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# Fast Trajectory Optimization for the Time Delay Effect on Supercavitating Flight

Original scientific paper

Utilizing supercavitation to achieve drag reduction, a supercavitating vehicle (SV) could travel 3~5 times faster than conventional underwater vehicles. Aiming at pursuing high stability and better performance for the SV, research on trajectory optimization for the SV in vertical plane was carried out. The principle of independence of cavity section expansion was applied for modelling of the time delay supercavity.

Dynamic equations incorporated with complex force models of SVs were analyzed in detail, on the basis of which a mathematical model of the optimal diving trajectory was established. Subsequently, the Time Delay Gauss Pseudospectral Method (TDGPM) was developed to achieve fast optimization. The solving process was significantly accelerated by converting the optimal control problem into nonlinear programming (NLP) problem.

The time delay effect of supercavitating flight was taken into account by interpolation of variables on all discretized nodes. Numerical solutions for different optimal diving trajectories and corresponding manoeuvres were obtained based on the proposed method using 20 Gauss-Legendre discretization nodes. Results of rapid optimization of the supercavitating trajectories indicate the feasibility and fast convergence of TDGPM.

**Keywords:** *gauss pseudospectral method, supercavitating vehicle, time delay effect, trajectory optimization*

## Brza optimizacija putanje superkavitirajućeg leta pod utjecajem vremenskog pomaka

Izvorni znanstveni rad

Iskorištavanjem superkavitacije za smanjenje otpora, superkavitirajuća vozila (SV) mogla bi se kretati 3-5 puta brže od konvencionalnih podvodnih sredstava. U cilju postizanja visoke stabilnosti i boljih svojstava za SV u ovom su radu provedena istraživanja o optimizaciji putanje SV. Primijenjen je postupak modeliranja vremenski odložene superkavitacije na temelju načela neovisnosti razvijanja kavitacijskih presjeka.

Matematički model optimalne putanje ronjenja ustanovljen je na osnovi iscrpne analize složenih modela sila inkorporiranih u dinamičke jednadžbe. Za postizanje brze optimizacije razvijena je metoda 'Time Delay Gauss Pseudospectral Method' (TDGPM). Postupak rješavanja znatno se ubrzao pretvorbom problema optimalnog nadzora u problem nelinearnoga programiranja (NLP).

Utjecaj vremenske odgode pri superkavitirajućem letu uzet je u obzir interpolacijom varijabli u sve diskretizirane čvorove. Numerička rješenja za različite optimalne putanje ronjenja i odgovarajuće manevre dobivena su na osnovi predložene metode korištenjem 20 Gauss-Legendreovih točaka diskretizacije. Rezultati brze optimizacije superkavitirajućih putanja pokazali su izvedivost i brzu konvergenciju metode TDGPM.

**Ključne riječi:** *Gaussova pseudospektralna metoda, optimizacija putanje, superkavitirajuća vozila, učinak vremenske odgode*

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## 1 Introduction

Utilizing natural/artificial supercavitating technology, a supercavity that is comparable with the vehicle size could be generated to envelope the body surface during a high speed underwater motion. Due to the separation of the solid body surface from the liquid environment, the drag could be dramatically reduced, which results in much higher speed of the vehicle. As a revolutionary drag reduction technology, supercavitation has been widely applied in fully-submerged vehicles in recent years, such as torpedo, underwater missile and projectiles [1-3]. However, owing to the complex flow behaviours that can be experienced in this two phase flow surrounding the vehicle, tremendous challenge will be induced in the dynamic modelling and motion manoeuvring of the SVs.

Plenty of efforts focusing on fundamental theories and practical experiments of supercavitating flow have been carried out to explore the characteristics of the flow field and the modelling of the hydrodynamic forces. The principle of independence of the cavity section expansion [4, 5], which is considered as the theoretical basis of the time delay effect of supercavitation, has been testified and widely used in the analysis of cavity dynamics. Achievements of experimental researches on supercavitating flight [6, 7] have led to better understanding of the hydrodynamic characters of supercavitating bodies. CFD techniques have been developed for the well prediction of the evolution process of supercavitation [8-11]. Based on deeper understanding of the inherent physics, numerous researches have been focused on establishing a dynamic model and developing motion control methods for SVs [12, 13]. In order to ensure the stability of supercavitating flight, robust control methods have been applied for the vehicle control [14-17] by considering both the nonlinear and time delay effects of supercavitating flight.

