MULTIPLE CRITERA METHODS WITH FOCUS ON ANALYTIC HIERARCHY PROCESS AND GROUP DECISION MAKING

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Abstract

Managing natural resources is a group multiple criteria decision making problem. In this paper the analytic hierarchy process is the chosen method for handling the natural resource problems. The one decision maker problem is discussed and, three methods: the eigenvector method, data envelopment analysis method, and logarithmic least squares method are presented for the derivation of the priority vector. Further, the group analytic hierarchy process is discussed and six methods for the aggregation of individual judgments or priorities: weighted arithmetic mean method, weighted geometric mean method, and four methods based on data envelopment analysis are compared. The case study on land use in Slovenia is applied. The conclusions review consistency, sensitivity analyses, and some future directions of research.

Key words: *multiple criteria decision making, analytic hierarchy process, group decision making, data envelopment analysis, management of natural resources*

1. INTRODUCTION

Operations research professionals have developed comprehensive and widely accessible multiple criteria models (MCM) (Belton and Stewart, 2002) to assist decision makers (DMs) facing the ever-present difficulties in seeking compromise or consensus between competing interests and goals. MCM imply a process of assigning values to alternatives that are evaluated along multiple criteria, and deal with the selection of the best alternative based on conflicting objectives. MCM vary across decision making problems: goal programming, multi-attribute utility theory (MAUT), conjoint analysis, ELECTRE, analytic hierarchy process (AHP), etc. The problems present a challenge for developing new multiple-criteria methodologies.

Managing natural resources is a group multiple criteria decision making problem, since it is sustainable, of multiple use, and develops several alternatives (scenarios, decisions). Each scenario is affected by several criteria such as economic, ecological, social, and educational. Several DMs, i.e. different social groups, identify and assess the management alternatives in order to determine the optimal alternative or to rank them. The decision support model for optimal management of natural systems can be illustrated by the diagram (Figure 1).

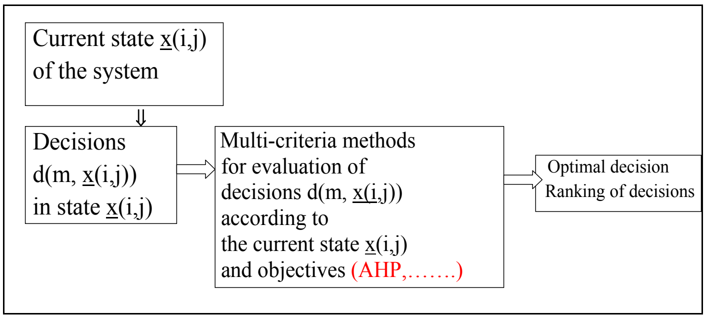

Figure 1: The diagram of optimal management of natural systems

The structure of the problem is hierarchical (Figure 2) with goal, criteria, sub-criteria and alternatives, which favors AHP.

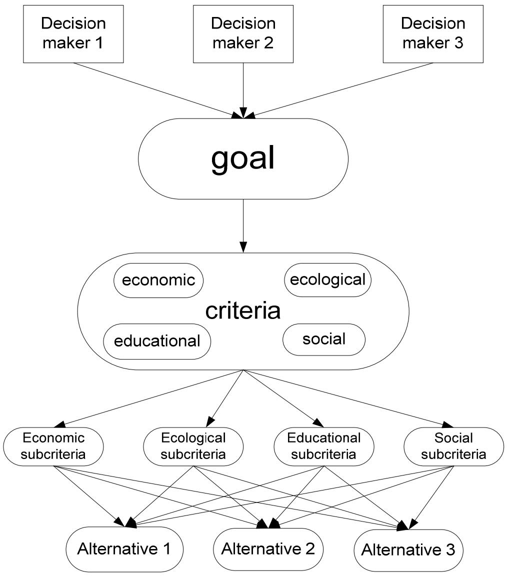

Figure 2: The hierarchical structure of the problem

In the paper we introduce the AHP and discuss the one DM's problem. For the derivation of the priority vector, three methods are presented: the eigenvector (EV) method (which is the best known method and is used in many applications), data envelopment analysis (DEA) method, and logarithmic least squares method (LLSM). Further, we discuss the group AHP where several DMs are included. In this case we compare six methods: weighted arithmetic mean method (WAMM), weighted geometric mean method (WGMM), and four DEA based methods: DEAW&C, DEA-WDGD, LP-GW-AHP, and WGMDEA. All presented methods are applied in the case study of land use in Slovenia. The conclusions discuss consistency, sensitivity analyses, and future research directions.

