IMPROVED APPROXIMATE CALCULATION OF BACKUP SUPPLY PROFITABILITY IN MV POWER DISTRIBUTION PLANNING

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Abstract:

In medium voltage optimal power distribution network routing, especially in the suburban and rural ones, it is often necessary to estimate additional construction costs of tie feeders enabling backup supply in comparison with a decrease in energy not supplied costs. This paper describes one fast approximation method used for this purpose. It is presumed that simultaneous installation of any number of manually or remotely controlled load-breaking disconnectors and circuit breakers can be installed in the network. In situations without and with backup supply, accurate and approximate expressions for the energy not supplied calculation of a medium voltage feeder are derived. The comparison of different optimal total costs, consisting of energy not supplied and switching devices costs, in planning period for cases without and with backup supply is the criterion used for tie feeder construction. The application of the method is shown on an example of 10 kV feeder.

1 Introduction

In medium voltage (MV) optimal power distribution network planning, reliability is taken into account either indirectly, through a preset network structure i.e. reliability level, or directly, by evaluating electricity supply interruption costs.

Different methods have been developed for determining the optimal number, type and locations of switching devices in an already routed network. They are based on minimizing total costs consisting of switching (device) costs and customer's supply interruption costs, e.g. [1-5], or on improving the reliability indices by determining optimal locations and types of a predefined number of switching devices [6,7].

Only a few papers dealing with the MV network configuration planning include reliability as a variable in the optimization process. In [8], energy not supplied costs and costs of switching devices are included in the same general approximation optimization procedure, based on a network programming method, along with substation and feeder costs. A problem of optimal MV network routing for a preset location of switching devices and operational practice, based on the minimization

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of total costs, consisting of investment, maintenance, resistive loss and energy not supplied costs, is solved by a simulated annealing method [9]. A very similar problem is solved in [10] using evolution programming. For preset MV feeder routes, a model for determining the optimal number, types and locations of switching devices as well as feeder types is presented in [11]. The actual optimization problem is solved using different methods (integer programming method, simulated annealing, genetic algorithm, etc.). Taking into account the construction costs of tie feeders, an optimization model for MV distribution network planning is solved using a tabu search method [12] or tabu search method and genetic algorithm [13].

The introduction of explicit consideration of reliability in the planning process, being more a characteristic of suburban and rural distribution networks, makes the problem even more complex. The high impact of the reliability on the optimal MV network configuration is reflected through additional construction costs of tie feeders, enabling backup supply, as potential elements of the future network configuration and a decrease in energy not supplied costs. Since the calculation is time-consuming even for one network configuration, it is important to calculate these expenses in the most timesaving and straightforward way.

A method for obtaining a very simple expression for comparing energy not supplied costs of a MV feeder in situations without and with backup supply is presented in [12]. It is an approximation method due to the assumption that the feeder is uniformly loaded. With the application of this method, a tie feeder construction can be easily justified and examined by making the whole planning process much faster. The method was based on an assumption that either disconnectors or load-breaking disconnectors can be installed in the network.

This paper describes an improved method of (taking into account also) simultaneous installation of circuit-breakers or load-breaking disconnectors, i.e. the general method for considering installation of any combination of load-breaking disconnectors and circuit breakers in MV network.

2 Energy not supplied of a MV feeder

Distribution networks almost always operate radially, although they can be constructed either as radial or meshed ones. Three basic reliability indices of distribution network are average failure rate λ_s, average outage time r_s and unavailability or average annual outage time U_s [14]. For the purpose of power distribution planning, one of the additional indices oriented to load and energy i.e. Energy Not Supplied Index is particularly important:

$$ENS = \sum_{i=1}^{n} P_i U_i \qquad (1)$$

where n represents the total number of network nodes and P_i is the average load connected in the node i which equals $P_i = P_{vi} LF_i$. P_{vi} and LF_i are peak load and load factor of node i.

Fig. 1 shows an example of 10 kV feeder consisting of n nodes (feeding node is not taken into account) and n sections. Each section i is described by the failure rate λ_i and the average outage time r_i, while each node i is described by the average load P_i.

A MV feeder or main line has almost always laterals. However, an appropriate substitution enables a reduction of every feeder with laterals to an equivalent feeder without laterals [15].

