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Stručni članak

The Application of Intelligent Techniques for Massreal Estate Appraisal

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ABSTRACT. The paper reviews the concept of mass appraisal of real estate, within which, besides defining the basic concepts, a comparative analysis is carried out of different international experiences related to this issue. The normative and institutional order of the subject area is analyzed in a test area, while the concept of evaluating spatial units as a base of massreal estate appraisal and the field of their use are also defined. The essence of a valuation model of spatial units is defined, based on the principle of case based reasoning (CBR) and logical aggregation (LA), and the mathematical basis for the proposed model is given for anticipating the average price of real estate within spatial units. In the proposed model, spatial units are described, that is, the method of their normalization and their granulation into groups. Individual attributes and groups are allocated appropriate weight by which their individual and group significance are defined within the framework of the integral model, and finally, testing of the model was carried out in a test area.

Keywords: evaluation of spatial units, mass appraisal of real estate value, case-based reasoning, logical aggregation.

1. Introduction

One of the fundamental reasons for establishing cadastral records, and thus the generator of geodesy, is the objective and fair administration of tax policy. From its conception to the present, the national spatial data infrastructure is one of the most important factors in the implementation of this policy. Adjusting to the real needs of their users, without deviating from their basic principles, cadastral records, particularly in developed countries, are constantly evolving. Valuing real estate is one of its attributes which has been expanding more and more in recent years.

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Real estate in the development of human civilization also has particular significance and can be considered as multi-spectral. Ownership of land or buildings, or their use, has a direct existential connection with their owner. That connection is often materialized as a relationship with the historical past or family inheritance, existential security, or position.

Alone its value is, in broad terms, the monetary value which the real estate can make on the market at the moment of its sale, and on the other hand it is the total resulting value of all of the most significant attributes that affect its value.

Astronomical leaps and unexpected collapse of the real estate market are just a few of the factors which support the thesis that real estate value is an integral function of the value of its attributes, some of which often remain unknown even to the greatest experts.

One of the preconditions to successfully modelling the process of mass appraisal of real estate value is defining the field of its application, and selecting the criteria and methodology for the appraisal are directly related to the size of its domain. Namely, the greater the population of buildings for the mass appraisal, as a result of the expansion of its field of application, the more complicated and complex the number of criteria, the methodology and the modelling become. Hence, the integral model must be broken down into its own subsystems, in order for it on one hand to be universally applicable, and on the other hand, to a model which not only recognizes the relevant attributes of each property being valued, but also assigns characteristics to the area in which that property is located.

The universal model of mass appraisal of real estate value in a particular country must in essence be functional, practically applicable, consistent and adaptable to the real conditions and trends in the real estate market. It must also recognize all relevant factors which influence the price of real estate in each spatial unit, and at the same time preserve all of the essential features of that area and use them in the process of determining the average price of real estate within it. Thus, one important task in modelling the mass appraisal of real estate is assessing the influence of an area on the real estate within it, as well as describing the given area with a sufficient number of attributes in order to determine the relation between the values of these attributes and average property prices in each spatial unit.

By describing the middle set of attributes, and using the known data on the average prices of property within it, a base of known cases can be formed–knowledge that will serve as a standard for pricing properties in any area, which can also be described by means of an identical set of attributes. Using logical aggregation or aggregate measures of similarity, that is by determining the measure of similarity between the assessed spatial units and representatives from the list of known cases, the most likely value of the average price of real estate can be anticipated. Namely, we can consider an aggregate measure of similarity as a point in n-dimensional real vector space, where the distance of the rectangular projection point from the point of origin for each axis is equal to the value of a partial measure of similarity for each attribute. On the basis of the distance from the point of origin, the overall measure of similarity to the list of known cases for the spatial unit concerned can be determined, and on the basis of the measure of similarity, the average price of property within it can be anticipated.

Based on analysis of the real estate market, on the national spatial data infrastructure and on relevant data from other sources, it is possible on one hand to assess the value of some spatial units for all of the defined attributes, and on the other hand on the basis of the market to define the average price of property in it. On the foundation of this data a database of knowledge can be formed, or a base of known cases which can still be used for anticipating the price of property in other spatial units. Namely, by using interpolative Boolean algebra and logical aggregation as aggregation operators for measuring similarity, one or more similar spatial units are found on the basis of the input data values of all attributes, based on predefined criteria, the average prices of which are used as a basis for anticipating the most probable average price of property in a given spatial unit.

Valuation of property usually takes place in developed market economies, and Serbia, realistically, is only at the beginning of this. Methodological inconsistency, institutional conflict and uncoordinated jurisdiction are only some of the characteristics which inevitably imply the previous statement for the test area.

The field of application of the assessment results is a multidimensional space using this information mostly for economic purposes. Providing funding, investment decision making, statistical or financial reporting, making business decisions, legal practice, business insurance and implementation of fiscal policy are just some of the areas for which estimating real estate value provides data, on the basis of which important and often difficult decisions are made. The concept of market value is as difficult to unambiguously define a show well the procedure for estimating suitable real estate is performed.

2. Models of Mass Appraisal of Real Estate Value

As with most estimates that a man makes each day, so it is with estimating the value of real estate. Behind the overall rating which is defined either numerically or semantically (linguistically), in fact, hides, a multi-criterial analysis and the optimization of different criteria. Although the individual is not usually aware of them, and although they would be hard to define, the criteria taken into account on that occasion are not only numerous, but in this thinking process their ranking is also carried out. However, an attempt to carry out an objective estimate in one thinking process on the basis of different criteria would more often than not be futile without some of the known methods of operational research – soft computing – that is, artificial intelligence.

