# CALCULATION OF NAVIGATIONAL PARAMETERS IN MERIDIAN, EQUATOR AND PARALLEL SAILING BY MEANS OF RELATIVE COORDINATES AND CORRELATION FACTORS 

## Izračun navigacijskih parametara za plovidbu po meridijanu, ekvatoru i paraleli pomoću relativnih koordinata i korelacijskih faktora

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## Summary

Meridian, equator and parallel sailing are seen as special sailing due to general regularity that can be unequivocally defined by relative coordinates $\Delta \varphi$ and $\Delta \lambda$. Based on the relative coordinates $\Delta \varphi$ and $\Delta \lambda$, and the correlation factors ( $\Psi_{1}, \Psi_{2}, \Psi_{3}, \Psi_{4}$ ), this paper infers equations for calculating the distance a vessel sails across the Meridian, Equator and Parallel, on condition that the Earth is approximated by the sphere of the unit radius $R_{1}$, selected radius $R$ or selected ellipsoid determined by the semi-axes $a$ and $b$.

The detailed method of determining the loxodrome course is explained on the basis of the sign of the corresponding relative coordinate. The result is completely defined navigational parameters for Meridian, Equator and Parallel sailing.

Key words: Meridian, Equator and Parallel sailing. Relative coordinates. Correlation factors.

## Sažetak

Plovidba po meridijanu, ekvatoru i paraleli zbog općih se zakonitosti može smatrati posebnim slučajevima plovidbe koji se mogu jednoznačno definirati pomoću relativnih koordinata $\Delta \varphi$ i $\Delta \lambda . U$ ovom radu, na osnovi relativnih koordinata $\Delta \varphi \quad i \quad \Delta \lambda \quad i$ odgovarajućih novouvedenih korelacijskih faktora ( $\Psi_{1}, \Psi_{2}, \Psi_{3}, \Psi_{4}$ ) koji su egzaktno definirani, izvedene su jednadžbe za izračun udaljenosti koju brod prijeđe pri plovidbi po

[^0]meridijanu, ekvatoru i paraleli pod uvjetom da se oblik Zemlje aproksimira oblikom kugle jediničnog radijusa $R_{1}$ kugle odabranog radijusa $R$ ili oblikom odabranog elipsoida zadanog poluosima a $i$ b. Detaljno je obrazložen i postupak određivanja općeg loksodromskog kursa plovidbe na osnovu predznaka odgovarajuće relativne koordinate, čime su u potpunosti definirani navigacijski parametri za plovidbu po meridijanu, ekvatoru i paraleli.

Ključne riječi: Plovidba po meridijanu, ekvatoru i paraleli. Relativne koordinate. Korelacijski faktori.

## 1. Introduction Uvod

On account of universal laws, Meridian, Equator and Parallel sailing represent specific cases in navigation. From the nautical standpoint there are two basic models in marine surface navigation regarding the running a ship on a given trackline;
1.1. Navigation in an unchangeable i.e. constant course ( $C=$ const.) where the course remains unchanged in time unit $(\Delta \mathrm{C}=0)$. Here three significant cases are possible:

- the course line being uninterruptedly at the same constant angle ( $\alpha=$ const.) intersecting all Earth's meridians on condition that $\alpha$ is not a right angle,
- the course line coinciding with the Earth's meridian, i.e. the angle between the course line and the Earth's meridian is $\alpha=000^{\circ} 00^{\prime} 00^{\prime \prime}$ or $\alpha=180^{\circ} 00^{\prime} 00^{\prime \prime}$,
- the course line coinciding with the equator or the parallel so that the course line and the Earth's meridians intersects at a right angle i.e. $\alpha=090^{\circ} 00^{\prime} 00^{\prime \prime}$ or $\alpha=$ $270^{\circ} 00^{\prime} 00^{\prime \prime}$.

