A Mathematical Model and Numerical Simulation of the Static Stability of a Tractor

Ivan DEMŠAR [1(✉)]
Rajko BERNIK [2]
Jože DUHOVNIK [1]

Summary

Tractor overturning remains a serious problem in agricultural activities. A detailed analysis of problems and causes leading to a tractor overturn have revealed that by improving tractor's static stability we can positively influence the safety as early as during the concept phase. We designed a mathematical model and a numerical simulation of the static stability of a tractor with an oscillating front axle in relation to its position on a slope. It was followed by analysing the changes of individual parameters, such as the position of the centre of gravity, the wheelbase, the wheel track width and the height of the oscillating axle mounting point, and their impact on tractor's static stability in relation to its position on a slope. Results show that manipulating these parameters can significantly increase tractor's static stability. A better static stability is directly proportional to improved dynamic stability, resulting in a better safety in a view of the tractor overturn, particularly while working on a sloping terrain.

Key words

static stability, tractor, slope, analysis, mathematical model, simulation

[1] University of Ljubljana, Faculty of Mechanical Engineering, Department of Engineering Design and Transportation Systems, 1000 Ljubljana, Aškerčeva 6, Slovenia
✉ e-mail: ivan.demsar@lecad.fs.uni-lj.si
[2] University of Ljubljana, Biotechnical Faculty, Department of Agronomy, 1000 Ljubljana, Jamnikarjeva ul.101, Slovenia

Received: December 12, 2011 | Accepted: July 10, 2012

Introduction

Tractor overturn accidents remain a serious problem in the agricultural industry. Statistics show (Liu and Ayers, 1999; National Safety Council, 2001; Žlender and Juvan, 2002; Bernik and Dolenšek, 2006) that most fatal accidents happen when a tractor overturns. Tractor stability and the reduction of injuries related to tractor rollovers were areas addressed by many researchers (Spencer, 1978; Ross and DiMartino, 1982; Murphy et al., 1996). Several safety systems have been developed to protect the driver and soften the consequences in case of a tractor overturn. Rollover protective structure (Springfeldt and Thorson, 1987; Springfeldt, 1993) with its rollover bar and safety cab is the best known and widely used system. Later, it was followed by the safety belt and AutoROPS (Etherton at al., 2004) system. After introducing these measures, the number of fatal accidents as a result of a tractor overturn has decreased (Springfeldt, 1996). However, the number of overturns themselves has stayed the same.

According to the literature (Liu and Ayers, 1998; Bernik, 2004), there are two types of stability. When a tractor is standing still it is a matter of static stability and when it is moving it is a matter of dynamic stability. Considering that, accidents usually occur when a tractor is moving so, it is easy to conclude that dynamic stability is the issue. Besides the static stability, the key parameters, affecting dynamic stability are factors from the environment, such as a rough terrain, potholes, washboards, stones etc. as well as subjective factors, particularly speed and driving style (Spencer and Gilfillan, 1976).

The main impact on static stability of the tractor has the position of the centre of gravity. According to the ISO 789-6 (1982) the distance from the rear axle and tractor's symetric plane to the centre of gravity can be determined by looking into the ratio beetwen wheels load. Fabbri and Moalari (2004) evaluated a static procedure to measure the centre of gravity height. Significant impact on the stability in the longitudinal direction has the wheelbase (Gilfillan, 1970; Abu-Hamdeh and Al-Jalil, 2004). Gravos et al. (2011) have shown that the track width has a significant impact on the stability of the tractor when moving sideways to the slope.

The purpose of this work was to develop a mathematical model and numerical simulation to study how changes in key parameters such as the position of the centre of gravity, wheelbase, the wheel track width and the height of the oscillating front axle mounting point affected the tractor's static stability in relation to its position on a slope.

Material and methods

Nomenclature

A = the rear right wheel's support point with the ground

B = the rear left wheel's contact point with the ground

C = front left wheel's contact point with the ground

C_1 = projection of the front left wheel's contact point

D = front right wheel's contact point with the ground

D_1 = projection of front right wheel's contact point

E = mounting point of the oscillating front axle

T = tractor's centre of gravity

$\vec{G}$ = gravity force

l = tractor's wheelbase

l_T= distance between the centre of gravity and the rear axle

b = wheel track width

b_T = shift of the centre of gravity from the tractor's symmetry plane (to the right, in the direction of driving)

h_E = height of the oscillating front axle mounting point

h_T = distance between the centre of gravity and the ground

β = angular movement of a tractor on a slope

β_{kA} = critical angular movement of a tractor on a slope – rolling over point A