The number of criteria on the basis of which to describe or estimate a property is very large, and together they are intended to represent their constructional, economic, social, ecological and other aspects. We are contemporaries of the fact that not only for the time function but also for the location, the evaluation criteria are subject to certain modifications, that is, adapting the basic requirements of the average buyer on the property market.

It is not surprising that buildings which are identical in terms of their technical construction criteria have different market values in different locations. There are also places or even regions which, because of their basic economic, demographic, sociological and other characteristics have identical real estate values for similar properties. Looking only through the prism of estimation, and frequency of operation, supply and demand, and therefore the price of real estate show that there are other relevant criteria that significantly influence the value, which will be discussed in the following consideration.

It is also important to emphasize the fact that certain features that affect property value in one place may be irrelevant in another place. For example, if a populated area is near a highway, by means of which it has a good connection with a major national or international road corridor, the value of land in its vicinity directly relates to its distance from that highway, a potential factor which can justify investment in it economically. However, if there is no such traffic route in the vicinity of a populated area, then it is an irrelevant criterion for all property in that area. A similar example can be given for the water, rail, sport recreation, spa, air and educational facilities which characterize or do not characterize a place.

2.1. Cost method

One of the basic principles of this methodological approach is to determine the objective price, that is the cost of building the property, that is replacement theory. The logical foundation of this approach is reflected in the statement that the uninformed buyer in the open real estate market could pay a higher price for a property than the actual cost of its construction. It can be seen immediately that in this methodological approach the time factor is neglected, as well as many other components, some of which can and others which cannot be defined quantatively, that is in terms of currency units, and which would definitely be used up during its construction, and fictively miss the profit from the eventual use of the existing building.

Bearing in mind its basic methodological approach, the cost method can be considered as effective when estimating the value of newly constructed buildings or buildings which are not affected by the time factor, or for buildings whose exploitation period does not have to be considered as a component that affects its depreciation. This method could also be considered as suitable when estimating property for which it is difficult to assess the benefit gained from its being rented out, or for cases in which the assessor does not have all the information about the cost of similar or identical properties in the locality, that is, when it is impossible from the base of known cases to find any kind of analogy with the property in question, and on the basis of that draw a conclusion about its value.

The cost method is used for estimating industrial, agricultural and other real estate, as well as being the dominant and only method for state administration procedures and court proceedings (Miladinović 2009).

2.2. Mass appraisal of real estate value

Consistently following the basic idea of establishing cadastral records and the corresponding requirements of modern society where real estate is a very important aspect of everyday life, the modern cadastral records of developed countries across the whole world are introducing models of mass appraisal (mass estimation) of real estate within the framework of their everyday activities (Miladinović 2009).

Modern technical and technological developments, the development of information technology in the administration of large amounts of data in real time, modern methods of acquiring large amounts of good quality and relevant geospatial data, operational research methods, artificial intelligence and multicriteria optimization have created the necessary preconditions for the mass appraisal of real estate value.

Georeferencing a large amount of heterogeneous spatial data, its statistical quality control, as well as relational searches within geoinformational systems have only further accelerated and facilitated the application of mass appraisal of real estate, made opportunities in the functioning of tax administration and facilitated its connection with other public bodies.

The intention of implementing of tax policy in modern society is that taxation on the basis of property ownership is based on determining the actual, objective market value in the shortest possible time period. One of the requirements is also for the best possible quality control methodology on the basis of statistical evaluations, as well as the need for a flexible and universal system, practically applicable for the implementation of fiscal policy.

Mass appraisal finds its role primarily in forming initial or reference values, while the methodology of individual estimates is used to survey the peculiarities and characteristics of a particular property in order to define its real value, which serves as the tax base. For quality and objective implementation of fiscal policy, in addition to the qualitative methods of mass that is individual estimation, an indispensible component is the administrative skill necessary for managing the human and physical resources of the tax department and the quality assurance at every level of the mass appraisal process.

One of the fundamental differences between mass and individual appraisal is that the mass model has to examine the wider context and defines the characteristics which influence the formation of real estate values over a larger area. Thus, appraisal refers to group rather than to individual properties, and the task of the assessor is to identify and recognize the common features of a large number of properties, that is, he must be capable of developing, supporting and explaining standardized adjustments to the model of appraisal, as well as using class, type of building, environmental and other groups of real estate.

The next important difference between these two methods is that the quality control of the appraisal results is carried out in different ways. Namely, individual estimates relate to a specific property which an individual or a small group of individuals are interested in, the result of which can be compared with the research or analysis for similar, that is, comparable sales of similar property.

For mass appraisal, quality testing is carried out using statistical methods. The subject of mass appraisal is a larger amount of real estate, and therefore the number of interested clients is larger – tax payers, who must on one hand be satisfied or at least unharmed, and on the other hand the model must be consistent and universally applicable throughout the whole area in which the tax policy is implemented under the auspices of an identical legal framework.

Unlike individual appraisal, it is almost impossible in this day and age to consider the methodology of mass appraisal without computer support. It not only reviews relevant criteria more objectively in a wider context, but is also fast and efficient in its application and has effective control over its implementation as well as being reliable in the results of the model applied.

The mass appraisal system, whether computerized or manual, consists of four subsystems:

1. Data management system
2. Sales analysis system
3. Appraisal system
4. Administrative system.

The four subsystems are interdependent. The appraisal system, for example, uses information held in the sales analysis system, and the data management system produces output documents necessary to the administrative system for printing tax bills.