To achieve aggressive manoeuvrability and better performance during flight, trajectory optimization is of great importance for overall design of the SVs [18-20]. Considering the time delay effect and the nonlinearity of supercavitating flight dynamics, it would be too complicated to solve the optimal control problem of supercavitating flight by indirect method. Meanwhile, the most direct method could be particularly time consuming in solving the optimal control problem of SV.

In this paper, the time delay gauss pseudospectral method is developed for rapid optimization of the supercavitating diving manoeuvre. The feasibility and advantage of the algorithm are considered. In section II, the flight equations and hydrodynamic forces of supercavitating flight are described in detail. The mathematical model of optimal control problem and the TDGPM algorithm are introduced in section III. Section IV presents the numerical results obtained by TDGPM, which also validates the fast convergence of the algorithm. The advantages of TDGPM and future works on the optimal control of SV will be summarized in section V.

## 2 Flight dynamics for SV

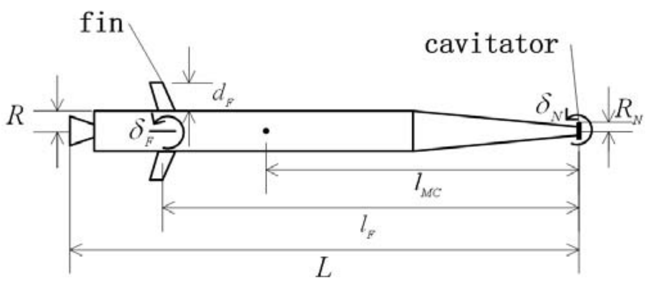



Figure 1 **Configuration of supercavitating vehicle**
Slika 1 **Konfiguracija superkavitirajućeg vozila**

For the purpose of generating a supercavity, a cavitor with proper shape and size is installed in front of a supercavitating body. As a result, the configuration of the SV presents great difference from that in the traditional underwater vehicles with a streamline shape. After the cavitator, the vehicle body is usually constructed by cylindrical and conical parts. Optional fins are mounted at the aft body. Thus, the traditional streamline shape is no longer essential for SVs. Typical geometry of the vehicle is shown in Figure 1, where $R$ denotes the maximum radius of the body hull, $L$ is the length of the vehicle, $l_{MC}$ is the distance between the cavitator and the mass centre of the vehicle. $R_N$ is the cavitator radius, $d_F$ is the length of fins, and $l_F$ the position where fins are mounted. $\delta_N$ and $\delta_F$ are the deflecting angles of the cavitator and the fins.

### 2.1 Motion equations

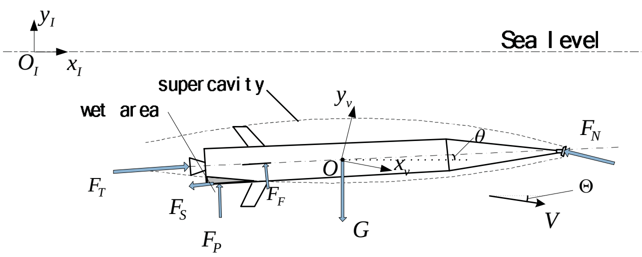



Figure 2 **Forces acting on supercavitating vehicle**
Slika 2 **Sile koje djeluju na superkavitirajuće vozilo**

Dynamic model of supercavitating flight in the vertical plane is used in this paper. As shown in Figure 2, an earth fixed reference $O_I x_I y_I$ which could be treated as an inertial reference, is established. The origin $O_I$ is located at any convenient position on the sea level. The axis $O_I x_I$ is placed horizontally and points to the direction that the vehicle is launched. The axis $O_I y_I$ points upward, i.e. in the opposite direction of gravity. The body fixed velocity reference $Ox_v y_v$ is set, with origin $O$ placed at the mass centre of the vehicle. The axis $Ox_v$ always points to the direction of the velocity of the vehicle.

Being enclosed in a supercavity, the buoyancy of SV is neglectable. The main forces acting on the supercavitating body are schematically illustrated in Figure 2, where $F_N$ denotes cavitator force, and $F_F$ fin forces. The interaction between the vehicle and the cavity wall contains planing force $F_P$ and skin friction $F_S$. $F_T$ is the thrust of motor, and $G$ is the gravity force. For simplicity, $F_P$ and $F_S$ are supposed to act at the aft of the body. $F_N$ and $F_F$ are centralized at the pressure centre of the cavitator and fins, respectively.