β_{kB} = critical angular movement of a tractor on a slope – rolling over point B

β_{kC1} = critical angular movement of a tractor on a slope – rolling over point C_1

β_{kD1} = critical angular movement of a tractor on a slope – rolling over point D_1

γ= angle of tractor's static stability

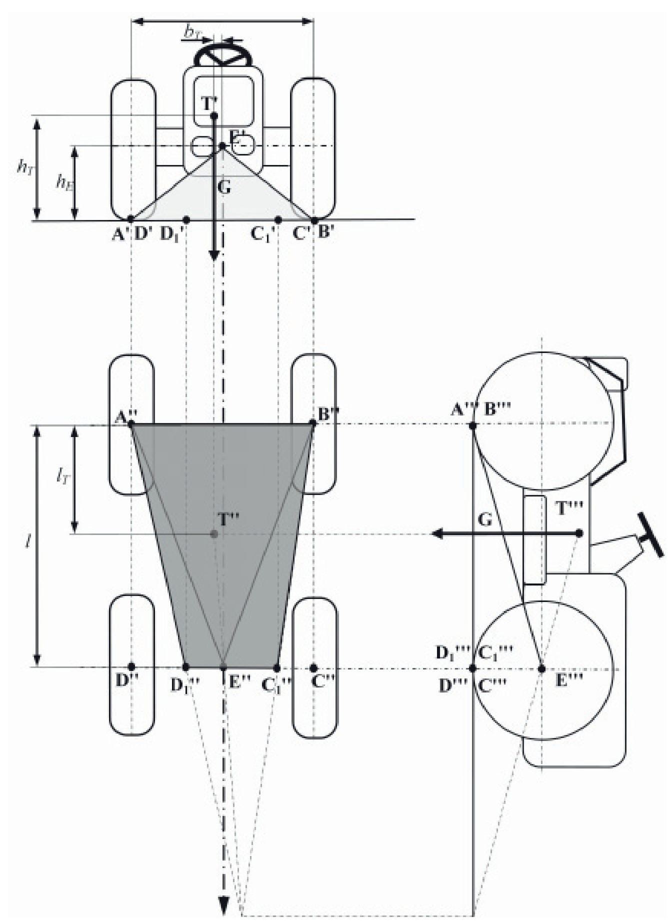

Figure 1. Supports for a tractor with an oscillating front axle

Static instability, leading to a tractor overturn occurs when the line of force $\vec{G}$, passing through the centre of gravity T, passes over the support plane ABC_1D_1 (Figure 1). The line between longitudinal stability and lateral stability (rolling over points A, B, C_1 and D_1) is marked with corresponding angles β_k. They are determined with geometry means (equations 1, 2, 3, 4)

$$\beta_{kA} = \tan^{-1}\left(\frac{\frac{b}{2}-b_T}{l_T}\right) \qquad (1)$$

$$\beta_{kB} = \tan^{-1}\left(\frac{\frac{b}{2}+b_T}{l_T}\right) \qquad (2)$$

$$\beta_{kC_1} = \tan^{-1}\left(\frac{\frac{l_F\left(\frac{b}{2}+b_F\right)}{l+l_F}+b_F+b_T}{l-l_T}\right) \qquad (3)$$

$$\beta_{kD_1} = \tan^{-1}\left(\frac{\frac{l_F\left(\frac{b}{2}-b_F\right)}{l+l_F}+b_F-b_T}{l-l_T}\right) \qquad (4)$$

with:

$$l_F = \frac{h_E(l-l_T)}{h_T-h_E} \quad \text{and} \quad b_F = \frac{b_T\cdot h_E}{h_T-h_E}.$$

Longitudinal static stability

Longitudinal static stability becomes an issue when a tractor rolls over either the rear wheels (segment $\overline{AB}$) or front wheels (segment $\overline{C_1D_1}$) (Figure 1). Generally speaking, when tractor's position is between $-\beta_{kB}$ and β_{kA} (rolling over the rear wheels) in relation to the direction of a slope, tractor's static stability can be calculated using equation 5 (Figure 2a). When tractor's position is between $\pi-\beta_{kC_1}$ and $\pi+\beta_{kD_1}$ (rolling over the front wheels), static stability can be calculated using equation 6 (Figure 2b).