2. ANALYTIC HIERARCHY PROCESS

AHP is based on pairwise comparison matrices (1) for *n* criteria (or alternatives). The comparisons between two elements on the same level regarding the parent element are assembled, using the values from 1 to 9 from fundamental AHP scale (Saaty, 2006). Reciprocal values are designated to the inverse comparisons.

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1 \end{bmatrix} \tag{1}$$

The main theoretical problem is to determine the priority vector, which ranks the criteria (or alternatives). The comparison matrix *A* is consistent if $a_{ij}a_{jk} = a_{ik}$, for $i, j, k = 1, \ldots, n$. In this case, matrix *A* has the ratio form $a_{ij} = \frac{w_i}{w_j}$, $i,j=1,\ldots,n$ and $Aw = nw$, where *n* is its principal eigenvalue and $w = (w_1, \ldots, w_n)^T$. From the consistency condition it follows $A^k = n^{k-1}A$. If *A* is an inconsistent matrix, this is not true. In this case the priorities are derived in the form of the principal right eigenvector $Aw = \lambda_{max} w$, where λ_{max} is the largest eigenvalue of matrix *A*. This method is called EV method (Saaty, 1980).

Many other methods for deriving the priority vector are discussed in the literature. In the paper we will discuss only two additional methods: DEA and LLSM.

DEA (Charnes et al., 1978) method has the objective to maximize the efficiency of decision making units which convert multiple inputs into multiple outputs. It is based on linear programming (LP).

Wang and Chin (2009) proposed a LP method (2) for deriving priorities, and is based on DEA. Each row of the comparison matrix *A* is treated as a decision making unit, each column as an output and a dummy input value of one for all decision making units.

$$\begin{gathered} \max \quad w_0 = \sum_{j=1}^{n} a_{0j} x_j , \\ \text{subject to: } \sum_{j=1}^{n} \left(\sum_{i=1}^{n} a_{ij} \right) x_j = 1 , \\ \sum_{j=1}^{n} a_{ij} x_j \geq n x_i , \; i=1,\ldots,n, \\ x_j \geq 0 , \; j=1,\ldots,n. \end{gathered} \tag{2}$$

LLSM (Crawford and Williams, 1985) is a method for deriving priorities, which solves the optimization problem (3):

$$\min \sum_{i=1}^{n}\sum_{j>i}^{n}\left(\ln a_{ij} - \left(\ln w_i - \ln w_j\right)\right)^2 \tag{3}$$

Its solution is the normalized geometric mean of the rows of matrix A:

$$w_i = \frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}},\ i=1,...,n. \tag{4}$$

3. MULTIPLE CRITERIA GROUP DECISION MAKING

Decisions generally affect groups of people instead of isolated individuals. Thus, a problem emerges of how many individuals' preferences can be combined to yield a collective choice. Moving from a single DM to multiple DMs adds a great deal of complexity to the analysis. The problem is no longer selecting the most preferred alternative among the non-dominated solutions according to one individual's preference structure as conferred in the first part of this paper. Instead, the analysis is extended to account for conflicts among different interest groups: stakeholders, owners, managers, ecologists, and public, who have different goals or on the group participation process in which groups have common interests. Group decision making under multiple criteria includes preference analysis, utility theory, social choice theory, theory of voting, game theory, expert evaluation analysis, among other approaches. However, this paper is concerned with the AHP method where the group valuation of alternatives is searched through compromise, voting, consensus or aggregating methods: the aggregation of individual priorities and the aggregation of individual judgments.

