# THE GOLDEN SECTION AS A SOURCE OF CONSISTENCY IN 20TH CENTURY MUSIC[1]

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The thesis consists of three major parts that deal with the most important aspects of the Golden Section phenomenon: the historical background, the theoretical and analytical presumptions and the means of application of the Golden Section[2] and the Fibonacci sequence[3] in 20th century music analysis and composition.

## *The historical background*

Although the concept of the Golden Section has been present in Western thought since the age of Classical Antiquity, there is no existing evidence of its relationship with music until the mid-19th century. The Golden Section concept appeared as a purely mathematical concept, a geometrical construction, as mentioned in some fragments of Pre-Socratic texts and the sixth book of Euclid's *Elements*, where it is referred to as the division of a line into extreme and mean ratio. Although its geometrical construction happens to be a simple procedure, the analogous arithmetical division is far more complicated and its results cannot be expressed by a rational number. The latter provoked a deep crisis in ancient Greek mathematical thought, which was founded on the premise that all visible and invisible things and ideas could be expressed in terms of the relationships of natural numbers.

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[2] The Golden Section is a division of a line such that the smaller segment of the line relates to the larger one as the larger one relates to the sum of the two. The ratio between them is usually referred to as the Golden Ratio.

[3] The Fibonacci series is an additive numerical row whose limits of consecutive quotients of neighbouring integers converge towards the Golden Ratio.

The dodecahedral vision of the structure of the Universe as portrayed in Plato's *Timaeus* widely contributed to the mystical connotations associated with the Golden Section. Such a shape can be constructed only by using the geometric division into »extreme and mean ratio«, the one considered to be the most perfect and most beautiful, since it enables the integrity of the entire cosmic space. The concept itself has survived in the works of Classical Greek writers until the present day, although it has lost most of its cosmological, ethical and aesthetic significance.

The modern-age interest in the heritage of Classical Antiquity arose in the 19th century. Together with archaeological discoveries and the broad spread of Neoclassicism, a range of new aesthetic theories emerged, trying to establish a Neo-Platonist vision of beauty based on rational ethico-aesthetical criteria. Those science-oriented attempts were particularly frequent in the early 1830-ies in Germany, where the term »Golden Section« occurred in the modern sense of the word for the first time. In such an environment Adolf Zeising wrote his *New Theory of the Proportions of the Human Body* (1854), the first scientific study entirely based on the Golden Proportion. Zeising considered the mirror symmetry and the Golden Section to be the fundamental principles of all forms striving for perfection and beauty, equally present in human and natural creations. According to Zeising, these universal laws also govern the most pleasant harmonies, immanent in the interval of the »impure sixth« (ratio 0.6), which inclines either to the major or the minor variant depending on the mode. This aspect of his theory is restricted to the pitch and disregards general acoustic laws. In spite of that, Zeising influenced a large number of notable scientists, e.g. Gustav Theodor Fechner, who tried to apply Zeising's theory to the field of experimental psychology. Since the results of the experiments did not fulfill his expectations, Fechner declared Zeising's theory to be overestimated, but not irrelevant. Nevertheless, Fechner's studies only encouraged further psychometric research of the Golden Section.

By the 1860-ies the concept had entered music historiography and analysis. Although inspired by Zeising, these studies were focused not on the intervallic, but on the formal aspects of it. Emil Naumann was the first to recognize the Golden Section in the proportional structure of classical sonata forms, considering it the pinnacle of the development of music in general (*Die Tonkunst in der Cultur-Geschichte*, 1869). Such an attitude became common among music professionals by the beginning of the 20th century when Gustav Ernest spoke on the occasion of the 1902 annual meeting of the Royal Music Association in London. He praised the superiority of the Golden Proportion (which he referred to as »the Zeising law«), based on the analyses of selected Beethoven sonata forms, emphasizing that the great composers of the Classical period could not have been acknowledged with the concept as an aesthetic determinant and that its presence, therefore, should be considered to be unconscious proof of their ingeniousness. The criteria of Ernest's analytical method were set rather similarly to the ones we would use today and his report should thus be regarded as one of the earliest scientific proportional analyses of music. Such a clear analytical method could have easily

been converted to a compositional tool for establishing formal and structural balance.