$$\gamma_{-\beta_B\le\beta\le\beta_{kA}} = \tan^{-1}\left(\frac{\frac{l_T}{\cos(\beta)}}{h_T}\right) \qquad (5)$$

$$\gamma_{\pi-\beta_{C1}\le\beta\le\pi+\beta_{kD1}} = \tan^{-1}\left(\frac{\frac{l-l_T}{\cos(\beta)}}{h_T}\right) \qquad (6)$$

Lateral static stability

Lateral static stability is defined as the point where a tractor tips over to its side (segments $\overline{AD_1}$ and $\overline{BC_1}$ respectively) (Figure 1). When tractor's angle in relation to the direction of a slope is between β_{kA} and $\pi-\beta_{kD_1}$ (tipping over to its right side) (Figure 3b), tractor's static stability can be described by the equation 7. When tractor's angle is between $-\beta_{kB}$ and $-\pi+\beta_{kC_1}$ (tipping over to its left side) (Figure 3a), tractor's static stability is described by the equation 8.

$$\gamma_{\beta_{kA}\le\beta\le\pi-\beta_{kD1}} = \tan^{-1}\left(\frac{l_T\cdot\cos\left(\beta_{kA}+\tan\left(\frac{l+l_F}{\frac{b}{2}+b_F}\right)\right)}{\cos(\beta_{kA})\cdot\cos\left(\beta+\tan\left(\frac{l+l_F}{\frac{b}{2}+b_F}\right)\right)h_T}\right) \qquad (7)$$

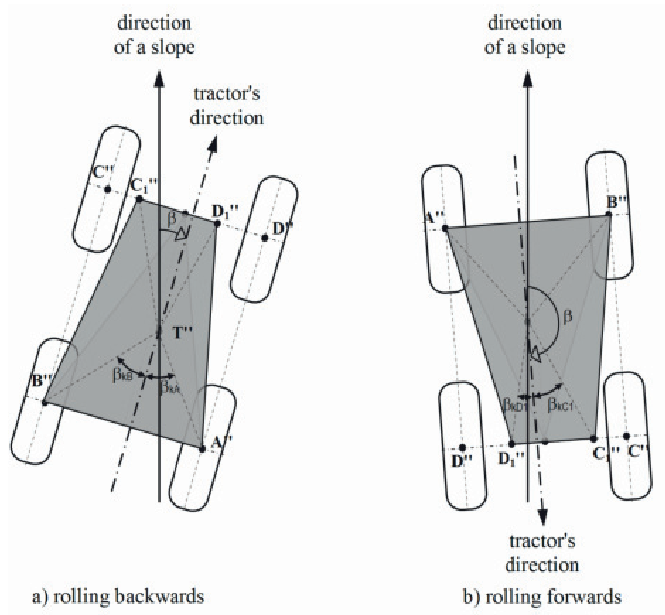

Figure 2. Longitudinal static stability limit

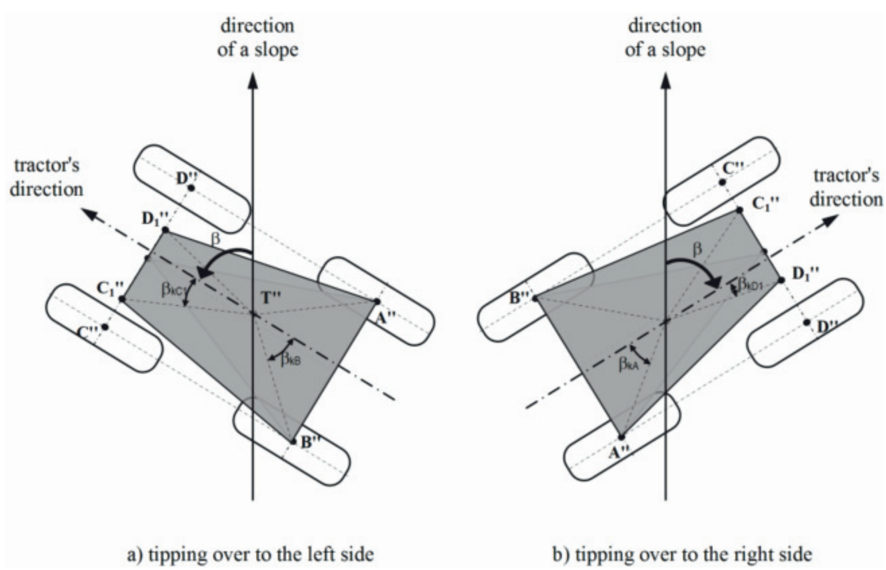

Figure 3. Lateral static stability limit

$$\gamma_{-\pi+\beta_{kC1}\le\beta\le-\beta_{kB}} = \tan^{-1}\left(\frac{l_T \cdot \cos\left(\beta_{kB} - \tan^{-1}\left(\frac{l+l_F}{\frac{b}{2}-b_F}\right)\right)}{\cos(\beta_{kB})\cdot\cos\left(\beta - \tan\left(\frac{l+l_F}{\frac{b}{2}-b_F}\right)\right)h_T}\right) \quad (8)$$

