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STRUCTURAL STIFFNESS OPTIMIZATION WITH RESPECT TO VIBRATION RESPONSE

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Summary

This paper presents a novel technique for structural stiffness optimization with respect to vibration response. Assuming the regularity of the structure mass matrix and the asymptotic stability of the state space system representing the structure, optimization criteria are defined in terms of the H_∞ norm of the system. In doing so, a special structure of system matrices is utilized to eliminate some of optimization variables. To overcome the high computational cost of the algorithm and make it applicable for optimization of large-scale structures, a novel reduced-order optimization algorithm is proposed. A numerical example which clearly illustrates the applicability and efficiency of the proposed optimization procedure is presented.

Key words: *vibration control, nonlinear optimization, bearing stiffness*

1. Introduction

High demands in modern structural engineering, notably higher demands for reliability, durability and safety, decreased noise, high power/weight ratio, longer operational life, higher operating speeds, and so on, have resulted in higher importance of vibration attenuation strategies. If high performance is needed, active vibration control comprising sensors, controllers and actuators may be utilized [1,2,3,4,5]. The use of passive vibration control, however, may be preferred in many instances due to its effectiveness, low cost, reliability and robustness. Passive vibration attenuation encompasses a wide variety of methods which, by the use of passive devices, suppress structural vibrations and noise [6,7,3]. We may broadly classify such techniques into damping, isolation and stiffening, although numerous other (often non-exclusive) classifications are possible. Isolation and stiffening usually involve a sort of structural element stiffness optimization in order to prevent vibration propagation or to shift the structure resonant frequency beyond the excitation frequency band. Recently, an increasing interest in posing the problem of structural vibration optimization as an optimal control problem has been shown – see for instance [8,9]. Such an approach quantifies the structure vibration response in terms of suitably chosen system norms, and the resulting optimal control problem is subsequently solved using numerical techniques.

The structural stiffness optimization framework proposed in this paper falls into this broad category of control-oriented methods, i.e. it is based on H_∞ optimality condition for state-space systems. It is the modification of the algorithm presented in [10], originally conceived as an algorithm for simultaneous optimization of mass, damping and stiffness parameters for multiple tuned mass damper devices. In contrast to the original algorithm, the proposed algorithm assumes the regularity of the structure mass matrix. A potential drawback of the algorithm is its increased computational cost, which is a common issue associated with control-oriented, as well as some other structural optimization methods. To overcome this drawback, a reduced-order optimization technique is proposed as well, thus adapting the original optimization framework to optimization for large-scale structures.

In this paper, the following notation is used. Let R denote the set of real numbers and $\mathbf{I}$ the identity matrix. For a matrix $\mathbf{A}$, $\mathbf{A}^{\mathrm{T}}$, $\mathbf{A}^{*}$ and $\sigma_{\max}(\mathbf{A})$ denote its transpose, conjugate transpose and maximum singular value (spectral norm), respectively. We define $\mathrm{He}(\mathbf{A})$ as an abbreviation for $\mathbf{A}+\mathbf{A}^{\mathrm{T}}$. For a vector $\mathbf{v}$, $\|\mathbf{v}\|$ denotes its Euclidean norm. We use $\mathbf{A}>\mathbf{B}$ ($\mathbf{A}\geq\mathbf{B}$) and $\mathbf{A}<\mathbf{B}$ ($\mathbf{A}\leq\mathbf{B}$) to denote, respectively, positive and negative (semi)definite ordering of symmetric matrices $\mathbf{A}$ and $\mathbf{B}$. A space of all signals $\mathbf{w}(t)$ such that $\int_0^{\infty}\|\mathbf{w}(t)\|^2\,\mathrm{d}t<\infty$ is denoted $L[0,\infty)$.