The above-mentioned may be defined as Loxodromic sailing.
1.2. Navigation in a changeable i. e. variable course where the course is changed in time unit as the course line intersects the Earth's meridians at different angles. This method of navigation can also include two significant cases:

- the change of course in a time unit is constant, i.e. $\Delta \mathrm{C}=$ const.
- the change of course in a time unit is variable, i.e. $\Delta \mathrm{C} \neq$ const.

By the above definitions Meridian, Equator and Parallel sailing may be considered as Loxodromic sailing as the conditions given in item 1.1. are fulfilled. In this type of sailing two basic navigational parameters are determined:

- the general loxodromic course ( $\mathrm{C}_{\mathrm{L}}$ ),
- the loxodromic distance between the points of departure and destination $\left(\mathrm{D}_{\mathrm{L}}\right)$.


## 2. Determining the general loxodromic course Određivanje općeg loksodromskog kursa

The general loxodromic course in Meridian, Equator and Parallel sailing can be determined by means of relative coordinates defined as follows:

$$
\begin{align*}
& ( \pm \Delta \varphi)=\left( \pm \varphi_{2}\right)-\left( \pm \varphi_{1}\right) \\
& ( \pm \Delta \lambda)=\left( \pm \lambda_{2}\right)-\left( \pm \lambda_{1}\right) \tag{1}
\end{align*}
$$

Where:
$\varphi_{1}$ - latitude of departure point
$\lambda_{1}$ - longitude of departure point
$\varphi_{2}$ - latitude of destination point
$\lambda_{2}$ - longitude of destination point
( $\varphi_{1}, \varphi_{2}, \lambda_{1}, \lambda_{2}$ ) - absolute geografic coordinates
$(\Delta \varphi, \Delta \lambda)$ - relative geografic coordinates expressed in angular measure and the sign ( $\pm$ )

In Meridian sailing $\Delta \lambda=0$, and the sign $\Delta \varphi$ determines the general loxodromic course. If the sign $\Delta \varphi$ is positive (+) i.e. North, then the general loxodromic course is $C_{L}=000^{\circ} 00^{\prime} 00^{\prime \prime}$; if the sign $\Delta \varphi$ is negative (-) i.e. South, then the general loxodromic course is $C_{L}=$ $180^{\circ} 00^{\prime} 00^{\prime \prime}$.

In the Equator or Parallel sailing $\Delta \varphi=0$, the sign $\Delta \lambda$ determines the general loxodromic course. If the sign $\Delta \lambda$ is positive (+) i.e. East, then the general loxodromic course is $C_{L}=090^{\circ} 00^{\prime} 00^{\prime \prime}$, and if the sign $\Delta \lambda$ is negative $(-)$ i.e. West, then the general loxodromic course is $C_{L}=$ $270^{\circ} 00^{\prime} 00^{\prime \prime}$.

## 3. Calculation of loxodromic distance Izračun loksodromske udaljenosti

In specific sailing cases loxodromic distances between two points can be determined by calculation, and the method of distance calculation will be shown in cases the shape of Earth is approximated to the figure of a sphere of a unit radius $\mathrm{R}_{1}$, to the figure of sphere of selected radius $R$ and the figure of a selected ellipsoid determined by the semi-axes $a$ and $b$.

### 3.1. Meridian sailing Plovidba po meridijanu

From navigational standpoint Meridian sailing should fulfil the following conditions: $\Delta \lambda=0, \Delta \varphi \neq 0$. The loxodromic distance run by the ship in Meridian sailing can be calculated as follows:

### 3.1.1. The earth as a sphere of unit radius $R_{1}$ Zemlja kao kugla jediničnog radijusa $\boldsymbol{R}_{1}$



Figure 1. Section of the Earth as a sphere of unit radius $\mathrm{R}_{1}$ in Meridian sailing
Slika 1. Presjek Zemlje kao kugle jediničnog radijusa R1 pri plovidbi po meridijanu

The loxodromic distance ( $\mathrm{D}_{\text {Lmsp }}$ ) between the points $\left(P_{1}\right)$ and $\left(P_{2}\right)$ expressed in nautical miles can be calculated by means of the following equation:

$$
\begin{equation*}
D_{\mathrm{Lmsp}}=\Psi_{1}\left|\Delta \varphi^{\prime}\right|=\Psi_{1}\left|\left(\varphi_{2}-\varphi_{1}\right)^{\prime}\right| \tag{2}
\end{equation*}
$$