Results and discussion

Comparison between the measured and simulated tractor's static stability

To verify the mathematical model and numerical simulations of a static stability of a tractor, we designed a characteristic model of a tractor design, featuring an oscillating front axle with known design parameters (Table 1). With test bench we measured the actual static stability of a characteristic model of a tractor in relation to the angular movement on a slope. The results are shown in Table 2.

Table 1. Values for design parameters of a characteristic tractor model

Design parameter	Value [mm]
Wheelbase (l)	121.0
Track width (b)	96.0
Height of the oscillating front axle mounting point (h_E)	28.0
Distance between the centre of gravity and the ground (h_T)	44.5
Distance between the centre of gravity and the rear axle (l_T)	52.2
Shift of the centre of gravity from the tractor's symmetry plane to the right (b_T)	0.0

A comparison between the collected results of the measurements and the simulation confirmed that the selected process of numerically simulating static stability of a tractor with an oscillating front axle was suitable (Figure 4).

Analysis of the effects of changing design parameters on tractor's static stability

Simulating static stability of a tractor at different values of individual parameters, we analysed the effects of changing individual parameters of the tractor design on tractor's static stability. Figure 5 shows simulations of static stability for the Reform MOUNTY 65 mountain tractor (Schrottmaier at al., 2004). Its characteristic values are shown in Table 3.

Individual uninterrupted sections represent the limits of tractor's static stability in terms of the type of overturns - forwards, to the left, backwards, to the right and forwards again.

Table 3. Values of design parameters for the Reform MOUNTY 65 mountain tractor

Design parameter	Value [mm]
Wheelbase (l)	2195.0
Track width (b)	1580.0
Height of the oscillating front axle mounting point (h_E)	352.0
Distance between the centre of gravity and the ground (h_T)	680.0
Distance between the centre of gravity and the rear axle (l_T)	960.0
Shift of the centre of gravity from the tractor's symmetry plane to the right (b_T)	9.0

Tractor's static stability in relation to the changes of the wheelbase

The range of tractor's static stability is changing due to changes of its wheelbase. The wheelbase was changed in the range between 1500 and 3000 mm with a constant weight distribution between the rear (56.3%) and the front (43.7%) axle. In order to assess tractor's static stability, simulations for the wheelbases of 1500 mm, 1800 mm, 2500 mm and 3000 mm were carried out. The graph in Figure 6 shows that increasing the wheelbase at a constant load has the biggest influence on improvements in longitudinal static stability. At the same time, the range of longitudinal static stability is being reduced together with a longer wheelbase. Changes of the wheelbase have a minimal effect on lateral static stability.

Tractor's static stability in relation to the changes of the track width

Increasing the track width is the best known way of improving tractor's stability. Turning wheel rims is the most widely used method of changing the track width. Figure 7 shows a graph representing simulations of tractor's static stability with track width symmetrically changing from 1200 mm to 2000 mm. Similarly to the changes of the wheelbase, this example showed that increasing the track width has the biggest effect on improvement of tractor's lateral stability and simultaneously on the range of lateral static stability .

Tractor's static stability in relation to the changes of the height of the front axle mounting point

With existing design solutions of standard tractors, produced by established manufacturers, the height of the oscillating front axle mounting point is the same as the radius of the front wheels. Changing the size of wheels also means different mounting points of the oscillating front axle but at the same time, the height of the centre of gravity changes, too. Changing the height of the oscillating front axle mounting point without significantly changing the height of the centre of gravity would

Table 2. Static stability of a characteristic tractor model

Angular movement of a tractor (β) [°]	Tractor static stability (γ) [°]												
	0	15	30	45	60	75	90	105	120	135	150	165	180
Mesurement 1	48.9	50.2	53.0	57.6	47.1	41.3	39.0	38.9	41.7	47.1	52.1	57.0	56.1
Mesurement 2	49.0	50.0	52.9	57.1	46.8	41.1	39.2	39.1	41.1	45.2	50.2	57.0	56.2
Mesurement 3	49.2	49.9	53.0	57.3	46.6	41.4	39.1	39.3	40.8	46.6	52.3	57.1	56.1
Average	49.0	50.0	53.0	57.3	46.8	41.3	39.1	39.1	41.2	46.3	51.5	57.0	56.1