The dynamic equations of the supercavitating flight in the $Ox_v y_v$ reference could be written as

$$\begin{cases} m\left(\dfrac{d\vec{V}}{dt} + \omega_z \times \vec{V}\right) = \vec{F}_N + 2\vec{F}_F + \vec{F}_P + \vec{F}_S + \vec{G} \\ I_z \dfrac{d\omega_z}{dt} = M_N + M_F + M_P + M_S \end{cases} \tag{1}$$

where $\vec{V}$ is the velocity, $\vec{\omega}$ is the angular velocity, $m$ is the mass of the vehicle. $I_z$ is the moment of inertia of the body about the mass centre, $M_i\left(i = N, F, P, S\right)$ represents the moments of cavitator, fin, planing, and skin friction forces, respectively. To be noticed, for the supercavitating flight in the vertical plane, both of the horizontal fins provide hydrodynamic forces.

### 2.2 Modelling of time delay supercavity

Forces acting on the SV depend on the dynamic model of the supercavity, which affects the hydrodynamic coefficients and the wetted area of the body. Thus, it is of great importance to obtain the parameters of the supercavity properly.

According to the principle of independence of the cavity section expansion, each cross section of the supercavity expands relatively to the trajectory of the cavitator which happens almost independently from the following or the previous body motion [4]. The process is dependent only upon the cavitator motion states at the moment when it passes the plane of the considered section.

The section radius of the cavity can be calculated as [4, 22]

$$R(\triangle t) = R_m \sqrt{1 - \left(1 - \frac{R_N^2}{R_m^2}\right)\left(1 - \frac{\triangle t}{t_m}\right)^{2/\kappa}} \tag{2}$$

$\Delta t$ is the interval between the current time and the moment that the cavitator passed through this section, $\kappa = 0.85$ is the empirical constant. $R_m$ is the maximum radius of the cavity section, which can be determined by

$$R_m = R_N \sqrt{\frac{C_d}{k\sigma}} \tag{3}$$

$\sigma$ is the nondimensional cavitation number. For disk cavitator, $C_d = 0.82(1+\sigma)$ is the supercavitating drag coefficient, $k = \left(1+50\sigma\right)/\left(1+56.2\sigma\right)$ is a correctional coefficient.

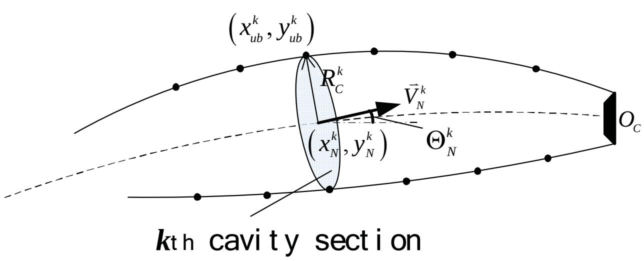



Figure 3 **Time delay supercavity model**
Slika 3 **Model kavitacije s vremenskom odgodom**

Given the cavitator track and motion states, as illustrated in Figure 3, radius and orientation of any cavity section at any moment could be solved. Furthermore, the coordinates of the upper boundary of the $k^{\text{th}}$ cavity section are calculated by

$$\begin{bmatrix} x^k_{ub} \\ y^k_{ub} \end{bmatrix} = \begin{bmatrix} x^k_N \\ y^k_N \end{bmatrix} + R^k_C \begin{bmatrix} -\sin\Theta^k_N \\ \cos\Theta^k_N \end{bmatrix} \tag{4}$$

The coordinates $\left[x^k_{ub}, y^k_{ub}\right]^T$ and $\left[x^k_N, y^k_N\right]^T$ indicate the upper boundary position and the cavity section centre, $R^k_C$ is the instant cavity radius of the section, $\Theta^k_N$ is the inclination angle of the cavitator when it passes through the section.

Thus, with the data of certain number of cavity sections, the accurate outline of supercavity could be obtained by interpolation. Coupled with the relative position of the vehicle, the wetted position of the hull could be calculated, which is used to define the interaction between the vehicle and the supercavity.