There is always a circuit breaker at the beginning of the feeder, while load-breaking disconnectors or circuit breakers are assumed to be installed along the feeder. If the switching device is installed in one of the sections, it is assumed that it is placed at its beginning. The outages are presumed to be single failures since the probability of multiple failures is negligible. In case that backup supply is available, it is considered to be at the end of the feeder.

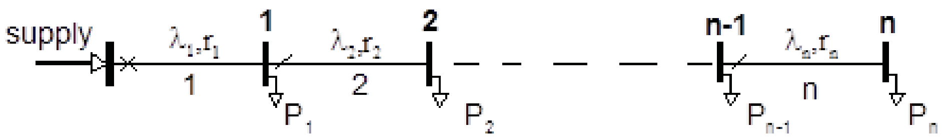

Figure 1. MV feeder

The Energy Not Supplied Index i.e. energy not supplied ENS substantially depends on feeder sectionalizing capability, types of installed switching devices and on backup supply. So, the failure of the section *i* is reflected on customers with different average outage times. The customers, who upon identification and isolation of an affected section *i*, have service facility even before the repair, are affected with average duration r_{A1i} or r_{A2i}. If there are only load-breaking disconnectors between the affected section and supply substation, the failure will affect all customers located upstream to the first switching point from the affected section towards supply substation with average duration r_{A2i}. However, if there are also circuit breakers installed, the customers upstream from the first circuit breaker from the affected section to supply substation will not be affected in duration r_{A2i} but in duration r_{A1i}. In the case when backup supply is in place, the failure will affect the customers located downstream from the first switching point from the affected section towards the end of the feeder in duration r_{A2i}.

For customers who do not have the possibility of power supply before repair of the section affected by fault, the failure has average duration time r_{Bi}.

In general, average outage times i.e. impact of the failure of the section *i* on the service interruption, are described with these expressions:

$$r_{A1i}=0 \tag{2}$$

$$r_{A2i}=r_{Ii}+r_{N1i}+r_{N2i} \tag{3}$$

$$r_{Bi} = r_{Ii} + r_{ITi} + r_{Pi} + r_{N2i} \tag{4}$$

where:

r_{Ii} - time for notification and location of fault and isolation of the section or incident-affected part of the network (h);

r_{N1i} - supply restoring time upon fault isolation, for the part of network where it is practicable (h);

r_{N2i} - post-fault regular supply restoration time (h);

r_{ITi} - time of establishing accurate fault location in an isolated affected part of the network (h);

r_{Pi} - repair time (h).

With regard to the assumption that only load-breaking disconnectors and circuit breakers are installed in the network, in (3) the time r_{N2i}=0, because in the process of establishing normal supply conditions there is no reason for circuit breaker being activated in the substation feeder bay and additional supply interruption. Remote control of switching devices significantly reduces durations r_{Ii} , r_{N1i} and r_{N2i}. By installing fault indicators, the duration r_{ITi} becomes considerably shorter, while in networks without remote controlled switching devices a portion of r_{Ii} is additionally reduced.

Taking into account r_{N2i}=0 in (3), the difference between r_{Bi} and r_{A2i} equals:

$$r_{\Delta i} = r_{ITi} + r_{Pi} + r_{N2i} - r_{N1i} \approx r_{ITi} + r_{Pi} \tag{5}$$

2.1 MV feeder equipped with *k* switching devices one of which is a circuit breaker

A MV feeder with *n* nodes and sections is shown in Fig. 2. In *k* sections i.e., sections $d_1,d_2,\ldots,d_k$ (e.g. d_3=7 means that the third switching device is located in the section 7) switching devices are installed, one of which is a circuit breaker located in section d_x, in section $1\leq x\leq k$, while all others are load-breaking disconnectors.