Where:
$\Psi_{1}$ - First correlation factor i.e. the value by which the corresponding relative coordinate is multiplied whose explicitly expressed value is different from zero in order to calculate the loxodromic distance between the point of departure $\left(\mathrm{P}_{1}\right)$ and point of destination $\left(\mathrm{P}_{2}\right)$ in Meridian,

Equator and Parallel sailing if the shape of Earth is approximated to a sphere of unit radius $\mathrm{R}_{1}$.

$$
\begin{equation*}
\Psi_{1}=\frac{R_{1}}{\rho^{\prime}} \tag{3}
\end{equation*}
$$

Radius $\mathrm{R}_{1}$ in angular minutes for the Earth as a unit sphere amounts to:
$\mathrm{R}_{1}=\frac{360^{\circ} 60^{\circ}}{2 \pi}=\frac{180^{\circ} 60^{\circ}}{\pi}$
The value $\rho^{\prime}=\frac{180^{\circ} 60^{\prime}}{\pi}$,
then is $\psi_{1}=\frac{R_{1}}{\rho^{\prime}}=\frac{\left(\frac{180^{\circ} 60^{\prime}}{\pi}\right)}{\left(\frac{180^{\circ} 60^{\prime}}{\pi}\right)}=1$, therefore

$$
\begin{equation*}
D_{\text {Lmsp }}=\psi_{1}\left|\Delta \varphi^{\prime}\right|=\left|\Delta \varphi^{\prime}\right|=\left|\left(\varphi_{2}-\varphi_{1}\right)^{\prime}\right| \tag{4}
\end{equation*}
$$

In this case the loxodromic distance expressed in nautical miles ( $1 \mathrm{M}=1852 \mathrm{~m}$ ) between the points $\left(\mathrm{P}_{1}\right)$ and ( $P_{2}$ ) on the sphere of unit radius $R_{1}$ is equal to the absolute value of angular measure of the relative coordinate $\Delta \varphi$ expressed in angular minutes, because the correlation factor $\Psi_{1}=1$, and the arc length of any of the meridians whose centre angle is $1^{\prime}$ equals the value of 1 nautical mile. The above-mentioned is demonstrated by relation (4).

### 3.1.2. The Earth as a sphere of selected radius $R$ Zemlja kao kugla odabranog radijusa $R$



Figure 2. Section of the Earth as a sphere of selected radius $\mathbf{R}$ in Meridian sailing
Slika 2. Presjek Zemlje kao kugle odabranog radijusa $R$ pri plovidbi po meridijanu

The loxodromic distance ( $\mathrm{D}_{\mathrm{Lmspl}}$ ) between the points $\left(P_{1}\right)$ and ( $P_{2}$ ) expressed in nautical miles can be calculated as follows:

$$
\begin{equation*}
D_{\text {Lmspl }}=\Psi_{2}\left|\Delta \varphi^{\prime}\right|=\Psi_{2}\left|\left(\varphi_{2}-\varphi_{1}\right)^{\prime}\right| \tag{5}
\end{equation*}
$$

Where:
$\Psi_{2}$ - Second correlation factor i.e. the value by which the corresponding relative coordinate is multiplied whose explicitly expressed value is different from zero to calculate the loxodromic distance between the point of departure ( $\mathrm{P}_{1}$ ) and the point of destination $\left(\mathrm{P}_{2}\right)$ in Meridian, Equator and Parallel sailing, if the shape of Earth is approximated to the figure of sphere of selected radius R .

$$
\begin{equation*}
\Psi_{2}=\frac{R}{\rho^{\prime} 1852} \tag{6}
\end{equation*}
$$

Where:
$R=\sqrt[3]{a^{2} b}=6370283 \mathrm{~m}$ - radius of Earth's sphere expressed in metres being of a volume equal to the volume of the ellipsoid by Bessel's figure of Earth's ellipsoid.
a - equatorial semi-axis of ellipsoid, b - meridian semiaxis of ellipsoid
$R=\sqrt{\frac{\pi}{2} \int_{0}^{2} \mathrm{MN} \cos \varphi \mathrm{d} \varphi}=6370290 \mathrm{~m}$ - radius of Earth's sphere expressed in metres having a surface equal to the surface of the Earth's ellipsoid according to Bessel's figures.