Apart from the Classical sonata forms, the Golden Section has also been discovered among diverse formal patterns. Studies by Roy Howat, Robert Orledge, Deborah Mawer and others show the presence of this proportion in the works of Claude Debussy, Erik Satie and Maurice Ravel. Although there is no evidence of their intentional use of the Golden Section, there are many indications that they may have acknowledged the concept through the writings of Zeising, then-recent formal analyses and certain social circles. It is highly possible that the Golden Section concept became a widely known means of artistic design in the first decades of the 20th century, appearing in the works of Gustav Mahler, Leoš Janáček and Paul Hindemith. Béla Bartók's opus seems to be the most famous example, in light of Ernő Lendvai's analytical studies. Lendvai claimed that Bartók used this proportion consciously to articulate the musical time and the intervallic structure of his music. His discoveries have been taken for granted for decades, until László Somfai refuted a significant part of his writings, since he could not find any proof of it in Bartók's rich collection of written documents and compositional sketches. Nevertheless, a high degree of structural organization in his music presents testimony to his extraordinarily refined sense of balance and proportion.

A fresh wave of interest in the topic arose after World War II, when Humanity attempted to renew faith in the orderliness of the Universe through restoration of Ancient, Humanist and Renaissance thought. Three major theoretical studies appeared in this context, re-introducing the harmonic and the Golden Section theories as principal proportional guidelines in the early postwar aesthetics: Le Corbusier's *Modulor* (1948), Rudolf Wittkover's *Architectural Principles in the Age of Humanism* (1949) and J. H. Douglas Webster's *Golden Mean Form in Music* (1950). The last one, based on the theory of organic growth, reveals the presence of the concept in a wide range of examples, from simple traditional tunes to the most developed symphonic forms.

Le Corbusier's *Modulor* has left a particularly strong impact on the New Music of the 20th century. Deeply rooted in the Ancient Greek philosophy, it proclaimed architecture on a human scale established on a set of regulating procedures based on the Golden Ratio. Being one of Le Corbusier's assistant engineers, Iannis Xenakis composed his *Metastaseis* (1953) under the strong influence of his supervisor. He has also left an extensive analytical study of the score, where he elaborated the means of application of the Golden Ratio in the structure of this work and presented arguments for his choice. That is probably the earliest confession of a composer who intentionally used proportional design as a compositional procedure to increase the symbolic weight of his music.

The consistency of the *Modulor* also inspired Paul Gredinger to write *Das Serielle* (1955), an attempt at mapping Le Corbusier's geometric serialism from the architectural space to the structure of music on the way to total parametric organization. Serial music proved fertile ground for the application of the Fibonacci

series, especially to the parameters of duration and pitch. Karlheinz Stockhausen has used the Fibonacci sequence in a large number of his scores, but, unlike Xenakis, he was less interested in its symbolic value than in the effect of its mathematical properties.

With the spread of serial techniques, the Fibonacci sequence became a common compositional tool. Along with works by Xenakis and Stockhausen, the Fibonacci sequence may also be recognized in the serial compositions of Luigi Nono, Ernst Křenek and many others. Applications of this and similar arithmetic series to the parameters of music represent an easy way to achieve proportionality and a dynamic sort of symmetry, balancing between the traditional strict symmetry and its unpredictable opposite. By the end of the 20th century the Golden Section and the Fibonacci sequence have become legitimate parts of the Western compositional practice and found their place in numerous specialist writings, as well as in a range of composition textbooks.