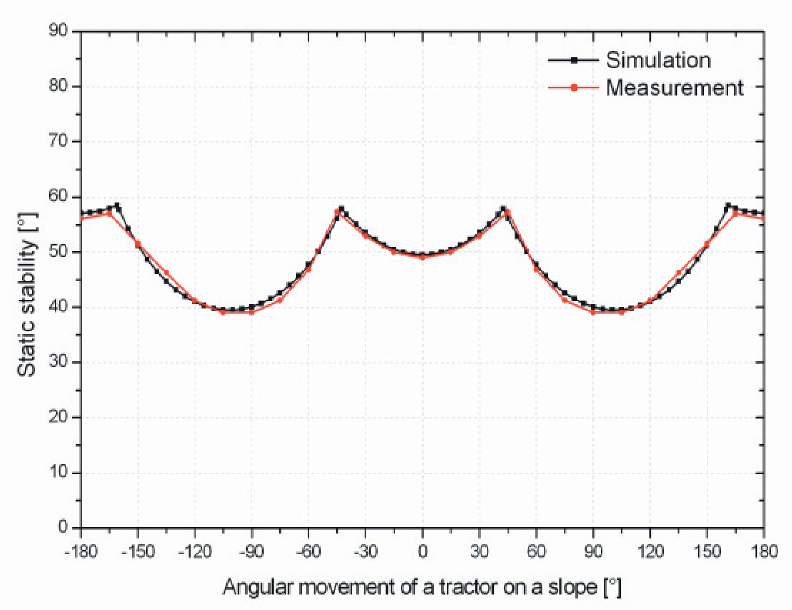

Figure 4. Comparing static stability of the characteristic tractor model

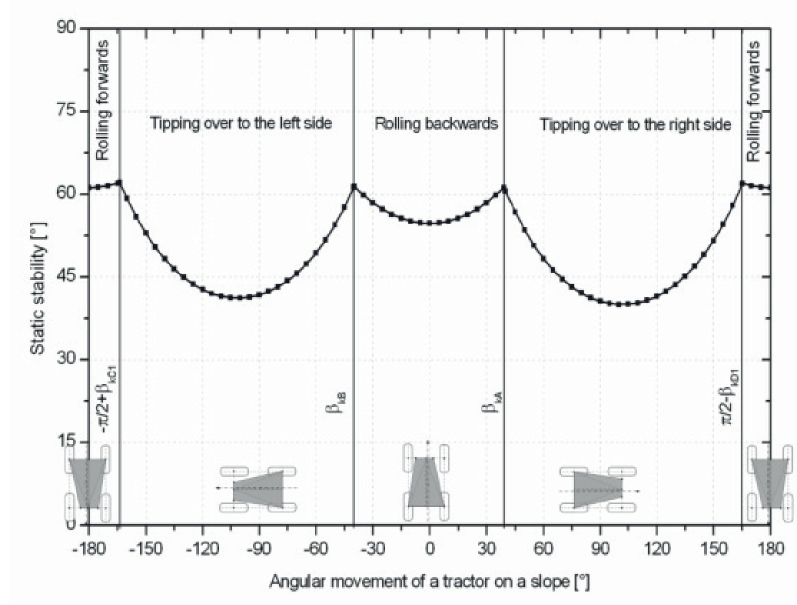

Figure 5. Simulating static stability of the Reform MOUNTY 65 mountain tractor

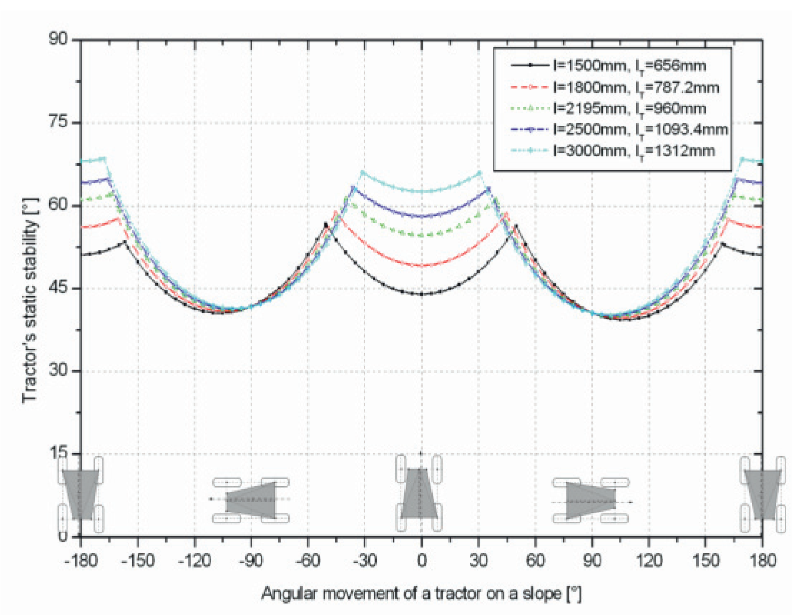

Figure 6. Simulating tractor's static stability in relation to the changes of the wheelbase