If for example, R is selected by Bessel's figures of Earth then $\Psi_{2}=1,00056 \ldots>1$.
3.1.3. The Earth as a selected ellipsoid determined by the semi-axes $a$ and $b$ Zemlja kao odabrani elipsoid zadan poluosima aib


Figure 3. Section of the Earth as a selected ellipsoid with semi-axes $\mathbf{a}$ and $\mathbf{b}$ in Meridian sailing
Slika 3. Presjek Zemlje kao odabranog elipsoida s poluosima a i b pri plovidbi po meridijanu

The loxodromic distance ( $\mathrm{D}_{\text {Lmel }}$ ) between the points $\left(\mathrm{P}_{1}\right)$ and $\left(P_{2}\right)$ is equal to the length of arc meridian i.e.

$$
\begin{equation*}
D_{\text {Lmel }}=\int_{\varphi_{1}}^{\varphi_{2}} M d \varphi=\frac{a\left(1-e^{2}\right)^{\varphi_{2}}}{1852} \int_{\varphi_{1}}\left(1-e^{2} \sin ^{2} \varphi\right)^{-\frac{3}{2}} d \varphi \tag{7}
\end{equation*}
$$

If the expression $\left(1-e^{2} \sin ^{2} \varphi\right)^{-\frac{3}{2}}$ is developed by the binomial model, we obtain:

$$
\begin{align*}
& \left(1-e^{2} \sin ^{2} \varphi\right)^{-\frac{3}{2}}=1+\frac{3}{2} e^{2} \sin ^{2} \varphi+  \tag{8}\\
& \frac{15}{8} e^{4} \sin ^{4} \varphi+\frac{35}{16} e^{6} \sin ^{6} \varphi+\ldots
\end{align*}
$$

Instead of the sine power the multiple angle cosines can be used by the trigonometric models:

$$
\begin{align*}
& \sin ^{2} \varphi=\frac{1}{2}-\frac{1}{2} \cos 2 \varphi \\
& \sin ^{4} \varphi=\frac{3}{8}-\frac{1}{2} \cos 2 \varphi+\frac{1}{8} \cos 4 \varphi  \tag{9}\\
& \sin ^{6} \varphi=\frac{5}{16}-\frac{15}{32} \cos 2 \varphi+\frac{3}{16} \cos 4 \varphi-\frac{1}{32} \cos 6 \varphi
\end{align*}
$$

If expressions (9) are inserted into equation (8), it results in:

$$
\begin{align*}
& \left(1-\mathrm{e}^{2} \sin ^{2} \varphi\right)^{-\frac{3}{2}}=\mathrm{A}-\mathrm{B} \cos 2 \varphi  \tag{10}\\
& +\mathrm{C} \cos 4 \varphi-\mathrm{D} \cos 6 \varphi+\ldots
\end{align*}
$$

Where:

$$
\begin{align*}
& A=1+\frac{3}{4} e^{2}+\frac{45}{64} e^{4}+\frac{350}{512} e^{6}+\ldots \\
& B=\frac{3}{4} e^{2}+\frac{60}{64} e^{4}+\frac{525}{512} e^{6}+\ldots \\
& C=\frac{15}{64} e^{4}+\frac{210}{512} e^{6}+\ldots  \tag{11}\\
& D=\frac{35}{512} e^{6}+\ldots
\end{align*}
$$

If for $e=\sqrt{\frac{a^{2}-b^{2}}{a^{2}}}$ numerical values are inserted e.g.
the Bessel's ellipsoid, we obtain:
$A=1,00503730604855, B=0,00504784924030$,
$C=0,000010563786831, D=0,000000020633322$.