In group decision making there are n criteria and m DMs. An $n \times n$ comparison matrix $A^{(k)} = (a_{ij}^{(k)})_{n\times n}$, $k=1,...,m$ and an individual priority vector $\mathbf{w}^k = \left(w_1^{\ k},\ldots,w_n^{\ k}\right)^T$ belong to each DM. The importance of k-th DM's opinion is denoted by α_k, for $k=1,...,m$, with $\alpha_k > 0$ and $\sum_{k=1}^{m}\alpha_k = 1$. The group priority vector is indicated by $\mathbf{w} = \left(w_1,\ldots,w_n\right)^T$.

In the group AHP weighted arithmetic mean method (WAMM) (Ramanathan and Ganesh, 1994) is usually used to aggregate individual priorities. EV method is applied for obtaining the priority vector of each DM. The group priority vector is synthesized by the weighted arithmetic mean:

$$w_i = \sum_{k=1}^{m}\alpha_k w_i^{\ k},\ i=1,...,n. \tag{5}$$

For aggregation of individual judgments weighted geometric mean method (WGMM) should be used (Aczél and Saaty, 1983). The weighted geometric mean complex judgment matrix $A^{WGM} = (a_{ij}^{\ WGM})_{n\times n}$, where each element is a weighted geometric mean of individual judgments $a_{ij}^{WGM} = \prod_{k=1}^{m} \left(a_{ij}^{(k)}\right)^{\alpha_k}$, is representing their group opinion. The group priority vector is obtained from A^{WGM} by the EV method.

Many other approaches exist in the group AHP, but we will restrict on the group AHP methods, which are based on DEA.

Wang and Chin (2009) proposed the new DEA methodology (DEAW&C), where the group priorities are obtained by solving the LPs (6):

$$\begin{aligned} &\max \quad w_0 = \sum_{j=1}^{n} \left(\sum_{k=1}^{m} \alpha_k a_{0j}^{(k)} \right) x_j \,, \\ &\text{subject to: } \sum_{j=1}^{n} \left(\sum_{k=1}^{m} \sum_{i=1}^{n} \alpha_k a_{ij}^{(k)} \right) x_j = 1 \,, \\ &\sum_{j=1}^{n} \left(\sum_{k=1}^{m} \alpha_k a_{ij}^{(k)} \right) x_j \geq n x_i \,, \ \text{i=1,...,n}, \\ &\qquad x_j \geq 0 \,, \ \text{j=1,...,n}. \end{aligned} \tag{6}$$

The method's main drawback is that it violates the reciprocal property.

Hosseinian et al. (2009a) suggested a method (DEA-WDGD), where only one LP has to be solved. The group priorities are the solution of the LP model (7):

$$\begin{aligned} &\max \qquad \sum_{i=1}^{n} w_i \,, \\ &\text{subject to: } \sum_{k=1}^{m} \left(\sum_{j=1}^{n} a_{ij}^{(k)} \right) v_k - w_i = 0 \,, \quad i=1,...,n, \\ &\qquad \sum_{i=1}^{n} w_i \leq 1 \,, \\ &\qquad \alpha_k v_{k+1} - \alpha_{k+1} v_k = 0 \,, \ k=1,...m-1, \\ &\qquad w_i \geq 0 \,, \ i=1,...,n, \\ &\qquad v_k \geq 0 \,, \ k=1,...,m. \end{aligned} \tag{7}$$

Its solution can be expressed as:

$v_k = \dfrac{\alpha_k}{\sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{j=1}^{n} a_{ij}^{(k)} \alpha_k}$, $k=1,...,n$ and $w_i = \dfrac{\sum_{k=1}^{m}\sum_{j=1}^{n} a_{ij}^{(k)} \alpha_k}{\sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{j=1}^{n} a_{ij}^{(k)} \alpha_k}$, $i=1,...,n$ (Grošelj and Zadnik Stirn, 2010a). It violates the reciprocal property.