1) Feeder without backup supply

In case that backup supply is not available, energy not supply of a feeder equals:

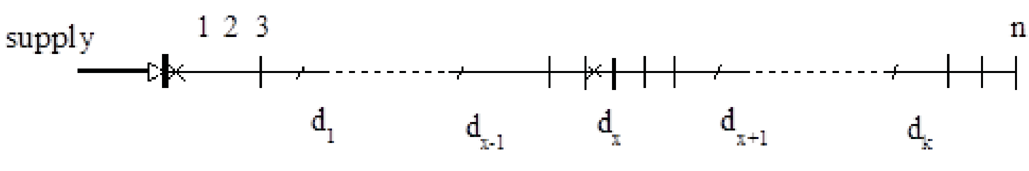

Figure 2. MV feeder with k switching devices, out of which the xth device is a circuit breaker

$$
\begin{aligned}
ENS_{nr-x} &= \sum_{i=1}^{d_1-1} \lambda_i r_{Bi} \sum_{i=1}^{n} P_i + e^{i\theta} \\
&+ \sum_{j=1}^{x-1} \sum_{i=d_j}^{d_{j+1}-1} \lambda_i (r_{A2i} \sum_{i=1}^{d_j-1} P_i + r_{Bi} \sum_{i=d_j}^{n} P_i) + \\
&+ \sum_{j=x}^{k-1} \sum_{i=d_j}^{d_{j+1}-1} \lambda_i (r_{A1i} \sum_{i=1}^{d_x-1} P_i + r_{A2i} \sum_{i=d_x}^{d_j-1} P_i + r_{Bi} \sum_{i=d_j}^{n} P_i) + \\
&+ \sum_{i=d_k}^{n} \lambda_i (r_{A1i} \sum_{i=1}^{d_x-1} P_i + r_{A2i} \sum_{i=d_x}^{d_k-1} P_i + r_{Bi} \sum_{i=d_k}^{n} P_i)
\end{aligned}
\tag{6}
$$

By introducing $r_{A2i}+r_{\Delta i}$ to replace r_{Bi} and by arranging (6) it is obtained:

$$
\begin{aligned}
ENS_{nr-x} &= \sum_{i=1}^{d_x-1} \lambda_i r_{A2i} \sum_{i=1}^{n} P_i + \\
&+ \sum_{i=d_x}^{n} \lambda_i r_{A2i} \sum_{i=d_x}^{n} P_i + \sum_{i=d_x}^{n} \lambda_i r_{A1i} \sum_{i=1}^{d_x-1} P_i + \\
&+ \sum_{i=1}^{d_1-1} \lambda_i r_{\Delta i} \sum_{i=1}^{n} P_i + \\
&+ \sum_{j=1}^{k-1} \sum_{i=d_j}^{d_{j+1}-1} \lambda_i r_{\Delta i} \sum_{i=d_j}^{n} P_i + \sum_{i=d_k}^{n} \lambda_i r_{\Delta i} \sum_{i=d_k}^{n} P_i
\end{aligned}
\tag{7}
$$

Assuming that the MV feeder is *uniformly* loaded i.e., that all nodes have the same load P, that all sections have the same failure rate λ and average outage times r_{A1}, r_{A2}, and r_{Δ}, (7) can be simplified as follows:

$$
\begin{aligned}
ENS'_{nr-x} &= n(d_x-1)\lambda r_{A2} P + \\
&+ (n-d_x+1)(d_x-1)\lambda r_{A1} P + \\
&+ (n-d_x+1)^2 \lambda r_{A2} P + \\
&+ n(d_1-1)\lambda r_{\Delta} P + \\
&+ \sum_{j=1}^{k-1} (d_{j+1}-d_j)(n-d_j+1)\lambda r_{\Delta} P + \\
&+ (n-d_k+1)^2 \lambda r_{\Delta} P
\end{aligned}
\tag{8}
$$

Because of the assumption of *a uniformly loaded* feeder, the expression (8) for energy not supplied calculation is *approximate*, but suitable for an analytical determination of optimal locations and number of switching devices. The procedure of determining average values for P, λ and r is described in [15]. The accuracy of the method is analyzed in [16].

The optimal locations for a preset number of switching devices on the MV feeder, in the case when backup supply is not available, can be determined by setting partial derivatives of the expression (8) with respect to d_j to zero. The obtained expressions for optimal locations are:

$$d_j = jA + 1 \quad j = 1, \ldots, x \tag{9}$$

$$d_j = jA + (j-x)B + 1 \quad j = x+1, \ldots, k \tag{10}$$

where:

$$A = f_A(j, n, k, x, r_{A1}, r_{A2}, r_{\Delta}) \tag{11}$$

$$B = f_B(j, n, k, x, r_{A1}, r_{A2}, r_{\Delta}) \tag{12}$$

For any value of x i.e. the circuit breaker position in range of load-breaking disconnectors, distances among switching devices before the circuit breaker, measured in sections, are mutually equal. The same goes for distances among switching devices behind the circuit breaker.