2. Optimization framework

Consider the following second order linear time invariant system that represents the structure

$$\begin{aligned}&\mathbf{M}\ddot{\mathbf{y}}(t)+\mathbf{D}\dot{\mathbf{y}}(t)+\mathbf{S}(\mathbf{s})\mathbf{y}(t)=\mathbf{B}_1\mathbf{w}(t),\\&\mathbf{z}(t)=\mathbf{C}_1\dot{\mathbf{y}}(t)+\mathbf{C}_2\mathbf{y}(t),\end{aligned}\tag{1}$$

where $\mathbf{M}\in\mathrm{R}^{q\times q}$, $\mathbf{D}\in\mathrm{R}^{q\times q}$ and $\mathbf{S}(\mathbf{s})\in\mathrm{R}^{q\times q}$ are the mass, damping and stiffness matrices, respectively, $\mathbf{B}_1\in\mathrm{R}^{q\times m}$ is the input matrix and $\mathbf{C}_1\in\mathrm{R}^{p\times q}$ and $\mathbf{C}_2\in\mathrm{R}^{p\times q}$ are the velocity and displacement output matrices, respectively. Time-dependent vectors $\mathbf{y}(t)$, $\dot{\mathbf{y}}(t)$, $\ddot{\mathbf{y}}(t)\in\mathrm{R}^{q}$, $\mathbf{w}(t)\in\mathrm{R}^{m}$ and $\mathbf{z}(t)\in\mathrm{R}^{p}$ are the displacement, velocity, acceleration, input and output vectors, respectively. The input vector is the force or displacement excitation, and we assume that it is steady-state, periodic and that $\mathbf{w}(t)\in L[0,\infty)$.

Structure stiffness matrix is assumed to be the affine function of stiffness parameters $\mathbf{s}=\{s_i\mid i=1,\ldots,l\}$, as follows:

$$\mathbf{S}(\mathbf{s})=\mathbf{S}_0+\sum_{i=1}^{l}s_i\mathbf{S}_i\,.\tag{2}$$

In order to accommodate actual stiffness parameters constraints due to design, technology and other requirements, we introduce The following general equality and inequality constraints of stiffness parameters, imposed by design, technology and other requirements, are introduced:

$$\begin{aligned}&\mathbf{h}(\mathbf{s})=0,\\&\mathbf{g}(\mathbf{s})\leq 0.\end{aligned}\tag{3}$$

Obviously, such constraints define a feasible set for stiffness parameters, as well as for stiffness matrices. Furthermore, we assume that $\mathbf{M}>\mathbf{0}$ and $\mathbf{S}(\mathbf{s})\geq\mathbf{0}$ for all $\mathbf{s}$ that satisfy the constraints (3).

The system (1) may be rewritten as a state space system

$$\begin{aligned}&\dot{\mathbf{x}}(t)=\mathbf{A}(\mathbf{s})\mathbf{x}(t)+\mathbf{B}\mathbf{w}(t),\\&\mathbf{z}(t)=\mathbf{C}\mathbf{x}(t),\end{aligned}\tag{4}$$

where

$$\mathbf{A}(\mathbf{s}) = \begin{pmatrix} -\mathbf{M}^{-1}\mathbf{D} & -\mathbf{M}^{-1}\mathbf{S}(\mathbf{s}) \\ \mathbf{I} & \mathbf{0} \end{pmatrix},\ \mathbf{B} = \begin{pmatrix} \mathbf{M}^{-1}\mathbf{B}_1 \\ \mathbf{0} \end{pmatrix},\ \mathbf{C} = \begin{pmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{pmatrix}, \tag{5}$$

are the system state-space matrices of appropriate dimensions, and

$$\mathbf{x}(t) = \begin{pmatrix} \dot{\mathbf{y}}(t)^{\mathrm{T}} & \mathbf{y}(t)^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}} \tag{6}$$

is the system state vector comprised of velocity and displacement vectors.

Assume that the system (1) is asymptotically stable for all $\mathbf{s}$ that satisfy the constraints (3), i.e. all eigenvalues of $\mathbf{A}(\mathbf{s})$ lie within the open left half of the complex plane. This implies that $\mathbf{z}(t) \in L[0,\infty)$ for all $\mathbf{w}(t) \in L[0,\infty)$. Furthermore, the stiffness parameter optimization problem may be considered as the H_∞ optimal control problem, namely to find $\mathbf{s}$ that minimizes a real scalar γ such that input and output signals of the system (1) satisfy

$$\int_{t=0}^{\infty} \left\| \mathbf{z}(t) \right\|^2 \mathrm{d}t < \gamma^2 \int_{t=0}^{\infty} \left\| \mathbf{w}(t) \right\|^2 \mathrm{d}t \tag{7}$$

for all $\mathbf{w}(t) \in L[0,\infty)$.