Hosseinian et al. (2009b) proposed LP-GW-AHP group method. The WGM is used for aggregating individual judgments. The group priorities are derived by solving the LP model (8):

$$\max z$$
$$\text{subject to } w_i \geq z,\ i=1,\ldots,n,$$
$$\sum_{j=1}^{n}\left(\prod_{k=1}^{m}\left(a_{ij}^{k}\right)^{\alpha_k}\right)v_j - w_i = 0,\ i=1,\ldots,n, \tag{8}$$
$$\sum_{i=1}^{n} w_i = 1$$
$$v_i - \tfrac{1}{\beta} w_i \geq 0,\ i=1,\ldots,n,$$
$$v_i - \tfrac{1}{n} w_i \leq 0,\ i=1,\ldots,n,$$
$$w_i \geq 0,\ v_i \geq 0,\ i=1,\ldots,n,$$

where $\beta = \min\left\{\max_i\left(\frac{1}{r_i}\sum_{j=1}^{n} a_{ij}^{WGM} r_j\right), \max_i\left(\frac{1}{c_i}\sum_{j=1}^{n} a_{ij}^{WGM} c_j\right)\right\}$ and $r_1,\ldots,r_n$ and $c_1,\ldots,c_n$ are the row sums and the column sums of group comparison matrix A^{WGM}, respectively.

We recommend our WGMDEA method (Grošelj and Zadnik Stirn, 2010a), which is similar to the DEAW&C model, but it uses WGM, which preserves the reciprocity, instead of WAM. The group priorities are the solution of LPs (9):

$$\max\ w_0 = \sum_{j=1}^{n}\left(\prod_{k=1}^{m}\left(a_{0j}{}^{(k)}\right)^{\alpha_k}\right)x_j\,,$$
$$\text{subject to: } \sum_{j=1}^{n}\left(\sum_{i=1}^{n}\prod_{k=1}^{m}\left(a_{ij}{}^{(k)}\right)^{\alpha_k}\right)x_j = 1\,, \tag{9}$$
$$\sum_{j=1}^{n}\left(\prod_{k=1}^{m}\left(a_{ij}{}^{(k)}\right)^{\alpha_k}\right)x_j \geq nx_i\,,\ i=1,\ldots,n,$$
$$x_j \geq 0\,, j=1,\ldots,n.$$

4. APPLICATION

The case study sources from the problem elaborated within an EU project: Mediterranean areas – an integrated framework for sustainable development (Kazana and Zadnik Stirn, 2005). One of the sub-problems was also how to manage the area of Panovec, the state forest in the immediate vicinity of Nova Gorica in Slovenia, which covers a total area of 384 ha. Optimal management of this renewable natural resource presents a multi-criteria decision problem from the economic, ecological, and social point of view, and where typically several decision makers are incorporated.

In this comprehensive problem alternatives were evaluated according to the merits of benefits, opportunities, costs, and risks. Let us take for illustration a small part (Table 1) of this problem, considering only benefits regarding the given scenarios.

Table 1: The criteria and sub-criteria of economic, ecological, and social benefits in project Panovec

BENEFITS	**economic**	**financial benefits**	forest production, new jobs (fast food kiosks, a nursery shop, guiding, workshops,...), grazing
		non-market benefits	(collecting firewood, picking mushrooms, berries, taking honey,chestnuts,...)
	ecological	**ecological benefits**	biodiversity, protection of nature
	social	**recreation**	walking, jogging, gymnastic, cycling, horse riding, hunting picnics, children playing
		education	observing the plants, watching and feeding the animals, workshops, meetings, natural heritage, information signs

With the research of Panovec we obtained through surveys the following preferences (pairwise comparisons) from Ministry of Agriculture and Forestry, and Municipality of Nova Gorica and gathered them in the pairwise matrix A (10). Thus, the decision problem is the problem of one DM.

$$A = \begin{bmatrix} 1 & 1 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 1 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{5} \\ 6 & 4 & 1 & 2 & 2 \\ 3 & 4 & \frac{1}{2} & 1 & 1 \\ 3 & 5 & \frac{1}{2} & 1 & 1 \end{bmatrix} \tag{10}$$

Priority vectors from comparison matrix A (10) were derived by the EV method, DEA method, and LLSM. The priorities are very similar and the ranking of the criteria is equal for all three methods. The priorities are shown in the graph (Figure 3).