By inserting (9) and (10) into (8) we obtain the minimum amount of energy not supplied for a preset number of k switching devices on the feeder, where the x^{th} device is a circuit breaker. By setting the derivative of the obtained expression with respect to x to zero, it follows that:

$$x = \frac{k+1}{2} \tag{13}$$

which means that the circuit breaker is optimally located on the feeder if the same number of load-breaking disconnectors is located before and behind it. Taking into account (13), the expressions (9) and (10) for optimal locations of switching devices are simplified:

$$d_j = \frac{jn}{k+1} + 1 \quad j = 1, \ldots, k \tag{14}$$

It is evident from (14) that distances are equal between switching devices when the position of circuit breaker is optimal. By inserting (14) into (8) we obtain the expression for minimum energy not supplied:

$$\begin{aligned}ENS'_{nr-x,\min} &= n^2 r_{A2}\lambda P - \\ &-\frac{n^2 x(k-x+1)}{(k+1)^2}(r_{A2}-r_{A1})\lambda P + \\ &+\frac{n^2(k+2)}{2(k+1)} r_\Delta \lambda P\end{aligned} \tag{15}$$

The first term of (15) depends on switching time in the network, while the third is determined by repair time. Because of circuit breaker installation (r_{A1}=0), the second term influences energy not supply decrease caused by switching. If a load-breaking disconnector was installed instead of a circuit breaker, it would not be contributed to the energy not supplied decrease.
By taking into account (13), the second term of expression (15) could be simplified. But this was not done in order to hold proper form for its generalization in the case of installation of any number of circuit breakers.

2) A feeder with backup supply

In case that backup supply is available, energy not supply of feeder equals:

$$\begin{aligned}&ENS_{r-x} = \sum_{i=1}^{d_1-1}\lambda_i(r_{Bi}\sum_{i=1}^{d_1-1}P_i + r_{A2i}\sum_{i=d_1}^{n}P_i) + \\ &+\sum_{j=1}^{x-1}\sum_{i=d_j}^{d_{j+1}-1}\lambda_i(r_{A2i}\sum_{i=1}^{d_j-1}P_i + r_{Bi}\sum_{i=d_j}^{d_{j+1}-1}P_i + r_{A2i}\sum_{i=d_{j+1}}^{n}P_i) + \\ &+\sum_{j=x}^{k-1}\sum_{i=d_j}^{d_{j+1}-1}\lambda_i(r_{A1i}\sum_{i=1}^{d_x-1}P_i + r_{A2i}\sum_{i=d_x}^{d_j-1}P_i + \\ &\qquad + r_{Bi}\sum_{i=d_j}^{d_{j+1}-1}P_i + r_{A2i}\sum_{i=d_{j+1}}^{n}P_i) + \\ &+\sum_{i=d_k}^{n}\lambda_i(r_{A1i}\sum_{i=1}^{d_x-1}P_i + r_{A2i}\sum_{i=d_x}^{d_k-1}P_i + r_{Bi}\sum_{i=d_k}^{n}P_i)\end{aligned} \tag{16}$$

By introducing $r_{A2i}+r_{\Delta i}$ to replace r_{Bi} and by arranging (16) we obtain:

$$\begin{aligned}ENS_{r-x} &= \sum_{i=1}^{d_x-1}\lambda_i r_{A2i}\sum_{i=1}^{n}P_i + \\ &+\sum_{i=d_x}^{n}\lambda_i r_{A2i}\sum_{i=d_x}^{n}P_i + \sum_{i=d_x}^{n}\lambda_i r_{A1i}\sum_{i=1}^{d_x-1}P_i + \\ &+\sum_{i=1}^{d_1-1}\lambda_i r_{\Delta i}\sum_{i=1}^{d_1-1}P_i + \\ &+\sum_{j=1}^{k-1}\sum_{i=d_j}^{d_{j+1}-1}\lambda_i r_{\Delta i}\sum_{i=d_j}^{d_{j+1}-1}P_i + \sum_{i=d_k}^{n}\lambda_i r_{\Delta i}\sum_{i=d_k}^{n}P_i\end{aligned} \tag{17}$$

