determined according to Equation (37),
- of the elevation of the station I_{H_i} determined according to Equation (38),
- of the point elevation of the reflector I_{H_l} determined according to Equation (39),
- of the Earth's radius I_R determined according to Equation (40).

$$\sigma_{S_k}^2 = I_{S_r}^2 + I_i^2 + I_l^2 + I_{H_i}^2 + I_{H_l}^2 + I_R^2 \tag{41}$$

$$\sigma_{S_k} = 5.3 \text{ mm}$$

9. Conclusions

This paper shows, both theoretically and on a practical example, the impact of accidental errors of parameters while determining geometric corrections of distances measured.

The measured distance used in the example had, after reductions for the meteorological corrections, a standard deviation of 4.6 mm. After the geometric corrections were entered, its standard deviation amounted to 5.3 mm. This means that the standard deviation from the example was increased by 15.2% due to the input of the geometric corrections.

This increase in the standard deviation is the result of the impact of the error of the multiplication constant $I_q = 2.6$ mm. This means that while entering the geometric corrections, full attention must be paid to the multiplication constant and its determination. The other influences are significantly lower (in the order of the one-hundredth of millimetre), while some influences, such as the impact of the error of the elevation of the station and the impact of the error in determining the Earth's radius, are so small that practically it is not necessary to take them into consideration while processing the distances measured during daily engineering work.

The determination of geometric corrections for the measured distances could be significant when performing a critical engineering task in which highly accurate measured distances are desired. This primarily refers to deformation monitoring in highly complicated terrain sites where accessibility is impossible. The equations shown enable simple implementation within appropriate software and the successful determination of the standard deviation of a measured distance, for which the geometric corrections had been determined, and this paper could serve as a general guideline for surveying professionals.

The equations, however, also enable the inverse procedure, or to be precise, they enable the a priori definition of the precision required for the measurement of auxiliary parameters in order to keep the total standard deviation of distances within permissible limits. Knowledge of the impacts enables their practical implementation during the optimization of distance measurement for achievement of the specified precision. The detection of the impacts that significantly affect the precision of the distance corrected for geometric sources enables the required precision of the measurement of auxiliary parameters to be defined, which bring such corrections to light.

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Analiza utjecaja pogrešaka pomoćnih veličina pri određivanju geometrijskih popravaka duljina

SAŽETAK. Rezultati mjerenja duljina opterećeni su sustavnim i slučajnim pogreškama. Utjecaj sustavnih pogrešaka smanjuje se ili eliminira najčešće unošenjem odgovarajućih popravaka ili metodom mjerenja. Veliki broj različitih izvora popravaka može se podijeliti u tri skupine, i to: meteorološke, geometrijske i projekcijske popravke. U ovom radu obrađene su samo geometrijske popravke. Određivanje geometrijskih popravka i eliminiranje njihovih utjecaja iz rezultata mjerenja zahtijeva poznavanje ili mjerenje različitih pomoćnih veličina. Pomoćne veličine su, također, opterećene slučajnim pogreškama koje prema zakonu o rasprostiranju pogrešaka utječu i na točnost duljine popravljene za geometrijske popravke. Ovo su opće poznate činjenice, ali iz autorima nepoznatih razloga, problematika izložena u ovom radu nije do sada tretirana u stručnoj literaturi. U radu je prikazana analiza utjecaja pomoćnih veličina na točnost mjerene duljine koja je popravljena za geometrijske popravke. Poznavanje točnosti ovako popravljene duljine je važno, jer su navedeni utjecaji svakodnevno prisutni pri realizaciji geodetskih zadataka iz područja preciznih geodetskih mjerenja, kao i pri umjeravanju elektroničkih instrumenata za mjerenje duljina. Također, prikazan je i detaljan primjer računanja utjecaja geometrijskih izvora i njihovih standardnih odstupanja na točnost elektronički izmjerene duljine. Stoga, rad može poslužiti i kao detaljne, opće upute za svakodnevnu profesionalnu primjenu prilikom preciznih mjerenja duljina.

Ključne riječi: *elektroničko mjerenje duljina, geometrijske popravke, standardno odstupanje.*

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