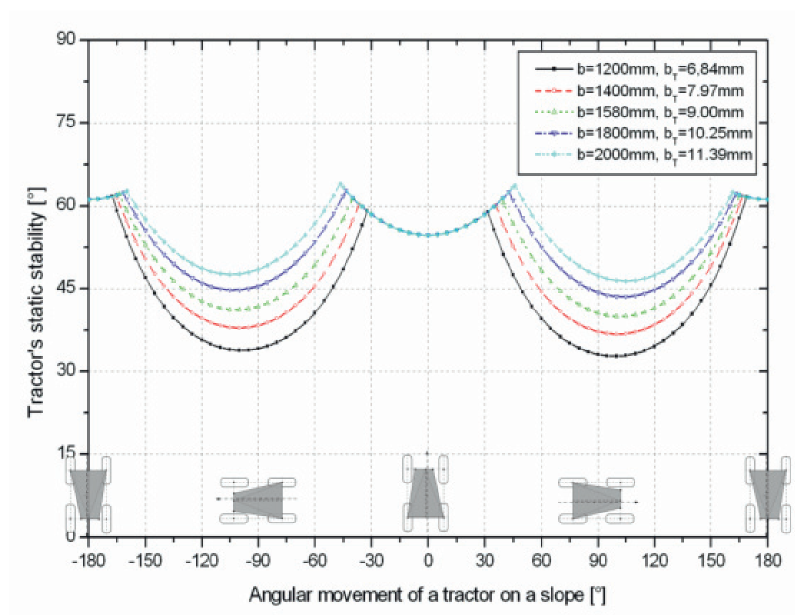

Figure 7. Simulating tractor's static stability in relation to the changes of the track width

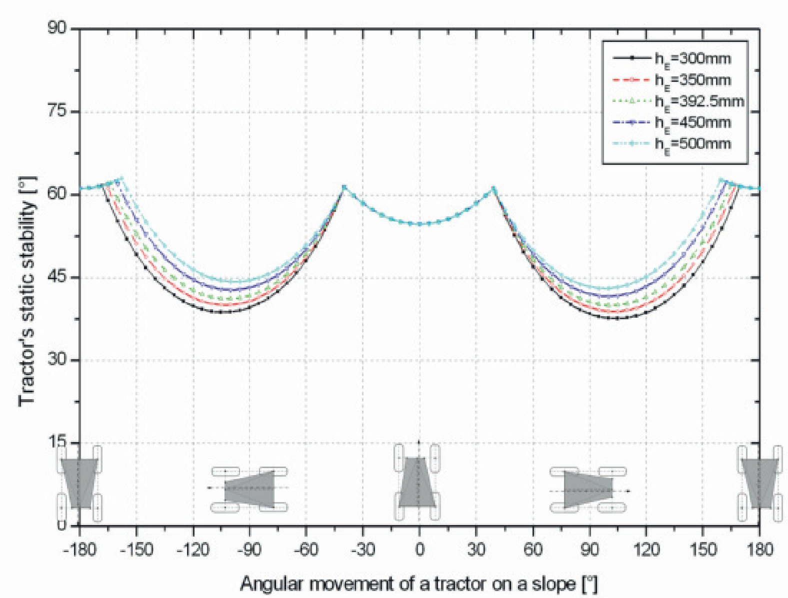

Figure 8. Simulating tractor's static stability in relation to the changes of the height of the front axle mounting point