If $A, B, C, D$ defined by relation (11) is inserted into equation (7), we obtain:
$D_{\text {Lmel }}=\frac{a\left(1-e^{2}\right)^{\varphi_{2}}}{1852} \int_{\varphi_{1}}(A-B \cos 2 \varphi+C \cos 4 \varphi-D \cos 6 \varphi+\ldots)($

After integration of the equation (12), there is:
$D_{\text {Lmel }}=\frac{a\left(1-e^{2}\right)}{1852}\left[\begin{array}{l}\frac{A}{\rho^{\circ}}\left(\varphi_{2}-\varphi_{1}\right)^{\circ}-\frac{B}{2}\left(\sin 2 \varphi_{2}-\sin 2 \varphi_{1}\right) \\ +\frac{C}{4}\left(\sin 4 \varphi_{2}-\sin 4 \varphi_{1}\right)-\frac{D}{6}\left(\sin 6 \varphi_{2}-\sin 6 \varphi_{1}\right)+\ldots\end{array}\right]$

By the method demonstrated so far in Meridian sailing it is possible to calculate the loxodromic distance between the latitudes $\varphi_{1} i \varphi_{2}$. expressed in nautical miles.

If en route on a meridian from the equator to the parallel $\varphi$, then $\varphi_{1}=0$, and $\varphi_{2}=\varphi$ may be integrated in the expression (13) thereby following:
$D_{\text {Lmelo }}^{\varphi}=\frac{a\left(1-e^{2}\right)}{1852}\left[\begin{array}{l}\frac{A}{\rho^{\circ}} \varphi^{\circ}-\frac{B}{2} \sin 2 \varphi \\ +\frac{C}{4} \sin 4 \varphi-\frac{D}{6} \sin 6 \varphi+\ldots\end{array}\right]$
where $\rho^{\circ}=\left(\frac{180}{\pi}\right)$

Now the third correlation factor can be defined:
$\Psi_{3}=\frac{a\left(1-\mathrm{e}^{2}\right)}{1852}$
$\Psi_{3}$ - Third correlation factor i.e. the value by which the corresponding relative coordinate is multiplied whose implicitly expressed value is different from zero to calculate the loxodromic distance between the $\left(\mathrm{P}_{1}\right)$ i $\left(\mathrm{P}_{2}\right)$ in Meridian sailing if the shape of the Earth is approximated to the selected ellipsoid determined by the semi-axes a and b.
$D_{\text {Lmel }}=\Psi_{3}\left[\begin{array}{l}\frac{A}{\rho^{\circ}}\left(\varphi_{2}-\varphi_{1}\right)^{\circ}-\frac{B}{2}\left(\sin 2 \varphi_{2}-\sin 2 \varphi_{1}\right) \\ +\frac{C}{4}\left(\sin 4 \varphi_{2}-\sin 4 \varphi_{1}\right)-\frac{D}{6}\left(\sin 6 \varphi_{2}-\sin 6 \varphi_{1}\right)+\ldots\end{array}\right]$

The length of the equator semi-axis a indicated in $\Psi 3$ must be expressed in metres.

### 3.2. Equator sailing Plovidba po ekvatoru

From the navigational standpoint in Equator sailing the following conditions should be fulfilled: $\varphi_{1}=\varphi_{2}=$ $\mathbf{0 0 ^ { \circ }} \mathbf{0 0 ^ { \prime } 0 0 ^ { \prime \prime }}, \Delta \varphi=0, \Delta \lambda \neq 0$. The loxodromic distance run by the ship in Equator sailing may be calculated as follows:

### 3.2.1. The Earth as a sphere of unit radius $R_{1}$ Zemlja kaon kugla jediničnog radijusa $\boldsymbol{R}_{1}$



Figure 4. Section of the Earth as a sphere of unit radius R1 in Equator sailing
Sika 4. Presjek Zemlje kao kugle jediničnog radijusa R1 prim plovidbi po ekvatoru