Frequency domain equivalent of the inequality (7), referred to as the bounded real lemma (BRL), is the system H_∞ norm condition

$$\left\| \mathbf{G} \right\|_\infty = \sup_{\omega \in \mathrm{R} \cup \infty} \sigma_{\max}(\mathbf{G}(\mathrm{i}\omega)) < \gamma, \tag{8}$$

where $\mathbf{G}(\mathrm{i}\omega) = \mathbf{C}(\mathrm{i}\omega\mathbf{I} - \mathbf{A}(\mathbf{s}))^{-1}\mathbf{B}$ is the system frequency response. Equivalently, (8) can be written as the matrix inequality

$$\begin{pmatrix} \mathbf{I} \\ \mathbf{G}(\mathrm{i}\omega) \end{pmatrix}^{*} \begin{pmatrix} -\gamma^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{G}(\mathrm{i}\omega) \end{pmatrix} < \mathbf{0} \tag{9}$$

for all $\omega \in \mathrm{R} \cup \infty$.

The scalar γ essentially quantifies the worst-case gain of the system, or in other words, the largest ratio of Euclidean norms of output and input signal amplitudes for all steady-state sinusoidal input/output signals and all frequencies.

2.1 Dissipativity inequalities and optimization criterion

Kalman-Yakubovic-Popov lemma is the fundamental result in dynamical systems theory that establishes equivalence between frequency domain inequality (9) and a linear matrix inequality (LMI) for the system state space realization [11]. Thus, the condition for the bounded realness of the system (1) may be expressed in terms of LMI involving system matrices (5), rather than as infinitely many inequalities (9) parametrized by ω, as follows.

Condition (9) holds true if and only if there exists a matrix $\mathbf{X} = \mathbf{X}^{\mathrm{T}} \in \mathrm{R}^{2q \times 2q}$ that satisfies the following LMI:

$$\begin{pmatrix} \mathbf{A}(\mathbf{s})^{\mathrm{T}}\mathbf{X} + \mathbf{X}\mathbf{A}(\mathbf{s}) & \mathbf{X}\mathbf{B} & \mathbf{C}^{\mathrm{T}} \\ \mathbf{B}^{\mathrm{T}}\mathbf{X} & -\gamma\mathbf{I} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} & -\gamma\mathbf{I} \end{pmatrix} < \mathbf{0} \tag{10}$$

Due to the fact that we are dealing with the second order system, which yields the special structure of matrices (5), some of the variables in $\mathbf{X}$ may be eliminated. We divide $\mathbf{X}$ according to the block structure of $\mathbf{A}(\mathbf{s})$ into

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 \\ \mathbf{X}_2^{\mathbf{T}} & \mathbf{X}_4 \end{pmatrix} \in R^{2q \times 2q}, \tag{11}$$

and apply some LMI transformations (for details, please refer to [10]) to eliminate the blocks containing $\mathbf{X}_4$. This results in the following LMIs:

$$\begin{pmatrix} \mathrm{He}\left(-\mathbf{X}_1\mathbf{M}^{-1}\mathbf{D} + \mathbf{X}_2\right) & \mathbf{X}_1\mathbf{M}^{-1}\mathbf{B}_1 & \mathbf{C}_1^{\mathrm{T}} \\ (\mathbf{M}^{-1}\mathbf{B}_1)^{\mathbf{T}}\mathbf{X}_1 & -\gamma\mathbf{I} & \mathbf{0} \\ \mathbf{C}_1 & \mathbf{0} & -\gamma\mathbf{I} \end{pmatrix} < \mathbf{0} \tag{12}$$

$$\begin{pmatrix} \mathrm{He}\left(-\mathbf{S}(\mathbf{s})\mathbf{X}_2\right) & \mathbf{X}_2^{\mathrm{T}}\mathbf{M}^{-1}\mathbf{B}_1 & \mathbf{C}_2^{\mathrm{T}} \\ (\mathbf{M}^{-1}\mathbf{B}_1)^{\mathrm{T}}\mathbf{X}_2 & -\gamma\mathbf{I} & \mathbf{0} \\ \mathbf{C}_2 & \mathbf{0} & -\gamma\mathbf{I} \end{pmatrix} < \mathbf{0} \tag{13}$$

Finally, the H_∞ optimal control problem may be formulated as

$$\min_{\mathbf{s},\mathbf{X}_1,\mathbf{X}_2} \gamma \tag{14}$$

such that

1. stiffness parameters $\mathbf{s}$ satisfy the constraints (3),
2. there exist $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}}, \mathbf{X}_2 \in \mathrm{R}^{q \times q}$ such that (12) and (13) hold true.