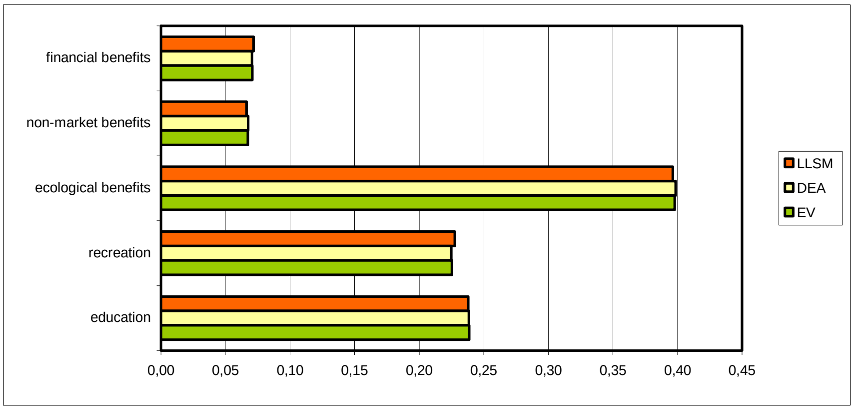

Figure 3: The priorities for benefits in project Panovec obtained by EV method, DEA method and LLSM

The ministry and municipality were not the only one who has been concerned about the management of Panovec. Stakeholders from the National Forest Service, headquarter Nova Gorica, Forest Enterprise Tolmin, and the Union of Forest Engineers generated the pairwise matrix B (11), The Union of local residents generated the matrix C (11) and representatives of university and forestry institute generated the matrix D (11) for the criteria:

$$B=\begin{bmatrix}1 & 6 & 3 & 2 & 3\\ \frac{1}{6} & 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{6}\\ \frac{1}{3} & 2 & 1 & \frac{1}{2} & 1\\ \frac{1}{2} & 4 & 2 & 1 & 2\\ \frac{1}{3} & 6 & 1 & \frac{1}{2} & 1\end{bmatrix},\ C=\begin{bmatrix}1 & 2 & 1 & \frac{1}{3} & \frac{1}{2}\\ \frac{1}{2} & 1 & 1 & \frac{1}{2} & 1\\ 1 & 1 & 1 & \frac{1}{3} & 1\\ 3 & 2 & 3 & 1 & 2\\ 2 & 1 & 1 & \frac{1}{2} & 1\end{bmatrix},\ D=\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{6} & \frac{1}{2} & \frac{1}{6}\\ 2 & 1 & \frac{1}{6} & \frac{1}{4} & \frac{1}{6}\\ 6 & 6 & 1 & 2 & 1\\ 2 & 4 & \frac{1}{2} & 1 & \frac{1}{2}\\ 6 & 6 & 1 & 2 & 1\end{bmatrix} \tag{11}$$

All four decision makers' opinions were equally important. The group priority vectors, derived by 6 group methods are presented in the Table 2.

Table 2: The priorities and the ranking of benefits sub-criteria, obtained by WAMM, WGMM; DEAW&C, DEA-WDGD, LP-GW-AHP and WGMDEA method

	WAMM		WGMM		DEAW&C		DEA-WDGD		LP-GW-AHP		WGMDEA	
financial benefits	0.1725	4	0.1390	4	0.1746	4	0.1599	4	0.1390	4	0.1389	4
non-market benefits	0.0823	5	0.0833	5	0.0810	5	0.0791	5	0.0833	5	0.0833	5
ecological benefits	0.2551	1	0.2541	2	0.2587	1	0.2569	2	0.2540	2	0.2542	2
recreation	0.2541	2	0.2733	1	0.2445	2	0.2430	3	0.2733	1	0.2733	1
education	0.2360	3	0.2503	3	0.2412	3	0.2611	1	0.2504	3	0.2503	3

By the WAMM the most important benefit is ecological benefits, closely followed by recreation. Education is ranked third. By the WGMM, the most important benefit's sub-criterion is recreation, followed by ecological benefits and education, very similar to the WAMM priorities. The priorities, gained by WGMDEA and LP-GW-AHP are equal to the priorities derived by WGMM. DEAW&C placed ecological benefits first, recreation second and education third. DEA-WDGD rankings are education, ecological benefits, and recreation.