In Serbia, the Law on State Survey and Cadastre envisages the introduction of mass appraisal of real estate, which should be an extremely significant event in this area and one which will give the initiative to regulate the other aspects of appraisal according to the regulations. Training personnel and establishing an association of appraisers and other institutions are tasks that lie ahead on the road to systematic and institutional regulation of this very important area.

3. Evaluation of Spatial Units based on CBR and LA

One of the goals of the scientific research in this paper is the recognition of the relevant features, that is, criteria of spatial units which affect the formation of the average price of real estate within them. The intensity or value of each of them speaks of the value of that locality from the aspect of the criterion in question, and the integration of all the individual values gives the total value.

A very important problem in determining the level of similarity is the aggregation (fusion) of a number of attributes into one globally representative aspect – the measure of similarity. In existing practice, the most commonly used method for summing up the weight coefficients of partial aspects is the aggregation technique. This approach is additive, and for all cases which are not additive, it is inadequate. For example, using a weighted sum for aggregation, even in cases with only 2 attributes (a, b), does not allow realization of the natural need for a requirement in which both attributes are important together. Among those who study the multi-attribute decision making method, this problem is known and as a solution, the theory of capacitance, known in the "phase-community" as *phase measures* and *phase integrals* (Mirković et al. 2006).

In this kind of approach, additivity is a "relaxed" monotonicity for which additivity is a special case. As a result, the range of possible applications for this approach is considerably broader, however from a logical point of view, monotonicity still remains a strongly limiting factor, since many logical functions by nature are not monotone. The generalized discrete Choquet integral is defined for a general measure – non monotonicity in general. This approach includes all logical and pseudo-logical functions, but allows the use of only one arithmetic operator for interpolation purposes – the *min* function. Interpolative realization of Boolean algebra (IBA) includes all logical functions and all interpolative operators (operators of a generalized product) (Mirković et al. 2006).

The proposed model, therefore, on the basis of measures of similarity between cases (spatial units) for which the average values of real estate are determined and representatives from the base of known cases anticipate the most likely average price. Measures of similarity are a type of evaluation function which transfers (maps) an abstract concept into a numerical value i.e. joins values together in a series of pairs, with the idea that a higher value indicates greater similarity, and that value is an aggregated, composite size.

Similarity, as a complex quantity, can be expressed by means of unique values which are reached by the aggregation of partial measures of similarity (for each of the selected attributes-criteria). The possibilities of the logical aggregation model extend the framework of the existing model. In most well-known models, only trivial attributes are used, without interaction between them (without the use of logical functions). Also, for the aggregation operator, most commonly just a normal product is used.

The logical aggregation model is generalized to allow the use of logical functions between attributes (from a finite set of possible functions for a finite number of attributes), as well as the use of different aggregation operators and different generalized products–subclasses of T-norms with the full axiom of non-negativity (Radojević 2006).

In addition, a measure of aggregation does not have to be either additive or monotone. Determining partial measures of similarity is a trivial task for the evaluator, since there are references for comparisons performed in one-dimensional space for the values of each individual attribute, so expert knowledge is not necessary in the way that it is for absolute (primary) evaluation.

The implementation of logical aggregation is possible within the concept of CBR in the phase of aggregating the data–determining the measure of similarity.

3.1. Model based on the Concepts of CBR and LA

Matrix $\boldsymbol{O}$ represents the knowledge base which consists of the spatial units – prototypes for which the average property prices in them are known. Thus, the cases consist of a set of attributes whose values are information carriers about the problem and corresponding average prices, and is the information carrier regarding the solution to the problem. Each prototype O_j represents a *case* – a vector which consists of normalized attribute values $a_i^j \in [0\ \ 1]$, and the average price of real estate c^j. The case index is j. The attributes are defined as:

$$\exists O_j \forall a_i^j,\ \ i \in (1, \dots k) \tag{1}$$

For determining the average price in spatial unit O_x it is necessary to normalize the attribute values in the same way as was done in the knowledge base. Then the measure of similarity μ^j is determined with prototypes from the knowledge base.

The measure of similarity is an aggregate size. It is a result of the aggregation of individual (partial) similarity measures for each attribute separately.

A partial measure of similarity is actually a measure of logical equivalence. An appropriate aggregation for the values of similarity measures is:

$$\mu_i^j = (a_i^j \Leftrightarrow a_i^x)^{\otimes} = [(a_i^j \cap a_i^x) \cup (Ca_i^j \cap Ca_i^x)]^{\otimes} = 1 - a_i^j - a_i^x + 2a_i^j \otimes a_i^x \quad \otimes := \min \quad (2)$$

Since this is aggregation of the same attribute (high positive correlation), for a generalized product, we use the *min* function. The aggregation operator carries out mapping:

$$Aggr: [0,\ 1]^2 \rightarrow [0,\ 1] \quad (3)$$

The total similarity measure μ^j is gained by aggregating partial (per attribute) measures of similarity:

$$\mu^j = (\cap_{i=1}^k \mu_i^j)^{\otimes} = \prod_{i=1}^k \mu_i^j, \quad \otimes := + \quad (4)$$

Since this is aggregation of different attributes (negligible correlation), for a generalized product we use an ordinary sum. The aggregation operator carries out mapping:

$$Aggr: [0,\ 1]^i \rightarrow [0,\ 1] \quad (5)$$

There is a possibility of allocating weight coefficients w_i to partial similarity measures μ_i^j. A relevant total measure of similarity is then determined by the expression:

$$\mu^j = \sum_{i=1}^k w_i \mu_i^j; \qquad \sum_{i=1}^k w_i = 1, \qquad w_i \geq 0, \qquad i = 1, \ldots k \quad (6)$$

There remains an open possibility of aggregating a hybrid-mixed type as required using interpolative pseudo-Boolean polynomials. The most similar prototype $O_j \in \boldsymbol{O}$ with O_x is the one with the highest value of similarity measure μ^j:

$$\exists O_x \forall c^j \in O_j,\ O^j \in \boldsymbol{O} \text{ where } j \text{ is chosen for } \mu^j = \max \quad (7)$$

In this way, the value of the average property price is taken for a spatial unit (case) from the knowledge base which is most similar to the input case, for which we want to find the average price. Then these price values are assigned to the case, the average price of which is required.