The loxodromic distance between the points $\left(\mathrm{P}_{1}\right) \mathrm{i}\left(\mathrm{P}_{2}\right)$ can be determined as follows:

$$
\begin{align*}
& \text { DLeqsp }=\Psi 1\left|\Delta \lambda^{\prime}\right|=\Psi 1\left|(\lambda 2-\lambda 1)^{\prime}\right|  \tag{16}\\
& \text { Being } \Psi 1=1 \text {, then DLeqsp }=\left|\Delta \lambda^{\prime}\right| \tag{17}
\end{align*}
$$

In this case as well the loxodromic distance between the points $\left(\mathrm{P}_{1}\right)$ i $\left(\mathrm{P}_{2}\right)$ on a sphere of unit radius $\mathrm{R}_{1}$ expressed in nautical miles is equal to the absolute value of angular measure of the relative coordinate $\Delta \lambda^{\prime}$ expressed in angular minutes being $\Psi_{1}=1$, and the length of arc equator with its corresponding centre angle $1^{\prime}$ equalling the value of 1 nautical mile. This is shown by relation (17).

### 3.2.2. The Earth as a sphere of selected radius $R$ Zemlja kao kugla odabranog radijusa $R$



Figure 5. Section of the Earth as a sphere of selected radius $\mathbf{R}$ in Equator sailing
Sika 5. Presjek Zemlje kao kugle odabranog radijusa $R$ pri plovidbi po ekvatoru

The loxodromic distance between the points $\left(\mathrm{P}_{1}\right) \mathrm{i}\left(\mathrm{P}_{2}\right)$ is equal to:

$$
\begin{equation*}
D_{\text {Leqspl }}=\Psi_{2}\left|\Delta \lambda^{\prime}\right|=\Psi_{2}\left|\left(\lambda_{2}-\lambda_{1}\right)^{\prime}\right| \tag{18}
\end{equation*}
$$

Where $\Psi_{2}$ - Second correlation factor.
3.2.3. The Earth as a selected ellipsoid determined by the semi-axes $a$ and $b$ Zemlja kao odabrani elipsoid zadan poluosima alb


Figure 6. Section of the Earth as a selected ellipsoid with the semi-axes $\mathbf{a}$ and $\mathbf{b}$ in Equator sailing
Slika 6. Presjek Zemlje ko odabranog elipsoida s poluosima a ib mri plovidbi po ekvatoru

The loxodromic distance between the points $\left(\mathrm{P}_{1}\right) \mathrm{i}\left(\mathrm{P}_{2}\right)$ is equal to the length of the equator arc:

$$
\begin{equation*}
\text { DLeqel }=\Psi 4\left|\Delta \lambda^{\prime}\right|=\Psi 4\left|(\lambda 2-\lambda 1)^{\prime}\right| \tag{19}
\end{equation*}
$$

Where:

$$
\begin{align*}
& \Psi_{4}=\frac{a}{\rho^{\prime}\left(1-e^{2} \sin ^{2} \varphi\right)^{\frac{1}{2}} 1852}= \\
& =\frac{a \pi}{10800\left(1-e^{2} \sin ^{2} \varphi\right)^{\frac{1}{2}} 1852} \tag{20}
\end{align*}
$$

$\Psi_{4}$ - Fourth correlation factor i.e. the value by which the absolute value of the relative coordinates $\Delta \lambda^{\prime}$ explicitly expressed in angular minutes is multiplied to calculate the loxodromic distance between the two points in Equator and Parallel sailing if the shape of Earth is approximated to the figure of a selected ellipsoid defined by the semi-axes a and b. In equation (20) the value of the equator semi-axis a should be expressed in metres.

### 3.3. Parallel sailing <br> Plovidba po paraleli

From the navigational standpoint for Parallel sailing it is advisable to fulfil the following conditions: $\varphi_{1}=\varphi_{2} \neq$ $00^{\circ} 00^{\prime} 00^{\prime \prime}, \Delta \varphi=0, \Delta \lambda \neq 0$.