5. CONCLUSIONS

In the presentation the central concern of solving a sophisticated natural resources problem is an integrated approach to multi criteria decision analysis. We discussed the idea of different methodologies to provide a meta-solution to multi-criteria and group decision making problem. Such considerations lead to the research needs in the area of MCM and group decision making. Special attention was paid to AHP, for one DM and for group decisions.

Further, we have to emphasize that the problem of consistency is important in AHP. The equation for calculation the priority vector from a pairwise matrix A is $Aw = \lambda_{\max} w$, where $\lambda_{\max} \geq n$. If A is consistent matrix, then $\lambda_{\max} = n$. Thus, the inconsistency is measured by consistency index $CI = \frac{\lambda_{max} - n}{n-1}$. If the consistency ratio $CR = \frac{CI}{RI}$, where RI is random index, is less than 0.1, then the inconsistency is acceptable.

The matrices in the application were of acceptable consistency, since the consistency ratios were $CR_A = 0.0172$, $CR_B = 0.0362$, $CR_C = 0.0420$, and $CR_D = 0.0316$, respectively.

In group AHP the problem of consistency of individual as well as aggregated pairwise comparison matrices is crucial. We (Grošelj and Zadnik Stirn, 2010b) presented a new proof of Xu's (Xu, 2000) (2000) theorem (Lin *et al.*, 2008), rejected it):

If comparison matrices of all DMs are of acceptable consistency, then the aggregated comparison matrix (weighted geometric mean) is also of acceptable consistency. We also discussed the conditions under which the aggregated comparison matrix is of acceptable consistency if not all individual comparison matrices are of acceptable consistency. We derived the upper bound (the lower is zero) for the consistency ratios of all DMs' comparison matrices.

Regarding the methods EV, DEA and LLSM for one DM there are really small differences in results in our example. But, the differences can grow larger as the inconsistency increases.

At group AHP, it is important to consider the consistency of individual and aggregated matrices, homogeneity of DMs, and finally DMs' opinions importance (their opinions are not always equally important).

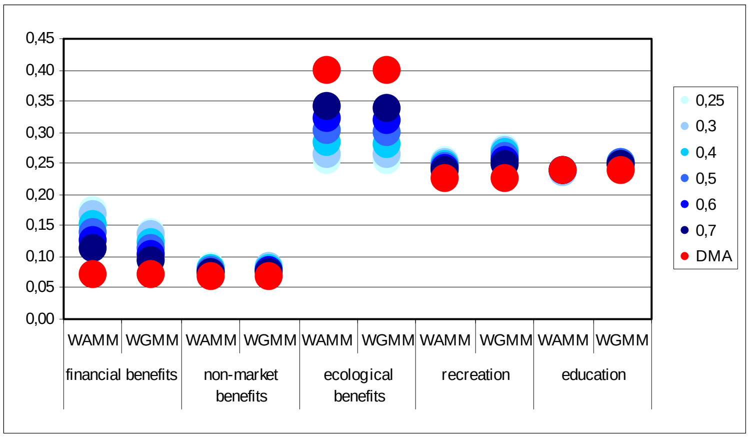

Figure 4: The sensitivity analysis for priorities regarding the importance of the DM_A' assuming that the importance of the other three DMs' opinions is equal.

The different importance of DMs' opinions is studied under sensitivity analysis. No theoretical studies have been conducted. The sensitivity analysis for our application is presented in Figure 4. We were changing the importance of DM_A's opinion, assuming that the importance of the other three DMs' opinions is equal and we were calculating group priorities. The priorities of DM_A and the group priorities, when the DM_A's opinion is equal (0.25) or more important (0.3, 0.4, 0.5, 0.6, 0.7) than the other three DMs' opinions, are shown in the graph (Figure 4). The group priorities were derived using WGMM and WAMM.

Further research is going also in the direction of cases when the pairwise comparisons between criteria (objects) are determined with the interval judgments (Wang et al., 2005, Liu, 2009).

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