### 2.3 Model of forces

#### 2.3.1 Cavitator force

Hydrodynamics of SV is mainly centralized in the cavitator. In the cavitator speed reference [16]

$$\vec{F}_N = \begin{bmatrix} F_{ND} \\ F_{NL} \end{bmatrix} = 0.5\rho V_N^2 C_d S_N \cos\alpha_N \begin{bmatrix} -\cos\alpha_N \\ \sin\alpha_N \end{bmatrix} \tag{5}$$

$F_{ND}$ and $F_{NL}$ denote the drag and lift part of the cavitator force. $\rho$ is the density of water, $V_N$ and $S_N$ are the speed and the section area of the cavitator, respectively. $\alpha_N$ is the cavitator's attack angle, which is related to the cavitator deflecting angle $\delta_N$.

The direction of the cavitator force is always vertical to the disk plane, the moment of cavitator force is

$$M_N = 0.5\rho V_N^2 C_d S_N l_{MC} \cos\alpha_N \sin\delta_N \tag{6}$$

#### 2.3.2 Fin forces

Hydrodynamic force acting on each horizontal fin is calculated by using the following model [12, 22]

$$\vec{F}_F = \begin{bmatrix} F_{FD} \\ F_{FL} \end{bmatrix} = 0.5\rho V^2 \begin{bmatrix} C_{FD}(w_F, \alpha_F) \\ C_{FL}(w_F, \alpha_F) \end{bmatrix} S_F \tag{7}$$

$C_{FD}$ and $C_{FL}$ are drag and lift coefficients of the fin. Both of the coefficients are the functions of fin wet part $w_F$ and the attack angle $\alpha_F$. The control of fins $\delta_F$ would affect the attack angle, which results in the changing of $C_F$. $S_F$ is the corresponding characteristic area when defining the force coefficients.

Fin force moment is calculated by

$$M_F = \left(F_{FL}\cos\Theta + F_{FD}\sin\Theta\right)\left(l_{MC} - l_F\right) \tag{8}$$

#### 2.3.3 Cavity-vehicle interaction

The supercavity model interpreted in 2.2 was applied in determining the wet position of the vehicle. The planing force and skin-friction force of the cavity-vehicle interaction could be expressed as [22]

$$F_P = 0.5\rho V^2 \left(\pi R_{cp}^2\right)\sin\alpha_P \cos\alpha_P \left[1 - \left(\frac{R_{cp} - R}{h_0 + R_{cp} - R}\right)^2\right]\left(\frac{R + h_0}{R + 2h_0}\right) \tag{9}$$

$$F_S = 0.5\rho V^2 \cos^2\alpha_P C_w S_w \tag{10}$$

where $\alpha_P$ is the planing angle, $R_{cp}$ is the cavity radius where planing occurs, $R$ is the radius of the vehicle, $h_0$ is the immersion depth, $S_w$ represents the wet area, $C_w$ is the coefficient of the skin friction. The planing force vector is vertical to the body axis, while the skin-friction points to the opposite direction of the body axis. The corresponding moments of the interaction are $M_P = F_P\left(l_{MC} - L\right)$ and $M_S = -F_S R$.

#### 2.3.4 Thrust and gravity forces

Both the thrust $\vec{F}_T$ and gravity $\vec{G}$ are set as constant in this paper. $\vec{F}_T$ acts along the symmetric axis of the body, while the $\vec{G}$ is in the opposite direction of the $O_I y_I$ axis. As the directions of these two forces pass the mass centre of the vehicle, the moments of thrust and gravity forces are zeros.

## 3 Trajectory optimization of SV

### 3.1 Optimal control problem of diving manoeuvre

Trajectory optimization is an optimal control problem which aims at determining the time histories of vehicle states and the associated vehicle control strategies that satisfy a certain number of constraints while minimizing a certain cost function. Therefore, there are two factors that need to be stated clearly. Firstly, the manoeuvres must be compatible with the vehicle dynamic equations. Certain limits, including boundary conditions and path constraints, should be satisfied as well. Secondly, the cost function should be established and minimized so that the solution is optimal in some extent.

Motion equations of supercavitating flight could be written in a simple form

$$\dot{X} = f(X,u,t) \tag{11}$$

where $X = \left[x, y, V, \Theta, \theta, \omega\right]^T$ indicates the motion states of the vehicle, and $u = \left[\delta_N, \delta_F\right]^T$ is the control vector. Referring to the deflecting angle of the cavitator and fins in this work, variable $t$ is the flight time, $t \in [0, t_f]$.