2.1.1 Numerical optimization procedure

Due to the fact that both $\mathbf{S}(\mathbf{s})$ and $\mathbf{X}_2$ in the top left block of (13) are variables, inequality (13) is a bilinear matrix inequality (BMI), which renders (14) a nonconvex optimization problem. Rather than applying a numerically very expensive global optimization procedure for tackling such problem, we opt for a local optimization using the readily available BMI optimization software [12]. Obviously, such a local optimization procedure yields a result that depends on the initial guess, as well as on the constraints (3).

The motivation for such a choice, apart from avoiding extensive numerical calculations, is that a good initial guess as to stiffness parameters is usually available – for example, initial parameters may be tuned to some sub-optimal configuration by means of some readily-available technique, or we may be dealing with some pre-existing sub-optimal design that needs further optimization. Additionally, stiffness parameters may be constrained to some small feasible set due to, for example, design requirement, in which case the local optimization hopefully results in finding the global optimum.

3. Reduced-order optimization for large-scale structures

Bounded realness condition, expressed in terms of inequalities (12) and (13), imposes the existence of matrices $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}}, \mathbf{X}_2 \in \mathrm{R}^{q \times q}$, whose dimensions q are equal to the dimensions of the mass, damping and stiffness matrices. Thus, additional $2q^2 + q$ variables are introduced. Furthermore, although $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{S}(\mathbf{s})$ may be sparse, there is no guarantee that $\mathbf{X}_1$ and $\mathbf{X}_2$ will be sparse as well – in fact, they are almost always dense. Obviously, for large-scale case where $q = 10^3$ or more, an optimization criteria based on inequalities (12) and (13) would be prohibitive from the computational point of view. Instead, we propose the following reduced-order optimization procedure:

1. Determine the initial stiffness parameters $\tilde{\mathbf{s}}$ that satisfy constraints (3).
2. For such constant $\tilde{\mathbf{s}}$, calculate a matrix $\mathbf{V} \equiv \mathbf{V}(\tilde{\mathbf{s}}) = (\mathbf{v}_1 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_r) \in R^{q \times r}$, such that $r << q$ and $\mathbf{v}_i \mid i = 1, \ldots, r$ are generalized eigenvectors of the matrix pair $(\mathbf{M}, \mathbf{S}(\tilde{\mathbf{s}}))$ that represent the structure critical vibration modes.
3. Apply a projection procedure to obtain reduced system matrices, as follows:

$$\mathbf{M}_r = \mathbf{V}^{\mathrm{T}}\mathbf{M}\mathbf{V}\,,\ \mathbf{D}_r = \mathbf{V}^{\mathrm{T}}\mathbf{D}\mathbf{V}\,,\ \mathbf{S}_r = \mathbf{V}^{\mathrm{T}}\mathbf{S}_0\mathbf{V} + \sum_{i=1}^{l}\mathbf{V}^{\mathrm{T}}\mathbf{S}_i\mathbf{V}\,, \mathbf{B}_{1r} = \mathbf{V}^{\mathrm{T}}\mathbf{B}_1\ \ \mathbf{C}_{1r} = \mathbf{C}_1\mathbf{V}\,,$$
$$\mathbf{C}_{2r} = \mathbf{C}_2\mathbf{V}\,. \tag{15}$$

4. Find a solution $\hat{\mathbf{s}}$ for the optimization problem (14) such that:
   a. stiffness parameters $\hat{\mathbf{s}}$ satisfy constraints (3),
   b. there exist $\mathbf{X}_{1r} = \mathbf{X}_{1r}^{\mathrm{T}}, \mathbf{X}_{2r} \in \mathrm{R}^{r \times r}$ such that (12) and (13) hold true for reduced system matrices (14).