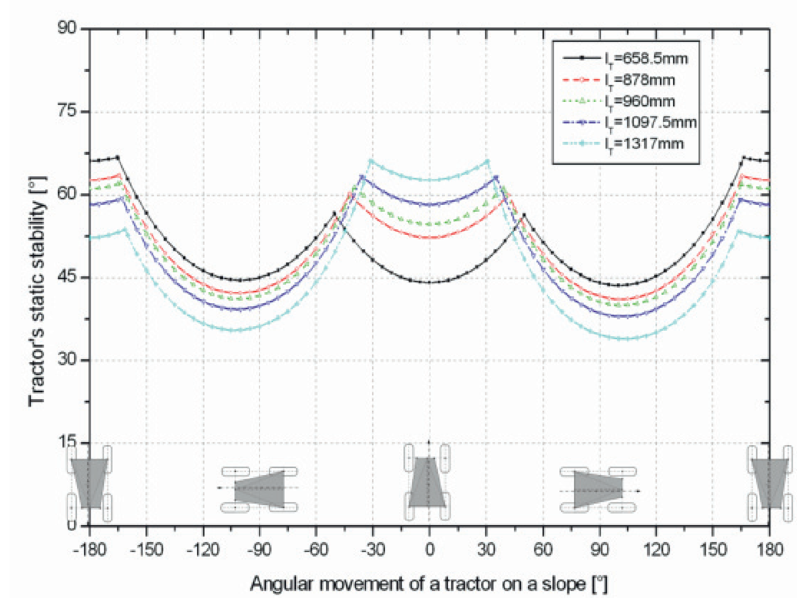

Figure 9. Simulating tractor's static stability in relation to the changes of the distance between the centre of gravity and the rear axle

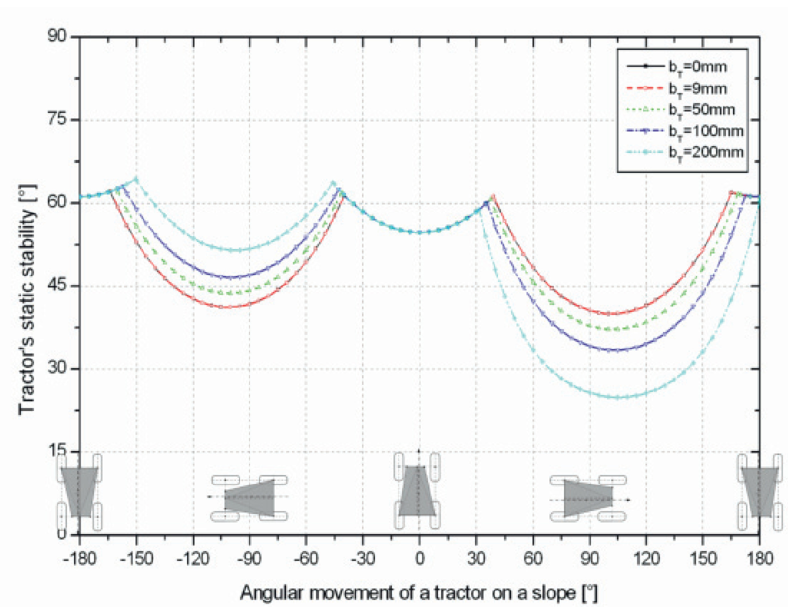

Figure 10. Simulating tractor's static stability in relation to the changes of the distance between the centre of gravity and tractor's symmetric plane

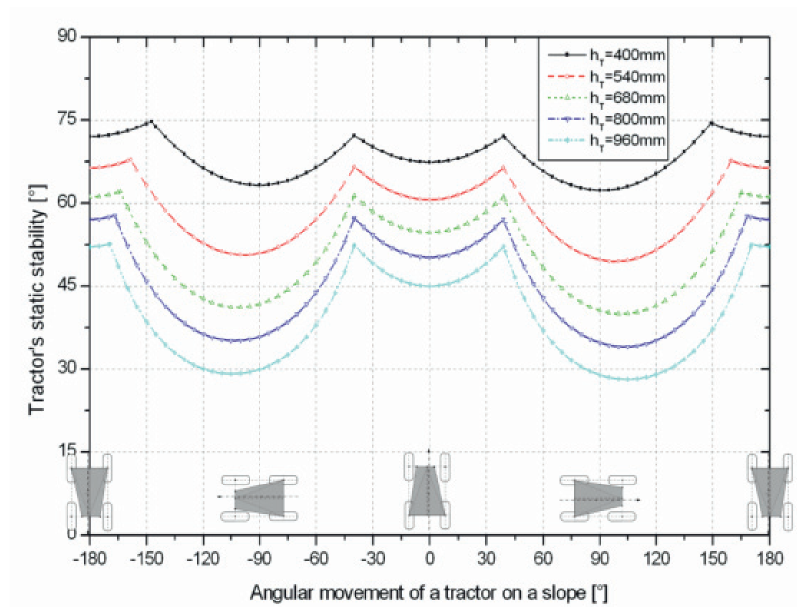

Figure 11. Simulating tractor's static stability in relation to the changes of the distance between the centre of gravity and the ground