Generally, the cost function of an optimal control problem could be expressed as

$$J = \Phi\left(X,t\right)\Big|_{\Gamma} + \int_0^{t_f} L\left(X,u,t\right)dt \tag{12}$$

The first term $\Phi$ defines the boundary cost at $\Gamma = \left\{0, t_f\right\}$, while the second is the integral term of the cost function.

In this paper, quick diving manoeuvre of supercavitating flight is considered as the primary task. To set the boundary cost $\Phi = 0$, and the integrated function $L = 1$, which yields

$$J = t_f \tag{13}$$

the initial and final states are set as trim conditions of level supercavitating flight. The upper and lower bounds are $X \in \left[X_{\min}, X_{\max}\right]$, $u \in \left[u_{\min}, u_{\max}\right]$.

### 3.2 Time delay gauss pseudospectral method

Indirect and direct methods have both been developed to solve optimal control problems with numerical techniques [23, 24]. Indirect methods typically use the calculus of variations to derive first-order necessary conditions for optimality. It requires a complicated thorough comprehension of the optimal control problem, which could be impracticable in some cases, e.g., supercavitating systems coupled with time delay effects. In contrast, direct methods convert the continuous-time optimal control problem into a nonlinear programming (NLP) problem, which could be efficiently solved by many algorithms [25].

In the case of the Gauss Pseudospectral Method (GPM), both flight state and control variables are discretized on Gauss-Legendre points (the number is supposed to be $N$) and approximated with global orthogonal interpolating polynomials. The differential dynamic equations' constraints were converted into algebraical ones at each single discretized point. The equivalence between solving the optimal control problem and computing the continuous-time variational conditions with the GPM has been proved in previous research [24], in which details about the discretization could be also found. GPM has been widely applied in the trajectory optimization processes in recent years [26].

Since the shape of supercavity depends on the historical motion of the cavitator, the planing force and skin friction at current moment are functions of the historical motion states. The time delay effect coupled in the supercavitating flight equations has brought difficulty in the calculation of the dynamic constraints at discretized points. To achieve fast solution of the trajectory optimization problem for the SV, the time delay GPM algorithm is developed as follows.

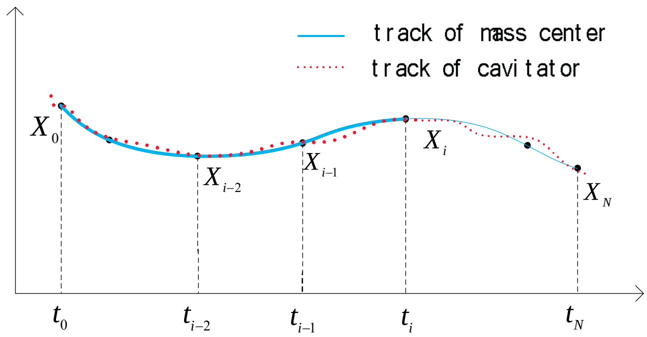


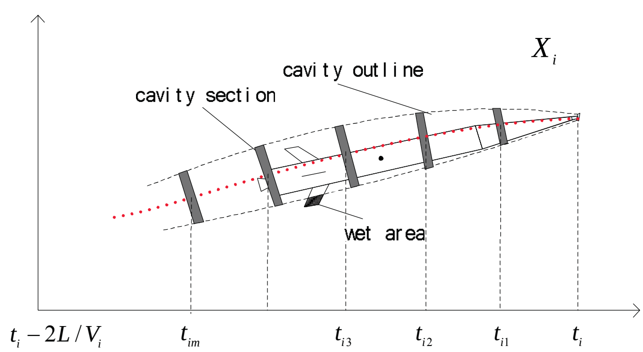

Figure 4 **Tracks presentation of vehicle mass centre and cavitator**
Slika 4 **Putanja središta mase vozila i kavitatora**

As shown in Figure 4, due to the time delay effect, the differential dynamic equations' value $\dot{X}_i$ in each discretized point is related to the former states $X_k (k < i)$. To deal with this effect, the trajectory of the vehicle mass centre is calculated with Lagrangian polynomial interpolation based on the discretized motion information of the vehicle. The motion states and control strategies during the flight time are obtained. The cavitator track, i.e. the cavity centre along the trajectory, could be derived according to

$$\begin{bmatrix} x_N \\ y_N \end{bmatrix} = \begin{bmatrix} x_{MC} \\ y_{MC} \end{bmatrix} + l_{MC} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \tag{14}$$

where the $\theta$ is the pitch angle of the vehicle at current moment.