3.2. The mathematical basis for the suggested model of evaluating spatial units

The mathematical model suggested on the basis of scientific research within the framework of this doctoral dissertation is based on the principle of case-based reasoning in which a combination of logic and the Euclidean (L2) norm are used.

Namely, anticipation of the average price values of real estate within the framework of a spatial unit is realized on the basis of aggregate measures of similarity between the spatial unit for which it is carried out and representatives from the base of known cases – knowledge. Aggregate similarity measures are the integration of individual – of partial measures of similarity for each of the attributes that describe all of the cases, and in this case a partial similarity measure is a measure of logical equivalence. As indicated above, an appropriate aggregation operator for the value of similarity measures is:

$$\mu_i^j = (a_i^j \Leftrightarrow a_i^x)^{\otimes} = [(a_i^j \cap a_i^x) \cup (Ca_i^j \cap Ca_i^x)]^{\otimes} = 1 - a_i^j - a_i^x + 2a_i^j \otimes a_i^x;\ \ \otimes := \min \quad (8)$$

The total – the aggregated measure of similarity can be represented as point (O^j) in n-dimensional real vector space, where the distance of the rectangular projection point from the point of origin for each axis is equal to the value of partial similarity measures for each of the attributes.

Using the Euclidean norm as an aggregation operator for measures of similarity actually calculates the measure of distance of point (O^j) from the point of origin as follows:

$$r^j = \left[\sum_{i=1}^k \mu_i^{j2}\right]^{\frac{1}{2}} \quad \text{where } k \text{ is the number of attributes} \quad (9)$$

In the proposed model there also remains the possibility of allocating weight coefficients to each of the squares of partial measures of similarity μ_i^j. Then the distance is defined with:

$$r^j = \left[\sum_{i=1}^k \mu_i^{j^2} w_i\right]^{\frac{1}{2}};\quad \text{where:}\quad \sum_{i=1}^{k} w_i = 1,\quad w_i \geq 0,\quad i = 1,\ldots k \quad (10)$$

Then the value of the total similarity measure is equal to the value of the logical negation of the distance.

$$\mu^j = (\neg r^j)^{\otimes} = 1 - r^j \quad (11)$$

The most similar prototype $O_j \in \boldsymbol{O}$ with O_x is the one with the greatest value of similarity measure μ^j, while the greatest value of similarity measure takes the case where the distance from the points r_i^j is the least.

Under the proposed model for every new case (spatial unit) which is valued, from the knowledge base an already known case is found to which it is most similar. As described above, the similarity measures will be normed values from the interval [0.1]. However, the question is which measure of similarity is a border case for which it can be said that a known case from the knowledge base can be used to anticipate the average price values of real estate for the case being assessed. Within the framework of the proposed model, in this doctoral dissertation, a minimal value of similarity measures of 0.8 is adopted. According to this, the principle is kept by which, on the basis of total measures of similarity, it is not necessary to find the most similar case, but rather in determining the average price value of real estate within the framework of evaluating spatial units, all known cases are used which, when evaluated, have a total similarity measure greater or equal to the given border values.

The estimated average value of real estate prices within the framework of the assessed spatial units is not only a simple arithmetic mean of the average prices for known cases from the knowledge base whose measure of similarity with assessment are greater than or equal to the border values, but rather, it is proportional to it. The anticipated value is achieved using the following formula:

$$PRICE = \sum_{i=1}^{n} price_i \frac{(\mu_i - 0.8)}{\omega}; \quad \text{where} \quad \omega = \sum_{i=1}^{n} (\mu_i - 0.8) \tag{12}$$

n – the number of cases from the knowledge base whose total measure of similarity to the case under assessment is greater than or equal to the border value.

4. Quantitative Definition of Parameters

In any procedure of multi-criteria evaluation, it is necessary, above all, to define the criteria on the basis of which the procedure will be carried out. Each of the criteria can have appropriate values whether they are expressed numerically or semantically. The value of each criterion represents a score, in this case, each spatial unit from the aspect of the criteria used to evaluate it.

In order for the process of evaluation, that is, for the assessment of the value of each individual criterion to be implemented, it is necessary for each of them to be described, that is, to define the method for their evaluation. The criteria used in this scientific research are selected on the basis of two basic postulates: to better describe the spatial units being assessed from the viewpoint of the real estate value within them, and for the data used to evaluate the attributes to be available from a reliable source so that when applied to future models, the same source can be used for the actualization and modification of the suggested model.

In creating a model of mass appraisal, the criteria for evaluating spatial units, which, according to their nature, are classified into four basic groups, which are: natural, social and economic characteristics, or criteria which can be considered as a corrective factor in mass appraisal.

The basic concept of establishing this model is that on the basis of the values of all criteria for each spatial unit, as a result of the estimation, a characteristic number normalized from 0 to 1 is achieved, which is basically a value criterion, that is, a ranked spatial unit which should correlate with the average value of the real estate within it.