The loxodromic distance run by a ship in Parallel sailing can be calculated as follows:

### 3.3.1. The Earth as a sphere of unit radius $R_{1}$ Zemlja kao kugla jediničnog radijusa $\boldsymbol{R}_{1}$



Figure 7. Section of the Earth as a sphere of unit radius R1 in Parallel sailing
Slika 7. Presjek Zemlje kao kugle jediničnog radijusa R1 pri plovidbi po paraleli

The loxodromic distance between the points $\left(\mathrm{P}_{1}\right) \mathrm{i}\left(\mathrm{P}_{2}\right)$ is equal to the length of arc parallel i.e.:
$D_{\text {Lpsp }}=\Psi_{1}\left|\Delta \lambda^{\prime}\right| \cos \varphi$, being $\Psi_{1}=1$, that is

$$
\begin{equation*}
D_{\text {Lpsp }}=\left|\Delta \lambda^{\prime}\right| \cos \varphi=\left|\left(\lambda_{2}-\lambda_{1}\right)^{\prime}\right| \cos \varphi \tag{21}
\end{equation*}
$$

3.3.2. The Earth as a sphere of selected radius $\mathbf{R}$ Zemlja kao kugla odabranog radijusa $R$


Figure 8. Section of the Earth as a sphere of selected radius $\mathbf{R}$ in Parallel sailing
Slika 8. Presjek Zemlje kao kugle odabranog radijusa $R$ pri plovidbi po paraleli

The loxodromic distance between $\left(\mathrm{P}_{1}\right)$ i $\left(\mathrm{P}_{2}\right)$ is equal to the length of arc parallel:

$$
\begin{equation*}
D_{\text {Lpspl }}=\Psi_{2}\left|\Delta \lambda^{\prime}\right| \cos \varphi=\frac{R}{\rho^{\prime} 1852}\left|\left(\lambda_{2}-\lambda_{1}\right)^{\prime}\right| \cos \varphi \tag{22}
\end{equation*}
$$

3.3.3. The Earth as a selected ellipsoid determined by the semi-axes $a$ and $b$ Zemlja kao odabrani elipsoid zadan poluosimaaib


Figure 9. Section of the Earth as a selected ellipsoid with semi-axes $a$ and $b$ in Parallel sailing
Slika 9. Presjek Zemlje kao odabranog elipsoida s poluosima a i b pri plovidbi po paraleli

The loxodromic distance between the points $\left(\mathrm{P}_{1}\right)$ and $\left(P_{2}\right)$ is equal to the length of parallel arc:

$$
\begin{equation*}
\mathrm{D}_{\text {Lpel }}=\Psi_{4}\left|\Delta \lambda^{\prime}\right| \cos \varphi \tag{23}
\end{equation*}
$$

where $\Psi_{4}$ - Fourth correlation factor.

## 4. Illustrative examples Demonstracijski primjeri

## Example 1.

Primjer 1.
En route on the meridian from point $P_{1}\left(\varphi_{1}=35^{\circ} 10^{\prime}\right.$ $\left.\mathrm{N}, \lambda_{1}=140^{\circ} 00^{\prime} \mathrm{E}\right)$ to point $\mathrm{P}_{2}\left(\varphi_{2}=03^{\circ} 05^{\prime} \mathrm{S}\right.$, $\lambda_{2}=140^{\circ} 00^{\prime} \mathrm{E}$ ). Calculation of navigational parameters:
1.1. Earth as a sphere of unit radius $\mathrm{R}_{1}$
$C_{L}=180^{\circ} 00^{\prime} 00^{\prime \prime}, D_{\text {Lmsp }}=2295$ nautical miles
1.2. Earth as a sphere of selected radius
$R=6370290 \mathrm{~m}$
$C_{L}=180^{\circ} 00^{\prime} 00^{\prime \prime}, D_{\text {Lmspl }}=2296,2915 \ldots$ nautical miles
1.3. Earth as Bessel's figures of ellipsoid
$C_{L}=180^{\circ} 00^{\prime} 00^{\prime \prime}, D_{\text {Lmel }}=2313,071 \ldots$ nautical miles