Note that the matrix $\mathbf{V}$, which normally depends on the stiffness parameters $\mathbf{s}$, is calculated for some initial $\tilde{\mathbf{s}}$ and kept constant throughout the rest of the optimization procedure. This brings us to the crucial requirement for the proposed procedure: the matrix $\mathbf{V}$ does not depend on the parameters $\mathbf{s}$ significantly. More accurately, columns of $\mathbf{V}(\tilde{\mathbf{s}})$ span a subspace that is a sufficient approximation of the column subspace of $\mathbf{V}(\hat{\mathbf{s}})$. This requirement, although not valid for the most general case, appears to be fulfilled for the vast majority of stiffness optimization problems we have encountered. We further clarify this, as follows.

Let $\delta\mathbf{s} = \tilde{\mathbf{s}} - \hat{\mathbf{s}}$ denote the differences in initial and optimal stiffness parameters, respectively, and assume that $\delta\mathbf{s} = \tilde{\mathbf{s}} - \hat{\mathbf{s}}$ is sufficiently small, i.e. the initial and optimal stiffness parameters are sufficiently close and/or constraints (3) keep the parameters within some sufficiently small set. Consequently, the columns of $\mathbf{V}(\hat{\mathbf{s}}) = (\hat{\mathbf{v}}_1 \quad \hat{\mathbf{v}}_2 \quad \cdots \quad \hat{\mathbf{v}}_r)$ may be viewed as perturbed generalized eigenvectors

$$\hat{\mathbf{v}}_i = \sum_{j=1, j \neq i}^{q} \frac{\mathbf{v}_j^{\,T}(\delta\mathbf{S})\mathbf{v}_i}{\lambda_i - \lambda_j}\mathbf{v}_j\,, \tag{16}$$

where $\delta\mathbf{S} = \mathbf{S}(\tilde{\mathbf{s}}) - \mathbf{S}(\hat{\mathbf{s}})$ is the stiffness matrix perturbation due to $\delta\mathbf{s}$, and $\lambda_i \mid i = 1, \ldots, q$ are the generalized eigenvalues that correspond to the generalized eigenvectors $\mathbf{v}_i \mid i = 1, \ldots, q$, as detailed in [13]. According to (16), columns of $\mathbf{V}(\tilde{\mathbf{s}})$ and $\mathbf{V}(\hat{\mathbf{s}})$ span the same subspace if the summation index q in (16) is replaced by r, or in other words, if the influence of the generalized eigenvectors $\mathbf{v}_j \mid j = r+1, \ldots, q$ on the perturbed generalized eigenvectors $\hat{\mathbf{v}}_i \mid i = 1, \ldots, q$ is neglected. Such influence is quantified by the constants

$$\varepsilon_j = \frac{\mathbf{v}_j^{\,T}(\delta\mathbf{S})\mathbf{v}_i}{\lambda_i - \lambda_j}\,, \tag{17}$$

for $j = r+1, \ldots, q$, which are small if $\delta\mathbf{S}$ is small and $\lambda_i - \lambda_j$ is large, i.e. for the generalized eigenvalue/eigenvector pairs that are further away from the eigenvector/eigenvalue pairs that represent structure critical vibration modes.

Therefore, the assumption that $\mathbf{V}(\mathbf{s})$ is (nearly) constant throughout the optimization procedure may be interpreted as follows: changes in stiffness parameters do not cause significant contribution of higher vibration forms to critical vibration forms of the system. This may be verified a posteriori by evaluating constants (17), or by checking the distance between $\mathbf{V}(\tilde{\mathbf{s}})$ and $\mathbf{V}(\hat{\mathbf{s}})$ by an appropriate measure, for example by calculating the maximum singular value $\sigma_{\max}(\mathbf{V}(\tilde{\mathbf{s}})-\mathbf{V}(\hat{\mathbf{s}}))$. If such a measure is significant, i.e. if $\mathbf{V}(\tilde{\mathbf{s}})$ and $\mathbf{V}(\hat{\mathbf{s}})$ are substantially different, one may simply choose a larger number of vibration forms for the order reduction. Another alternative, which is the research in progress, is a sort of iterative procedure that consists of several reduction and optimization sequences.