Each group of characteristics in itself has more individual criteria or sub-criteria, the value of which is also normalized within the framework of each group in intervals from 0 to 1. In other words, the evaluation is decomposed into several layers, and for each of these layers, normalization is carried out using a “bottom up” principle, until one spatial unit reaches a characteristic number.

For easier and more systematic presentation below, the sub-criteria used and the method of their normalization in individual groups are described first, followed by a presentation of the normalization of a characteristic number of spatial units as a result of mass appraisal.

4.1. Natural Characteristics

Among natural characteristics are those which in any environment, a man's life and work should not be able to influence. For the purposes of the scientific research in this paper, the following natural characteristics are used as criteria for the evaluation of spatial units:

1.1. The ecological aspect
1.2. Distance from the capital city
1.3. Geostrategic location
    1.3.1. Transport corridors
    1.3.2. Bordering countries
    1.3.3. Natural resources.

By considering these criteria in a slightly broader context, it is possible to come into conflict with the previous paragraph by stating that actually, man does indirectly influence these factors.

We are contemporaries of the growing impact man has on ecology, which is unfortunately mainly negative. The expansion of cities and migration from smaller communities to large urban centres inevitably leads to huge expansion and over time, a change in the distance between large centres and smaller communities must be considered. If in this regard, technical and technological progress is taken into account, then that physical distance or difference takes on another dimension. We are also contemporaries of the fact that at different time intervals the concept of geostrategic position has often changed in our region, and therefore the importance of its influence through history has had different intensities.

The result of the assessment, that is, evaluation of spatial units with respect to their natural features is a real number – a normalized value which takes a discrete value in the interval from 0 to 1.

A discrete value is obtained by breaking down the evaluation criterion from the perspective of natural features into sub-criteria, as previously shown. Each of these sub-criteria (ecological aspect, distance from the capital city and geostrategic position), also as a result of the evaluation of the local environment, has as a resulting normalized value in the interval from 0 to 1. Hence, the maximum value of each of these three sub-criteria is 1. The resultant from these three sub-criteria is calculated as the sum of these three values which are assigned appropriate weight (coefficients) as shown in the Table 1.

Table 1. *Decomposition of the criterion "natural characteristics".*

Criterion	Sub-criterion	Value	Coef.	Norm. Value.
1. Natural characteristics	1. ecological aspect	[0,1]	0.15	[0,0.15]
	2. distance from the capital city	[0,1]	0.5	[0,0.5]
	3. geostrategic location	[0,1]	0.35	[0,0.35]
			Sum=1	Sum max =1

4.2. Social Characteristics

The social characteristics of any spatial unit, as opposed to the natural ones, include all of those traits, that is, characteristics which greatly depend on the population and their activities in that area.

For the purposes of the scientific research in this paper it was necessary to distinguish those characteristics or criteria which, from the social aspect of a spatial unit, affect the value of the real estate in it. In this respect the criteria used to estimate or evaluate a spatial unit in the suggested model are:

2.1. Population density
2.2. Increase in the number of inhabitants
2.3. The number of employed
2.4. The educational aspect.

The evaluation value for spatial units from the aspect of social characteristics or criteria is a normalized value – a real number from the interval from 0 to 1, which is obtained as the sum of the normalized values of the sub-criteria broken down in the method previously shown.

Decomposition of social characteristics (criteria) into sub-criteria: population density, increase in the number of inhabitants, the number of employed and the educational aspect has the aim of evaluating a spatial unit in the best way possible to give the most reliable evaluation that is to obtain the most objective evaluation possible of their influence on real estate value.

The normalized value of each sub-criterion is given a corresponding weight which defines the influence or importance of each sub-criterion within the total assessment of the criterion in question.

The method of decomposition and evaluation of the criterion "social characteristics" is shown in the following Table 2.

Table 2. *Decomposition of the criterion "social characteristics".*

Criterion	Sub-criterion	Value	Coef.	Norm. Value.
2. Social characteristics	1. population density	[0,1]	0.15	[0,0.15]
	2. inc. in no of inhabitants	[0,1]	0.05	[0,0.05]
	3. number employed	[0,1]	0.4	[0,0.4]
	4. educational aspect	[0,1]	0.4	[0,0.4]
			Sum=1	Sum max =1

4.3. Industrial Characteristics

On similar way as previously presented in the procedure of evaluating spatial unit it could be analysed industrial characteristics as well. The method of decomposition and evaluation of the criterion "industrial characteristics" is shown in the following Table 3.

Table 3. *Decomposition of the criterion "industrial characteristics".*

Criterion	Sub-criterion	Value	Coef.	Norm. Value.
3. Industrial	1. average earnings	[0,1]	0.4	[0,0.4]
	2. agricultural development	[0,1]	0.1	[0,0.1]
	3. GDP	[0,1]	0.2	[0,0.2]
	4. road infrastructure	[0,1]	0.1	[0,0.1]
	5. tourism	[0,1]	0.2	[0,0.2]
			Sum=1	Sum max =1

5. The Results of Testing the Suggested Model

Within the framework of the scientific research in this paper, the suggested model for evaluating spatial units was tested on a sample of 30 towns equally spaced over the whole test area. The data used in testing the application of the model was taken mainly from official statistical data from the Serbian Statistical Office, or from other official public sources. It should also be noted that testing was not based on any data from 2009 or 2010.

The reason for this fact is the greatest world economic crisis since the Second World War, as a consequence of which were large fluctuations in the real estate market, as well as significant oscillations in the prices of the same. Significant price fluctuations are a direct consequence of this crisis, and not any other reason, whether economic, commercial or related to any other characteristic, and which considerably influence the market price.