require designing a special front axle mounting. We carried out numerical simulations of static stability at different heights of the oscillating front axle mounting point at a constant height of the centre of gravity. Comparing the results (Figure 8) has revealed that a higher oscillating mounting point has the biggest effect on improvements in lateral stability when driving down a slope and it increases the range of longitudinal static stability. When the height of the oscillating front axle mounting point is the same as the height of the centre of gravity points C_1 and D_1 become points C and D (Figure 1), and we get the effect of stiffly mounted wheels.

Tractor's static stability in relation to the changes of the distance between the centre of gravity and the rear axle

We designed a simulation for static stability of a tractor with a wheelbase of 2195 mm and changing weight distribution between the rear and the front axle: 70% at the rear and 30% at the front (l_T =658.5 mm), 60% at the rear and 40% at the front (l_T=878 mm), 56.3% at the rear and 43.7% at the front (l_T =960 mm), 50% at the rear and 50% at the front (l_T=1097.5 mm) and 40% at the rear and 70% at the front (l_T=1317 mm). Comparing the results in Figure 9 has revealed that a shorter distance of the centre of gravity from tractor's rear axle means lower longitudinal stability in case of overturning backwards. Lateral static stability and longitudinal static stability in case of overturning forwards increase with a shorter distance between the centre of gravity and the rear axle. The range of longitudinal static stability in case of overturning backwards increases while in case of overturning forwards it decreases but at a considerably slower rate.

The effect of changes in the position of the centre of gravity in the longitudinal direction also appears when different symmetrical devices are attached to a tractor. Devices, attached to the rear of a tractor decrease the distance of the centre of gravity from the rear axle while devices, attached to the front of a tractor, increase the distance.

Tractor's static stability in relation to the changes of the distance between the centre of gravity and tractor's symmetric plane

Compared to the track width, the effect of shifting the centre of gravity from the symmetric plane is negligible (0.6% for the Reform MOUNTY 65). Attaching devices to the side of a tractor can significantly contribute to decreasing lateral stability on the side where the device is attached. For this reason, we designed a numerical simulation for tractor's static stability, where the centre of gravity was shifted from the symmetric plane by 0 to 200 to the right, looking in the direction of a tractor. Results show that increasing the distance between the centre of gravity and the symmetric plane has the biggest effect upon the lateral static stability. Figure 10 shows simulations of static stability for the cases when the centre of gravity is shifted to the right, looking in the direction of driving. Shifting to the left (b_T<0) yields a mirrored picture.

Tractor's static stability in relation to the changes of the distance between the centre of gravity and the ground

Results in Figure 11 show that out of all the above-mentioned parameters, the distance between the centre of gravity from the ground has the biggest effect. We analysed the results of simulations where the centre of gravity's heights between 400 and 960 mm above the ground were monitored. It has been concluded that lowering the centre of gravity significantly contributes to improvements in static stability (lateral and longitudinal) over the entire range of tractor's angular movements on a slope.

Conclusion

On the basis of a numerical simulation, we analysed influences of individual design parameters on tractor's static stability in relation to its position on a slope. It has been concluded that the distance between the centre of gravity from the ground has the biggest effect on static stability. By lowering the centre of gravity, longitudinal as well as lateral static stability improve. The limiting factor here is the clearance height under a tractor.

Longitudinal static stability can be improved by increasing the wheelbase. This parameter also has a limiting factor because a longer wheelbase means less agility - a bigger turning circle. This problem can be partly solved by improving the steering system (bigger angle of lock or four wheel steering). With a constant wheelbase and in terms of overturning backwards, a similar effect can be achieved by increasing the distance of the centre of gravity from the rear axle. However, it results in a lower lateral as well as longitudinal static stability in case of overturning forwards.

Numerical simulations have shown that the problem of longitudinal static stability and partially also that of lateral static stability can be solved by adjusting the height of the oscillating front axle mounting point, particularly when driving down a slope. Positive effects of such change can be observed up to the point when the height of the oscillating front axle mounting point is the same as the height of the centre of gravity.

Things are similar for lateral static stability. Increasing the track width increases lateral static stability, however, there is a problem of roadways width. A solution to this problem has been found in the form of a fast continuously variable track width during the operation. Besides, the system allows an easy and quick adjustment of track width, depending on working conditions, contributing to time efficiency and quality of work.

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