i.e. assuming that the MV feeder is *uniformly* loaded:

$$\begin{aligned}ENS'_{r-x} &= n(d_x-1)\lambda r_{A2}P + \\ &+(n-d_x+1)^2\lambda r_{A2}P + \\ &+(n-d_x+1)(d_x-1)\lambda r_{A1}P + \\ &+(d_1-1)^2\lambda r_\Delta P + \\ &+\sum_{j=1}^{k-1}(d_{j+1}-d_j)^2\lambda r_\Delta P + \\ &+(n-d_k+1)^2\lambda r_\Delta P\end{aligned} \tag{18}$$

Comparing (7) to (17) and (8) to (18), it is evident that backup supply has no influence on the amount of energy not supplied due to switching in the network; however, it depends only on the part caused by fault repair duration.
Optimal locations for a preset number of switching devices on the MV feeder in a case that backup supply is available are determined in the same manner as in a case of no backup supply. Resulting expressions for optimal locations can be functionally expressed as:

$$d_j = jE+1, \quad j=1,\dots,x \tag{19}$$

$$d_j = jE+(j-x)F+1, \quad j=x+1,\dots,k \tag{20}$$

where

$$E = f_E(j,n,k,x,r_{A1},r_{A2},r_\Delta) \tag{21}$$

$$F = f_F(j,n,k,x,r_{A1},r_{A2},r_\Delta) \tag{22}$$

By inserting (19) and (20) into (18), we obtain the minimum amount of energy not supplied. By setting the derivative of the obtained expression with respect to x to zero, x takes the value defined by (13). This means that the optimal position of a circuit breaker, in range of load-breaking disconnectors, on the feeder is independent from backup supply existence. Since the circuit breaker performs its function, it is expected that this feature distinguishes it from load-breaking disconnectors, before using backup supply.

Considering (13), the expressions (19) and (20), for optimal locations of switching devices, take the value defined by (14), as in the case without backup supply. From (18) the expression for minimum energy not supplied is obtained as:

$$\begin{aligned} ENS'_{r-x,\min} &= n^2 r_{A2}\lambda P - \\ &-\frac{n^2 x(k-x+1)}{(k+1)^2}(r_{A2}-r_{A1})\lambda P + \\ &+\frac{n^2}{k+1} r_\Delta \lambda P \end{aligned} \tag{23}$$

The expressions (15) and (23) have only third term different since it refers to energy not supplied because of fault repair duration.

2.2 MV feeder equipped with k switching devices two of which are circuit breakers

Fig. 3 shows a MV feeder equipped with k switching devices, two of which are circuit breakers installed in sections d_x and d_y, $1 \le x < y \le k$.

1) Feeder without backup supply

In case that backup supply is not available, and assuming that the MV feeder is uniformly loaded, energy not supply of feeder equals:

$$\begin{aligned} ENS'_{nr-xy} &= n(d_x-1)\lambda r_{A2}P + \\ &+(d_y-d_x)(n-d_x+1)\lambda r_{A2}P \\ &+(n-d_y+1)^2 r_{A2}P + \\ &+(d_y-d_x)(d_x-1)\lambda r_{A1}P + \\ &+(n-d_y+1)(d_y-1)\lambda r_{A1}P + \\ &+n(d_1-1)\lambda r_\Delta P + \\ &\sum_{j=1}^{k-1}(d_{j+1}-d_j)(n-d_j+1)\lambda r_\Delta P + \\ &+(n-d_k+1)^2 \lambda r_\Delta P \end{aligned} \tag{24}$$

Similarly as in 2.1.1) we can determine that the optimal placement of switching devices is:

$$x = \frac{k+1}{3} \tag{25}$$

$$y = 2\frac{k+1}{3} \tag{26}$$

Optimally allocated switching devices are evenly placed on the feeder as in the case with only one circuit breaker. By inserting (14) into (24) we obtain the expression for minimum energy not supplied:

$$\begin{aligned} ENS'_{nr-xy,\min} &= n^2 r_{A2}\lambda P - \\ &-\frac{n^2\left[(k-y+1)y+(y-x)x\right]}{(k+1)^2}(r_{A2}-r_{A1})\lambda P + \\ &+\frac{n^2(k+2)}{2(k+1)} r_\Delta \lambda P \end{aligned} \tag{27}$$

The first and third term in expressions (15) and (27) for energy not supplied are equal irrespective of the type of the switching devices installed.