Another important reason lies in the fact that this period coincides with the adoption of a new planning and construction law in Serbia, which introduces significant changes within this field. One of the consequences of these changes is a pronounced decrease in the dynamic of constructing new residential and commercial buildings, partly due to slow and poor implementation of the law, and the non-existence of secondary legislation, and partly because of the mentioned economic crisis.

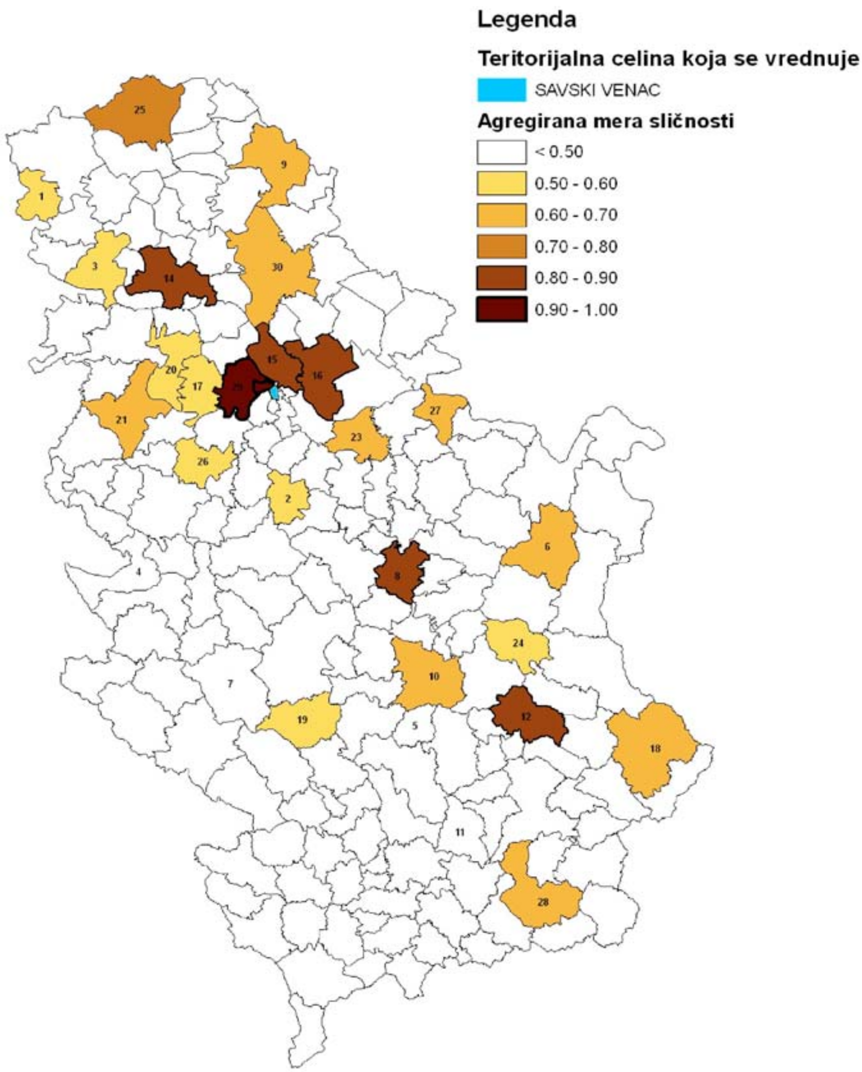
Also, one of the political decisions of the government of the Republic of Serbia, that by stimulating construction and intensive building of "social" housing, the prices of which will be significantly lower than the market value, it will revive its own economy in a period of world economic crisis. This decision significantly affects the creation of real estate market prices, which can also not be treated as usual market characteristics.

All of the above implies that using data from the real estate market from the past two years would call into question the quality of the model. At the same time it would burden the results with its use, as it relates to a very specific non-standard period which significantly deviates from the legal principles which would otherwise apply under normal circumstances.

Below are the clearly presented results of the valuation of spatial units, as well as the anticipated average values of a square metre of real estate in them. The principle established for the purpose of this scientific research is that on the basis of the list of known cases comprising 29 places described by a set of attributes, the price of a spatial unit being valued is based on the values of aggregate similarity measures between the spatial units in question and corresponding representatives from the list of known cases. Visualization of the results achieved was carried out on thematic maps of the test area, and is presented in concise tables. For the purpose of presentation the results of this investigation in the paper the results of evaluating of the spatial unit Savski Venac, in the Table 4 and corresponding Fig. 1.