4. Numerical example

As an illustrative example of the applicability and efficiency of the proposed optimization framework, the following problem is studied. Consider a power plant comprising of a turbine and a generator connected by a shaft depicted in Fig. 1. The shaft is supported by two bearing blocks, referred to bearing 1 and bearing 2., with radial stiffness parameters s_1 and s_2, respectively. Note that we have adopted a rather simple bearing model, reducing bearing block properties to a single parameter. This parameter, i.e. radial stiffness, will serve as the optimization variable. The parameters for the power plant are: shaft lengths a = 1938 mm, b = 7000 mm, c = 1310 mm, shaft diameter D = 900 mm, turbine mass m_T = 64000 kg, turbine moments of inertia I_{Tx} = 68000 kg m^2, I_{Tz} = 34000 kg m^2, generator mass m_G = 230000 kg, generator moments of inertia $I_{Gx} = 2\cdot 10^6$ kg m^2, $I_{Gz} = 10^6$ kg m^2, bearing stiffnesses $s_1 = 0.6667\cdot 10^9$ N m^{-1}, $s_2 = 0.6667\cdot 10^9$ N m^{-1}. Nominal power plant rotational speed is 187.5 revolutions per minute.

Vibrations of the plant are excited by two harmonic forces $f_1(t) = f_{10}\sin\omega t$, referred to as input 1, and $f_2(t) = f_{20}\sin\omega t$, referred to as input 2, acting perpendicular to the shaft at the turbine (input 1) and the generator (input 2). For such a vibration model, two outputs are defined as well: vibration displacements at the turbine and generator are referred to as output 1 and output 2, respectively.

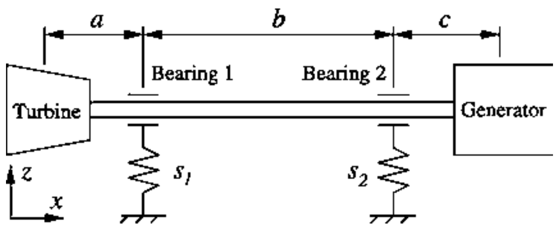

Fig. 1 Power plant vibration model

For simulation and optimization purposes, a finite element model comprising 10 shear deformable beam elements is used to model the shaft. The turbine and the generator are considered to be discrete mass/inertia elements. Shaft material properties are as follows: modulus of elasticity E = 210 GPa, mass density ρ = 7850 kg m^{-3}, the Poisson coefficient ν = 0.3. Damping is proportional to the Rayleigh damping coefficients α = 0.8319, $\beta = 4.2716\cdot 10^{-4}$, which are determined in the way that the modal damping ratios are 2% for the first three vibration modes.

Frequency response of such a model is shown as a dashed line in Fig. 2. Modal analysis of the structure is performed as well, and the results for modal frequencies and modal damping ratios for the first four vibration modes are presented in Table 1. Based on such results, we have identified the first three vibration modes as critical, i.e. the most contributory factors to the system vibration response. Therefore, we create a reduced order model using the projection matrices comprised of eigenvectors for the first three vibration modes, as described in section 3.

Table 1 Power plant natural frequencies and modal damping ratios

Mode	1	2	3	4
Frequency, Hz	4.85	8.78	10.62	1846
Mod. damp., %	2.01	1.93	2.05	2.84

In order to attenuate forced vibrations, we optimize turbine bearing stiffness parameters s_1 and s_2, taking into the account the following constraints:

$$\begin{aligned} &0.3333\cdot 10^9\,\mathrm{Nm}^{-1} \le s_1 \le 1\cdot 10^9\,\mathrm{Nm}^{-1}, \\ &0.3333\cdot 10^9\,\mathrm{Nm}^{-1} \le s_2 \le 1\cdot 10^9\,\mathrm{Nm}^{-1}. \end{aligned}, \quad (18)$$

In other words, the allowed range of bearing stiffness parameters is between 50 % and 200 % of their initial values.