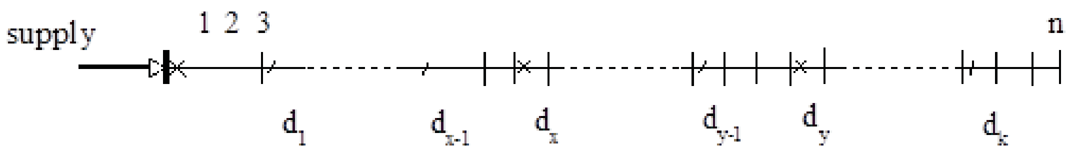

Figure 3. MV feeder with k switching devices, out of which the xth and yth device are circuit breakers

The second term depends on the number of circuit breakers installed and presents their impact on the energy not supply decrease by avoiding supply interruption for those customers located upstream the first circuit breaker in the fault location.

2) Feeder with backup supply

In case that backup supply is available, assuming that the MV feeder is uniformly loaded, energy not supplied of feeder equals:

$$\begin{aligned} ENS'_{r-xy} &= n(d_x-1)\lambda r_{A2}P + \\ &+(d_y-d_x)(n-d_x+1)\lambda r_{A2}P + \\ &+(n-d_y+1)^2 r_{A2}P + \\ &+(d_y-d_x)(d_x-1)\lambda r_{A1}P + \\ &+(n-d_y+1)(d_y-1)\lambda r_{A1}P + \\ &+(d_1-1)^2\lambda r_\Delta P + \\ &+\sum_{j=1}^{k-1}(d_{j+1}-d_j)^2\lambda r_\Delta P + \\ &+(n-d_k+1)^2\lambda r_\Delta P \end{aligned} \quad (28)$$

For this case also, it can be shown that optimal placement of the circuit breakers is defined by (25) and (26) and the optimal allocation of all switching devices on the feeder by (14). By inserting (14) into (28) we obtain the expression for minimum energy not supplied:

$$\begin{aligned} ENS'_{r-xy,\min} &= n^2 r_{A2}\lambda P - \\ &-\frac{n^2\left[(k-y+1)y+(y-x)x\right]}{(k+1)^2}(r_{A2}-r_{A1})\lambda P + \\ &+\frac{n^2}{k+1}r_\Delta\lambda P \end{aligned} \quad (29)$$

Comparing expressions (15), (23), (27) and (29) for minimum energy not supplied it is observed that their first term is always the same, the second term depends on the number of the circuit breakers and the third term depends on the backup supply regardless of the number of the circuit breakers on the feeder.

2.3 A MV feeder equipped with k switching devices b of which are circuit breakers

In order to generalize the second term, in the expression for minimum energy not supplied, which depends on number of the installed circuit breakers, we will use a simple example. Fig. 4 shows an example of a MV feeder consisting of n =24 nodes and k=5 switching devices, b=2 of which circuit breakers and remaining ones are load-breaking disconnectors.

For this case, second terms in expressions (27) and (29) for energy not supplied can be shown as:

$$\begin{aligned} &\frac{n^2\left[(k-y+1)y+(y-x)x\right]}{(k+1)^2}(r_{A2}-r_{A1})\lambda P = \\ &=\left[\frac{n}{k+1}(k-y+1)\lambda\cdot\frac{n}{k+1}yP + \right. \\ &\left. +\frac{n}{k+1}(y-x)\lambda\cdot\frac{n}{k+1}xP\right](r_{A2}-r_{A1}) \end{aligned} \quad (30)$$

Terms $\frac{n}{k+1}(k-y+1)$ and $\frac{n}{k+1}(y-x)$ represent the number of feeder sections between feeder end node and second circuit breaker i.e. feeder sections between the second and the first circuit breaker.