Table 4. *The results of evaluating of the spatial unit Savski Venac.*

No.	Town	Aggregate measures of similarity				Average prices	μ-0.8	Price
		Natural Characteristics	Social Characteristics	Social Characteristics	DEFINI TELY			
1.	Novi Beograd	0.943	0.985	0.853	0.939	1557	0.139	464.4
2.	Zemun	0.909	1.000	0.928	0.961	1323	0.161	456.4
3.	Savski Venac					2347		
4.	Palilula	0.961	0.781	0.934	0.867	1339	0.067	191.5
5.	Subotica	0.509	0.852	0.725	0.757	643	–0.04	
6.	Zrenjanin	0.568	0.679	0.772	0.688	620	–0.11	
7.	Kikinda	0.453	0.718	0.719	0.674	582	–0.13	
8.	Pančevo	0.744	0.816	0.844	0.811	770	0.011	17.58
9.	Apatin	0.478	0.267	0.828	0.536	534	–0.26	
10.	Bačka Palanka	0.476	0.346	0.766	0.530	600	–0.27	
11.	Novi Sad	0.608	0.912	0.855	0.842	1000	0.042	90.00
12.	Ruma	0.655	0.323	0.673	0.523	744	–0.28	
13.	Pećinci	0.689	0.256	0.721	0.533	560	–0.27	
14.	Šabac	0.600	0.662	0.695	0.660	707	–0.14	
15.	Ub	0.502	0.446	0.610	0.511	555	–0.29	
16.	Smederevo	0.659	0.417	0.837	0.619	560	–0.18	
17.	Veliko Gradište	0.451	0.674	0.593	0.611	586	–0.19	
18.	Aranđelovac	0.481	0.495	0.641	0.541	618	–0.26	
19.	Jagodina	0.555	0.924	0.715	0.801	570	0.001	1.10
20.	Bor	0.505	0.671	0.729	0.660	400	–0.14	
21.	Sokobanja	0.483	0.543	0.668	0.573	850	–0.23	
22.	Bajina Bašta	0.394	0.233	0.690	0.448	790	–0.35	
23.	Ivanjica	0.550	0.273	0.592	0.450	760	–0.35	
24.	Raška	0.504	0.380	0.652	0.500	750	–0.30	
25.	Kruševac	0.531	0.674	0.730	0.666	602	–0.13	
26.	Niš	0.534	1.000	0.728	0.846	750	0.046	74.17
27.	Blace	0.497	0.282	0.601	0.444	500	–0.36	
28.	Pirot	0.524	0.592	0.706	0.616	655	–0.18	
29.	Medveđa	0.481	0.233	0.648	0.447	450	–0.35	
30.	Vranje	0.524	0.665	0.695	0.649	563	–0.15	
						ω =	0.466	1295.3



Spisak teritorijalnih celina korišćenih u modelu

1, APATIN	7, IVANJICA	13, NOVI BEOGRAD	19, RAŠKA	25, SUBOTICA
2, ARANĐELOVAC	8, JAGODINA	14, NOVI SAD	20, RUMA	26, UB
3, BAČKA PALANKA	9, KIKINDA	15, PALILULA	21, ŠABAC	27, VELIKO GRADIŠTE
4, BAJINA BAŠTA	10, KRUŠEVAC	16, PANČEVO	22, SAVSKI VENAC	28, VRANJE
5, BLACE	11, MEDVEĐA	17, PEĆINCI	23, SMEDEREVO	29, ZEMUN
6, BOR	12, NIŠ	18, PIROT	24, SOKOBANJA	30, ZRENJANIN

Fig. 1. *The thematic map of the results of evaluating of the spatial unit Savski Venac.*

In Table 5 is a comparative presentation of the average price values in the spatial units where the suggested model was tested. Also presented is a comparison of the average market prices and the price obtained by anticipation or by applying the suggested mathematical model, in which the percentage ratio of the same is given.

Table 5. *The results of testing of the proposed model of evaluating spatial units.*

No.	Town	Average prices	Values from the model	%	Increment the price in %
1.	Novi Beograd	1557	1636	105	5
2.	Zemun	1323	1505	114	14
3.	Savski Venac	2347	1295	55	–45
4.	Palilula	1339	1393	104	4
5.	Subotica	643	648	101	1
6.	Zrenjanin	620	562	91	–9
7.	Kikinda	582	594	102	2
8.	Pančevo	770	1001	130	30
9.	Apatin	534	642	120	20
10.	Bačka Palanka	600	647	108	8
11.	Novi Sad	1000	1228	123	23
12.	Ruma	744	616	83	–17
13.	Pećinci	560	644	115	15
14.	Šabac	707	672	95	–5
15.	Ub	555	668	121	21
16.	Smederevo	560	632	113	13
17.	Veliko Gradište	586	689	118	18
18.	Aranđelovac	618	636	103	3
19.	Jagodina	570	717	126	26
20.	Bor	400	597	149	49
21.	Sokobanja	850	634	75	–25
22.	Bajina Bašta	790	601	76	–24
23.	Ivanjica	760	611	80	–20
24.	Raška	750	629	84	–16
25.	Kruševac	602	563	94	–6
26.	Niš	750	1038	138	38
27.	Blace	500	661	132	32
28.	Pirot	655	670	102	2
29.	Medveđa	450	668	148	48
30.	Vranje	563	554	98	–2

As is evident from the above table, the proposed model of evaluating spatial units, based on the concept of case-based reasoning using interpolative Boolean algebra and logical aggregation, confirms the scientific hypothesis that the proposed model enables the anticipation of the average price of real estate in the test area which is in accordance with real (market) prices with a minimum amount of 75 to 80%, and it shows a very high level of agreement between market and anticipated prices obtained from the model.

By thorough analysis of the presented results, it can be concluded that in individual cases, the suggested model gives results which deviate significantly from the real (market) values, which will be cause for further explanation of these anomalies.

Savski Venac. Namely, one of the extremes which reflects the significant deviations is the example of the Savski Venac Municipality, whose anticipated average property value of 1295 makes up only 55% of the real – market value which amounts to 2347 euros. There are a number of real reasons for this kind of anomaly. One of these certainly lies in the fact that Savski Venac is the municipality with the highest average property price in the test area, with prices significantly higher than any in the test area. Because the proposed model is designed to anticipate a value based on measures of similarity with known cases from the knowledge base, that is, based on the value of the same cases, it is natural to expect that such anomalies arise. In other words, a knowledge base should be created so that each new unknown case is resolved on the principle of interpolation, since it is natural that in this type of model, the principle of extrapolation does not result in the desired outcome.