### *The theoretical and analytical presumptions*

To be aware of the impacts of the use of the Golden Section or Fibonacci numbers, one has to be familiar with their mathematical properties, advantages and limitations. For example, the adjacent members of the Fibonacci series are incommensurable and cannot be reduced to a common denominator within the same time signature. The ratios of adjacent members of the ascending Fibonacci row converge to an irrational number, φ (the Golden Ratio constant), suggesting the composer use a larger absolute value of the adjacent integers in order to approach the Golden Ratio more closely. By using the descending part of the sequence, one moves away from the Golden Ratio and approaches strict binary symmetry (1:1).

Furthermore, the effect of the dynamic symmetry of the Golden Section differs depending on whether the larger part precedes the shorter one (primary Golden Section) or vice versa (secondary Golden Section).

The irrationality of the Golden Ratio constant causes further problems with its application in a discrete medium: it can never be embodied with complete accuracy. Theorists and analysts are therefore often in dispute over the threshold of tolerance of the deviation from the ideal proportion and the determination of a measurement unit sensitive enough but not too small to indicate the significance of the proportion. Composers' theories and sketches can be very helpful in making such difficult decisions.

The division in the Golden Ratio is infinite and enables a multi-levelled automorphic mapping within a structure. This is often the case with Classical sonata forms, where the relation between the duration of the whole and the larger part (development + recapitulation) resembles the ratio between the larger and the shorter part (exposition) and can analogously be mapped onto deeper structural levels.

Finally, like any mathematical concept, the Golden Section is by its nature entirely abstract and could be applied onto virtually any parameter of music. However, probably due to its spatial origin, it is most closely related to the parameter of duration. Applications to the pitch (the tonal space) are considerably common, while the proportion of the spectrum happens only sporadically, mostly in the field of electronic sound synthesis. There are still no notable results regarding the possibility of proportioning the musical volume.

### *The means of application of the Golden Section and the Fibonacci sequence in musical analysis and composition*

The **musical time** is certainly the most interesting medium for the application of proportional design in composition and analysis.

The Golden Section could be a useful analytical tool in interpreting musical form. The method is analogous to the analysis of visual artifacts and is commonly used to determine the proportions of the **overall form**, respecting the most important musically relevant moments (boundaries of formal sections, major changes of the dynamics, orchestration…). Works that embody the Golden Ratio in their overall structure often bear the same proportion on multiple structural levels, showing its automorphic properties, as in Bartók's *Music for Strings, Percussion and Celesta* (1936) or the *Passacaglia for Strings* in g-minor (1957) by the Croatian composer Krsto Odak. The latter shows clearly discernible multiple divisions of the variation series: its main dynamic climax has been placed at the primary Golden Section of the overall form, which also reveals further analogous divisions. The theme itself is consequently shaped, having its dynamic pinnacle at the exact point of the Golden Section.

Since there is no evidence of intentional proportioning procedures for the majority of similarly shaped pieces, the Golden Proportion in such structures could be a result of intuitive design. This could possibly be related to the presence of the Golden Ratio in the traditional music of many European regions.

Composer's theories prove that many other works have been intentionally designed in accordance with the Golden Section or the Fibonacci sequence, particularly in the forms that do not originate from the traditional laws of tonality. In some cases the proportions of musical time segments play a decisive role and often remain the only factor of formal integrity. Stockhausen's moment forms like his wind quintet *Adieu* (1966) often embody interesting arithmetical patterns. The piece consists of eight sections — moments — whose duration is determined by the adjacent integers of the Fibonacci sequence, approaching the Golden Ratio. Some moments are divided by a single point, while the others show analogous divisions at lower structural levels.

In addition to the proportioning of formal elements, the segments of the Fibonacci sequence can be used to articulate the musical pace, as in Křenek's

*Quaestio temporis* (1958-59), where all metronome marks represent multiplications of the Fibonacci numbers, while the speed (tempo) and duration depend on the Fibonacci-related intervallic structure of the basic twelve-tone row. Křenek later continued to study the relationship between the pre-arranging of the musical material and pure chance. In his *Fibonacci Mobile* (1964), the large sections composed according to strict serial procedures should be performed in an aleatoric sequence. The intervallic series, tempos and tone durations are again determined by the consequent integers of the Fibonacci row.