Figure 5 **Real time cavity at the $i^{\text{th}}$ discretized point**
Slika 5 **Kavitacija u i-toj diskretiziranoj točki u stvarnom vremenu**



At each discretized node, estimating time intervals $\{[t_i - 2L/V_i, t_i]\}, i = 0,1,\ldots,N$ with certain number of sampling points $m$ is established. Based on the cavitator track, motion states at each $t_{ij}(j = 1,2,\ldots,m)$ are interpolated to calculate the real time cavity sections which define the accurate cavity shape and the wet condition of the vehicle (see Figure 5).

As described above, the numerical calculation of the optimal control problem could be stated as follows: to determine the discretized state variables $X_i$ and control variables $u_i$, which will minimize the cost function $J = t_f$ while satisfying the constraints.

(1) Dynamic constraints on discretized points

$$R_i \equiv \sum_{k=0}^{N} D_{ik} X_i - \frac{t_f - t_0}{2} f\left(X_i, u_i, \bar{\tau}_i; t_0, t_f, X_{k',(k'<i)}\right) = 0 \quad (i = 1,\cdots,N) \tag{15}$$

(2) Terminal state constraints

$$R_f \equiv X_f - X_0 - \frac{t_f - t_0}{2} \sum_{k=1}^{N} w_k f\left(X_k, u_k, \bar{\tau}_k; t_0, t_f, X_{k',(k'<k)}\right) = 0 \tag{16}$$

(3) Boundary conditions and other constraints during the flight

$$\phi\left(X_0, t_0; X_f, t_f\right) = 0 \tag{17}$$

$$C\left(X_k, u_k, \bar{\tau}_k; t_0, t_f\right) \le 0 \tag{18}$$

where $D \in R^{N\times(N+1)}$ is the differential matrix of the Lagrange interpolating polynomial, $w_k$ is the weight vector of Gauss integration, $\bar{\tau}_k$ indicates the Gauss-Legendre points in the time interval $[0, t_f]$.

## 4 Numerical results and analysis

Configuration parameters of the SV are listed in Table 1. During the supercavitating flight, the cavitation number $\sigma$ is set as constant, $\sigma = 0.03$. The initial motion conditions are set as $[x_0, y_0, V_0, \Theta_0, \theta_0, \omega_0] = [0, -10, 85, 0, 0, 0]$. The initial and terminal pitch constraints are both set as level trim flight [27].

Table 1 **Configuration parameters of supercavitating vehicle**
Tablica 1 **Parametri konfiguracije superkavitirajućeg vozila**

| Parameter | Description | Value & Unit |
|---|---|---|
| $R$ | Radius of vehicle | 0.0508 m |
| $R_N$ | Radius of cavitator | 0.0191 m |
| $L$ | Length of vehicle | 1.8 m |
| $L_{MC}$ | Position of mass center | 1.09 m |
| $l_F$ | Position of fins | 1.44 m |
| $l_F$ | Length of fins | 0.07 m |
| $m$ | Mass | 22.7 kg |

In the vertical plane, based on the dynamic equations, the trim flight condition for the SV is to solve the corresponding controls for the pitch angle $\theta_{trm} = 0$. Based on the configuration parameters, the solved thrust is $F_T = 3.89\,kN$, while the corresponding control angles are $\delta_{N\_trim} = -0.903°$ and $\delta_{F\_trim} = 0.770°$.

With the initial condition listed above, optimal trajectories and controls for minimal diving time manoeuvre to different depth are researched in this paper. In order to achieve the quick optimization, the NLP problem with a small number of Gauss-Legendre nodes ($N = 5$) is solved firstly to calculate the rough solution. Taking the rough solution as initial values, more accurate optimal trajectory and control strategies are obtained with $N = 20$. The optimization process could be finished in several minutes in a PC with CPU clock frequency of 3.3 GHz, which is much faster than other optimal techniques.