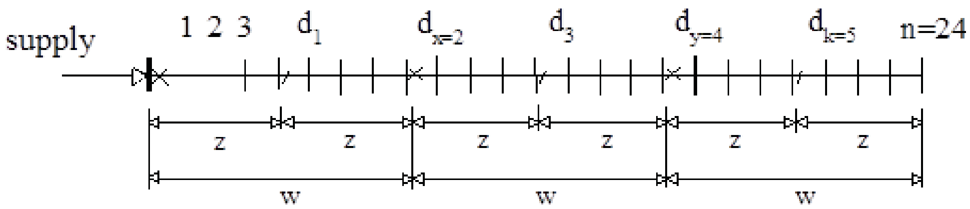

Figure 4. MV feeder with 5 switching devices, out of which the xth and yth device are circuit breakers, ($z=\frac{n}{k+1}=4$, $w=\frac{n}{b+1}=8$)

These terms have the same value and amount to $w=\frac{n}{b+1}$; their number is equal to the number of circuit breakers. The term $\frac{n}{k+1}y$ represents the number of customers between the supply substation and the second circuit breaker and the term $\frac{n}{k+1}x$ represents the number of customers between the supply substation and the first circuit breaker. Values of these terms are not equal and represent the multiples of $\frac{n}{k+1}x$ i.e. w. In a general case when a MV feeder is equipped with k switching devices, b of which are circuit breakers, $0 \le b \le k$, (30) can be expressed as:

$$\begin{aligned}&\sum_{i=1}^{b}\left[w\lambda \cdot wiP\right](r_{A2}-r_{A1})=\\&=\frac{n^2}{(b+1)^2}\lambda P(r_{A2}-r_{A1})\sum_{i=1}^{b} i=\\&=\frac{n^2 b}{2(b+1)}\lambda P(r_{A2}-r_{A1})\end{aligned} \tag{31}$$

Now the expression for minimum energy not supplied of a MV feeder equipped with k switching devices, b of which are circuit breakers, in the case without backup supply is:

$$\begin{aligned}ENS'_{nr-b,\min} &= n^2 r_{A2}\lambda P-\\&-\frac{n^2 b}{2(b+1)}(r_{A2}-r_{A1})\lambda P+\\&+\frac{n^2(k+2)}{2(k+1)}r_\Delta \lambda P\end{aligned} \tag{32}$$

and in the case with backup supply:

$$\begin{aligned}ENS'_{r-b,\min} &= n^2 r_{A2}\lambda P-\\&-\frac{n^2 b}{2(b+1)}(r_{A2}-r_{A1})\lambda P+\\&+\frac{n^2}{k+1}r_\Delta \lambda P\end{aligned} \tag{33}$$

3 Determining optimal number and types of switching devices on the feeder

Determining discounted costs of energy not supplied and switching devices in an observed planning period is described in [15]. Total discounted costs of energy not supplied and switching devices, with their optimum placement on the MV feeder, in the case without backup supply equals to:

$$\begin{aligned}T_{nr} &= \left[r_{A2}-\frac{b}{2(b+1)}(r_{A2}-r_{A1})+\right.\\&\left.+\frac{k+2}{2(k+1)}r_\Delta\right]n^2\lambda P c_e G(t)\\&+C(k-b)+Cf_p b\end{aligned} \tag{34}$$

where:
k - number of switching devices;
b - number of circuit breakers;
C - total investment and discounted maintenance costs for load-breaking disconnector ($);
f_p - ratio of circuit breaker investment and discounted maintenance costs to appropriate load-breaking disconnector costs;
c_e - price of energy not supplied [$/kWh].

The function $G(t)$ discounts costs occurring during the observed period t to present value and equals to:

$$G(t)=\frac{(1+p/100)^t-1}{(1+p/100)^t(p/100)} \tag{35}$$

where p is discount rate (%). By setting the partial derivatives of (34) with respect to k and b to zero an optimal number of total switching devices as well as circuit breakers on the MV feeder can be obtained with:

$$k_{nrOPT}=n\cdot\sqrt{\frac{\lambda P c'_e r_\Delta}{2C}}-1 \tag{36}$$

$$b_{nrOPT}=n\cdot\sqrt{\frac{\lambda P c'_e (r_{A2}-r_{A1})}{2C(f_p-1)}}-1 \tag{37}$$