## Example 2.

## Primjer 2.

En route on the parallel from point $\mathrm{P}_{1}\left(\varphi_{1}=41^{\circ} 00^{\prime} \mathrm{S}\right.$, $\left.\lambda_{1}=174^{\circ} 47^{\prime} \mathrm{E}\right)$ to point $\mathrm{P}_{2}\left(\varphi_{2}=41^{\circ} 00^{\prime} \mathrm{S}, \lambda_{2}=\right.$ $\left.072^{\circ} 10^{\prime} \mathrm{W}\right)$. Calculation of navigational parameters:
1.1. Earth as a sphere of unit radius $R_{1}$ $C_{L}=090^{\circ} 00^{\prime} 00^{\prime \prime}, D_{\text {Lpsp }}=5119,195 \ldots$ nautical miles
1.2. Earth as a sphere of selected radius
$R=6370 \quad 290 \mathrm{~m}$
$C_{L}=090^{\circ} 00^{\prime} 00^{\prime \prime}, D_{\text {Lpspl }}=5122,076 \ldots$ nautical miles
1.3. Earth as Bessel's figures of ellipsoid
$C_{L}=090^{\circ} 00^{\prime} 00^{\prime \prime}, D_{\text {Lpel }}=5133,171 \ldots$ nautical miles

## 5. Conclusion Zaključak

The basic navigational parameters that define Meridian, Equator and Parallel sailing can be calculated by means of relative coordinates and the corresponding correlation factors. Because of universal specifications Meridian, Equator and Parallel sailing may be considered as peculiar cases of navigation governed however by general laws:

- the value of one relative coordinate is always equal to zero and the value of the other relative coordinate is always different from zero,
- generally speaking, in all cases of that kind of sailing concerning a time unit the course is not changed ( $\Delta \mathrm{C}=$ 0 ), i.e. sailing occurs at a constant course ( $C=$ const.) what may be characterised as Loxodromic sailing,
- universally taken, the sign ( $\pm$ ) of the corresponding relative coordinate of a value different from zero determines the direction of sailing i.e. the general loxodromic course ( $\mathrm{C}_{\mathrm{L}}$ ),
- the loxodromic distance $\left(\mathrm{D}_{\mathrm{L}}\right)$ run by the ship in Meridian and Equator sailing can be calculated so that
the absolute value of the corresponding relative coordinate whose explicitly or implicitly expressed value is different from zero are multiplied by one of the corresponding correlation factors $\Psi_{1}, \Psi_{2}, \Psi_{3}, \Psi_{4}$ defined by a selected shape of Earth and the method of sailing (by Meridian, by Equator or by Parallel). In case of Parallel sailing to calculate the loxodromic distance it is necessary to multiply the explicitly expressed absolute value of the relative coordinate $\Delta \lambda^{\prime}$ by the correlation factor $\Psi_{4}$ and by the cosine of latitude along which the ship is sailing $(\cos \varphi)$. The value of the First correlation factor $\Psi_{1}=\frac{R_{1}}{\rho^{\prime}}=1$ as the shape of Earth is approximated to a sphere of unit radius $R_{1}$, while the value of the other correlation factors is calculated by means of the following equations: $\quad \Psi_{2}=\frac{R}{\rho^{\prime} 1852}, \quad \Psi_{3}=\frac{\mathrm{a}\left(1-\mathrm{e}^{2}\right)}{1852} \quad$ and

$$
\Psi_{4}=\frac{a \pi}{10800\left(1-\mathrm{e}^{2} \sin ^{2} \varphi\right)^{\frac{1}{2}} 1852}
$$

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[^0]:    "Dr. sc. Serđo Kos, Izvanredni profesor, Pomorski fakultet
    Sveučilišta u Rijeci, Studentska 2, 51000 Rijeka,
    E-mail: skos@brod.pfri.hr