On the other hand, the specificity of Savski Venac, and other Belgrade municipalities, lies in the fact that the administrative division of Belgrade into municipalities cannot be treated in the same way as other cases in the test area. The division into municipalities, in this case more than in other cases, represents an administrative boundary, but not a boundary that represents a change in ambient, conditions, quality, comfort, culture and lifestyle. By describing and consistently evaluating urban municipalities with a set of defined attributes and having in mind their territorial jurisdiction, it is very easy to arrive at significant errors in the application of the model.

Such consistent application of the evaluation of urban municipalities would ignore the fact that, for example, an important international traffic route which passes through the centre of Belgrade represents equal quality for all of the municipalities, although administratively speaking, that same traffic route passes through only a few of them. We can also observe the example of rivers and streams. The Sava and Danube rivers offer a great quality of life in Belgrade, and regardless of their administrative belonging to particular municipalities they represent not only the pride but the value of the whole city. Also, no person would say to the residents of a particular municipality that he lives in a place that is not a university centre, even though the whole university infrastructure is located in just a few municipalities.

Similar examples can be given for cultural historical monuments, important hotel accommodation, cultural, administrative, religious, sport and other urban centres. If we added to these facts the specificities of municipalities it could mean, for example, that an important university centre in one municipality in a street which is on its administrative border, is opposite the street to buildings that are in another municipality which does not have such an institution. By consistently applying the model, buildings situated several kilometres from an important university centre, yet belong to that municipality, will have the same value as buildings which have that kind of centre in their municipality, and those who administratively do not belong to that municipality, but are only a few steps from the important centre, will not have that quality, which is a more than obvious anomaly which should be taken into account during the evaluation process.

The example of Savski Venac hides a fact that is very important to take care of and have in mind during valuation. Namely, it is the municipality where the most attractive locations are situated, not only in Belgrade but in the whole of the test area, whose average property prices do not represent a quantitative manifestation of the quality of that property, but rather that the attractiveness of the location is more important than the place of living. Ownership of property in these locations speaks of the financial, sociological, civil or other status of the property owner, although by evaluating these locations according to a set of proposed attributes, the results would not reflect this. For this kind of location, an additional quality is the absence of everything which in any other place would make its quality. By analyzing the fact that in these locations there is no major infrastructure, or significant economic, administrative, retail, university, cultural or any other centre, it is possible to arrive at errors related to the value of real estate in them. However, it is only in large urban centres like Belgrade in which all that is "missing" in that location can still be found in the vicinity, that such absence can be referred to as a quality, since if all of there attributes could not be found within a reasonable distance, it would certainly not be considered as a quality. This can be seen in the example of some naturally more attractive locations on the periphery of Belgrade, where property prices are significantly lower and the location is not nearly as attractive.

Bearing all of this in mind in a model, particular attention should be paid to urban centres so as not to mislead a user into using the model literally, taking into account only the administrative and territorial aspects of municipalities.

6. Conclusions

The proposed model is tested on a limited sample, with its results confirming a high level of correlation between the market and anticipated average price values of real estate in the subject spatial units. Also, the tested model indicates the need for certain specifics of a local area to be taken into particular consideration, especially when some of the attribute values are extreme, and as such dominant in the formation of the value of real estate within that area.

This model is a good basis for solving the comprehensive problem of mass appraisal of real estate values, and its mathematical base provides a consistent approach to problem solving at all levels and should therefore be modelled and used at all levels in the formation of a comprehensive model of the mass appraisal of real estate values.

Testing the model confirmed the hypothesis that it is possible to conceive a model that, for the given input values of the attributes of spatial units, finds the most similar from the knowledgebase–prototypes, and on the basis of them assesses the most probable average value. If it does not find a value with the given measure of similarity, it is capable of expanding the knowledge base with the given case, so that over time, that base would become better and more reliable.

A model conceptualized in this way is consistent in everything with the basic assumptions of this scientific research and it should serve as a starting point in the creation of a model of mass appraisal of real estate in the Republic of Serbia, and as such it could be universally applicable, not only in the test area, but also in a wider context.

This model, like any other should be periodically reviewed, that is, it must comply with all relevant factors affecting the market value of real estate including economic, commercial, sociological, ecological, security and any other factors. Hence, the choice of relevant attributes in all models, taking into account the differences both in dependence on the level of generality and in the classification of real estate, should be considered in a time function. The proposed concept of an operative model provides flexibility and the possibility of recognizing changes in relevant criteria values that influence the value of real estate at a given moment.

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Primjena inteligentnih tehnika za masovnu procjenu nekretnina

SAŽETAK. U radu je razmatran pojam masovne procjene vrijednosti nekretnina u okviru kojega se, osim definiranja osnovnih pojmova, izvodi i paralelna analiza različitih međunarodnih iskustava vezanih uz ovu problematiku. Također, analizira se normativna i institucionalna uređenost predmetne cjeline na testnom području, ali se definira i pojam vrednovanja prostornih jedinica kao osnove masovne procjene vrijednosti nekretnina i polje njegove primjene. Definirana je osnova modela vrednovanja prostornih jedinica temeljenog na zaključivanju po principu slučaja (ZOS) i logičke agregacije (LA) te je dana matematička osnova predloženog modela za predviđanje prosječnih cijena nekretnina u okviru prostornih jedinica. U predloženom modelu opisane su prostorne jedinice, odnosno način njihove normalizacije te njihova granulacija po grupama. Pojedinim atributima i grupama dodijeljene su odgovarajuće težine kojima se definiraju njihovi pojedinačni i grupni značaji u okviru integralnog modela. Na kraju je izvedeno testiranje modela na testnom području.

Ključne riječi: *vrednovanje prostornih jedinica, masovna procjena vrijednosti nekretnina, zaključivanja po principu slučaja, logička agregacija.*

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