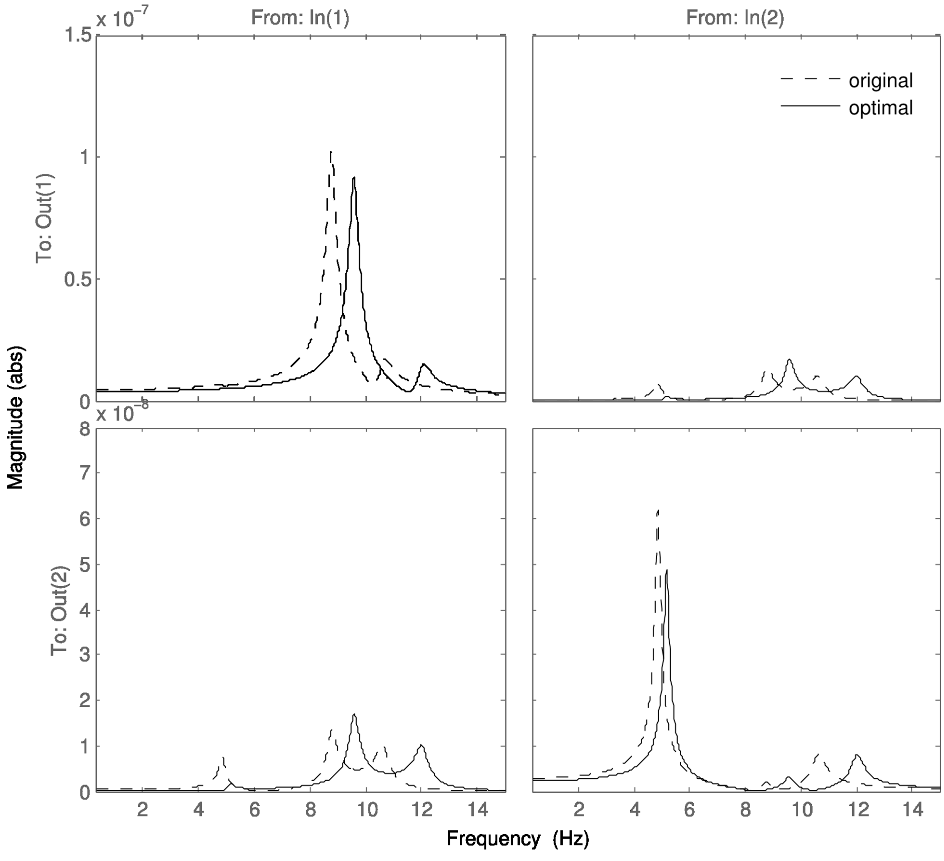

Fig. 2 Frequency response for theoriginal and the optimized power plant vibration model

After optimization, the following bearing stiffness parameters are obtained: s_1 = $0.93225\cdot 10^9$ N m^{-1}, $s_2 = 0.93131\cdot 10^9$ N m^{-1}. A frequency response for the optimized model is shown as a solid line in Fig. 2. As a result of optimization, the peak frequency response has been reduced from $10.2\cdot 10^{-8}$ to $9.13\cdot 10^{-8}$, or, in other words, by 10.49 %, for turbine vibrations (output 1) due to turbine excitation (input 1). This peak frequency response corresponds to the second vibration mode, which has shifted from 8.78 Hz to 9,57 Hz due to stiffer optimal bearings. The frequency response for generator vibrations (output 2) due to generator excitation (input 2) has been reduced from $6.11\cdot 10^{-8}$ to $4.89\cdot 10^{-8}$, or, in other words, by 19.97 %. This corresponds to the first vibration mode, which has shifted from 4.85 Hz to 5.17 Hz. The frequency response for turbine vibrations (output 1) due to generator excitation (input 2), as well as for generator vibrations (output 2) due to turbine excitation (input 1), has been reduced as well.

5. Conclusions

This paper presents a structural stiffness optimization framework based on the H_∞ optimality condition. The proposed approach assumes a constant and regular structure mass matrix, as well as asymptotic stability of the structural system for all feasible stiffness parameters. This may be guaranteed a priori for the majority of structural vibration problems. The main idea behind the proposed technique is local optimization, and consequently, its results are significantly influenced by the initial guess, constraints of stiffness parameters, as well as by the specific optimization problem. We do not consider the local optimization to be a serious drawback since a good initial guess for stiffness parameters is often available. To overcome the high computational cost of the algorithm and make it applicable for optimization of large-scale structures, we propose a novel reduced-order optimization algorithm. It comprises a modal projection of parameterized system matrices, and it is designed assuming small sensitivity of the subspace that represents critical vibration modes with respect to changes in stiffness parameters throughout the optimization. Note that any other reduction technique may be used instead, as long as it satisfies the condition of assumed small sensitivity of the subspace and preserves the structure of the system matrices.

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