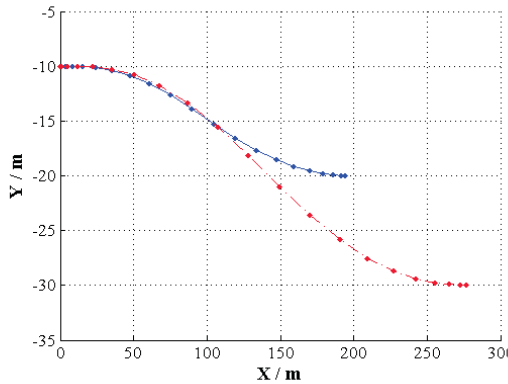



Figure 6 **Optimal trajectories for diving manoeuvre**
Slika 6 **Optimalne putanje za manevre ronjenja**

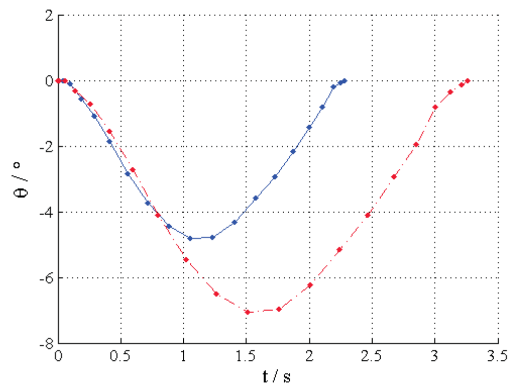
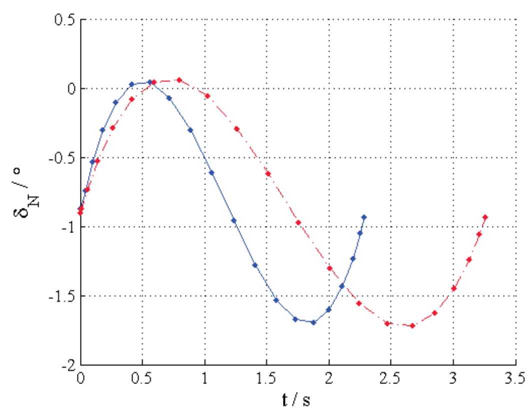



Figure 7 **Time history of flight pitch angle**
Slika 7 **Vremenska povijest kuta nagiba leta**

Figure 8 **Control strategies of the cavitator**
Slika 8 **Strategija kontrole kavitatora**



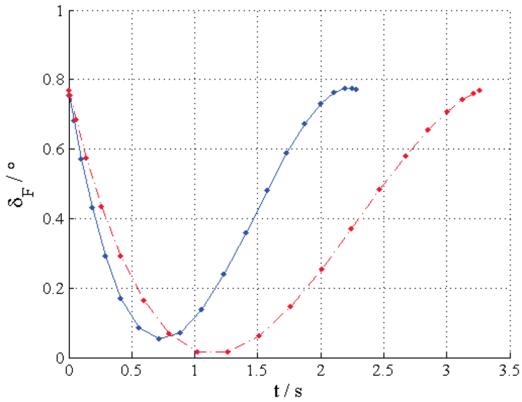



Figure 9 **Control strategies of the fins**
Slika 9 **Strategija kontrole stabilizatora**

The minimal diving time is $t_{m10} = 2.29\,s$ and $t_{m20} = 3.27\,s$ for different diving depth, i.e., $\Delta h = 10\ m$ and 20 $m$, respectively. The corresponding horizontal flight distance is $x_{10} = 193.8\ m$ and $x_{20} = 276.0\ m$. Figure 6 and Figure 7 present the optimal trajectories and the corresponding pitch of the supercavitating flight. For the minimal time diving manoeuvre, result shows that the maximal pitch angle increases with the diving depth. Figure 8 gives the cavitator control strategies in time history. For diving manoeuvres, the cavitator control history is approximated to sinusoid, with the initial and terminal value equal to the trim control angle. Figure 9 shows the control history of immersion fins. The trends of both cavitator and fin control history are similar while diving depth changes.

## 5 Conclusion

This paper presents performed research on optimal control problems for SV to achieve different diving manoeuvres. The dynamic equations and the mathematical model of the trajectory optimization for the supercavitating flight are established. In order to accomplish a quick optimization, the TDGPM algorithm is developed considering the time delay effect of supercavitating flight.

Numerical results of the optimal trajectory and the corresponding control strategies are obtained based on the proposed optimization method. The validity and advantages of the TDGPM are proved. In the vertical plane, the diving manoeuvre of SV requires proper cooperation of both the cavitator and fin controls.

Future researches could be extended to more complex scenarios, e.g. the optimization of 6DOF trajectories and other manoeuvres. The optimization framework and the TDGPM algorithm presented in this research are also suitable for application in other optimal control problems that are also combined with time delay effect.

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