Furthermore, the Fibonacci series is regularly used as a means of articulation of the musical time at its micro-level, particularly in compositions where the traditional symmetry of musical meter has partially or completely lost its significance. In his *IX. Piano Piece* (1955-61), Stockhausen determines durations of individual tones and groups of tones by Fibonacci numbers. They are often not exposed in their original order and thus do not transmit real proportional relations, but only a symbolic message of the Golden Section.

The integers of the Fibonacci sequence are nevertheless more often being exposed in their original order, keeping their proportional and additive properties. Thereby the consequent numerical values either approach to the Golden Ratio or digress from it, which makes such sequences a very convenient compositional tool for opening and closing sections. A descending segment of the Fibonacci sequence usually stands at the beginning of a piece of music, gradually digressing from the Golden Ratio (such as Xenakis' *Metastaseis* and *Zyia*). The other case — the exposure of an ascending segment of the row — is even more common, especially in the final, cadential sections of musical pieces, approaching the durational ratios to the Golden Section (Stockhausen: *Die Schmetterlinge spielen*, Xenakis: *Metastaseis*, *Khoaï* etc.). Mirror symmetry of introductory and closing sections is also not rare and often comes together with the treatment of consonance and dissonance. In his *Chronostasis* for strings and electronics (2003) the Croatian composer Vjekoslav Nježić combines the two: at the beginning of the piece the instrumental parts enter one by one, separated by the time sections whose durations correspond to the values of a descending segment of the Fibonacci sequence (21 — 13 — 8 — 5 — 3…). The level of dissonance grows simultaneously with the digression from the Golden Ratio, from the initial perfect unison through other perfect and imperfect consonances to the dissonant major second. At the end of the piece one hears the reverse procedure: the sound of the orchestra gradually reaches the perfect unison, the instrumental parts exit one by one and the time sections are prolonged according to the values of the Fibonacci sequence.

The incommensurability of the adjacent integers of the Fibonacci sequence may cause difficulties with its application to the parameter of duration in a polyphonic texture, especially when the composer chooses to preserve the traditional stability of a symmetrical musical meter. Xenakis managed to solve this problem in his *Metastaseis* (1953-54) by interpreting the rhythmic values in an original way, keeping the clarity of the standard notation. He superposed the

orchestral parts with different time signatures and approached the values of the Fibonacci numbers by subtracting durations of the superimposed notes. Xenakis named such derived time sections »the differential durations« (*durées différentielles*). A couple of years later Nono applied a similar system in the second movement of his *Il canto sospeso* (1955-56).

Compared to the parameter of duration, the other parameters of music are far less appropriate for proportional design. The practice of proportioning of the tone spectrum and dynamics is negligible and thus insignificant, while the application to the **pitch** appears in a number of valuable compositions.

In the tempered tuning system, composers often choose to approximate to the Golden Ratio by projecting a segment of the Fibonacci series onto the tonal space divided by half-tone intervals. This procedure most probably grew out of Lendvai's analyses of the intervallic structure of Bartók's and Kodály's musical material, especially the pentatonic scale and unique chord structures such as the so-called alpha-chord.

Lendvai also noticed a reflection of the Golden Ratio in the pitch compasses of some Bartók's melodies, e.g. in the relations of the thematic segments of his *Sonata for Two Pianos and Percussion* (1937). Although his intervallic theory seems to be even weaker in argumentation than its formal pair, it has left an undeniable impact on the composing technique. The projection of the Fibonacci series onto a half-tone axis has become a common compositional tool of both serialist and non-serialist composers and is often used along with the analogously treated parameter of duration.

From the foregoing exposure of the most significant historical, theoretical and practical aspects of the relation between the Golden Section and music, one can come to a general conclusion that this ancient mathematically conceived aesthetic concept, as well as its approximations by the integers of the Fibonacci sequence, offer not only a rich set of possibilities for future research, but also a handy and generous compositional tool for 21st century composers.