It should be emphasized that it is not profitable to install circuit breakers i.e. that is $b_{nrOPT} \le 0$ if

$$f_p \geq \frac{n^2 \lambda P c_e^{'} (r_{A2} - r_{A1})}{2C} + 1 \tag{38}$$

Total discounted costs of energy not supplied and switching devices in the case with backup supply are equal to:

$$T_r = \left[r_{A2} - \frac{b}{2(b+1)}(r_{A2} - r_{A1}) + \right.$$
$$\left. + \frac{1}{k+1} r_\Delta \right] n^2 \lambda P c_e^{'} + \tag{39}$$
$$+ C(k-b) + C f_p b$$

By setting the partial derivatives of (39) with respect to k and b to zero optimal numbers of switching devices on the MV feeder can be obtained:

$$k_{rOPT} = n \cdot \sqrt{\frac{\lambda P c_e^{'} r_\Delta}{C}} - 1 \tag{40}$$

$$b_{rOPT} = n \cdot \sqrt{\frac{\lambda P c_e^{'} (r_{A2} - r_{A1})}{2C(f_p - 1)}} - 1 = b_{nrOPT} \tag{41}$$

The optimal total number of switching devices is greater in the case with backup supply while the optimal number of circuit breakers is the same in both cases.

4 Costs and profitability of backup supply

The lowest overall discounted cost for the feeder without backup supply is obtained by inserting (34) and (37) into (34):

$$T_{nrOPT} = \frac{n^2}{2} \cdot \lambda P c_e^{'} (r_\Delta + r_{A2} + r_{A1}) +$$
$$+ n \cdot \sqrt{2 \lambda P c_e^{'} C} \cdot \left[\sqrt{r_\Delta} + \sqrt{(f_p - 1)(r_{A2} - r_{A1})} \right] - \tag{42}$$
$$- C f_p$$

while for a feeder with backup supply by inserting (40) and (41) into (39):

$$T_{rOPT} = \frac{n^2}{2} \cdot \lambda P c_e^{'} (r_{A2} + r_{A1}) +$$
$$+ n \cdot \sqrt{2 \lambda P c_e^{'} C} \cdot \left[\sqrt{2 r_\Delta} + \sqrt{(f_p - 1)(r_{A2} - r_{A1})} \right] - \tag{43}$$
$$- C f_p$$

In the MV power distribution network planning, the basic criterion for assessing the justifiability of constructing a tie line in order to ensure alternative supply is the difference between (42) and (43) i.e. the difference between overall optimal costs in situations without and with backup supply:

$$\Delta = \frac{n^2}{2} \lambda P c_e^{'} r_\Delta - n \sqrt{2 \lambda P c_e^{'} C r_\Delta} \cdot (\sqrt{2} - 1) \tag{44}$$

An expression for difference between optimal total costs, when MV feeder is equipped only with load-breaking disconnectors, in cases without and with backup supply is given in [15] and it is equal to expression (44). This means that the difference between optimal total costs does not depend on occurrence of a circuit breaker in the optimal solution.

The installation and maintenance costs of a tie line T_{CL} in the planning period t of the network [15], are:

$$T_{CL} = T_L^{'} l_{CL} \tag{45}$$

where:

l_{CL} - length of connecting line (km);

T'_L - investment costs and annual discounted maintenance costs of a line per unit of length ($/km).

If the condition:

$$\Delta > T_{CL} \tag{46}$$

is satisfied, then, the construction of line is justifiable.

It is presumed that the backup supply is provided with the construction of a tie line towards the closest feeder from the same or adjacent supply substations.

5 An example of the MV feeder

Fig. 5 shows an example of a 10 kV feeder. Basic data, including failure rate λ_i, outage times r_{A2i} and

$r_{\Delta i}$ as well as average loads P_i of the feeder are given in [14].

Total average load of the feeder is 1081 kW, total length is 12.1 km, total number of nodes is n_{tot}=24 and number of nodes of the equivalent feeder i.e. feeder without laterals is n=10.

Presuming that the feeder is uniformly loaded, the obtained average values of the approximation feeder parameters are λ= 0.121 (failure/year), r_{A2}= 1.0793 h, r_Δ=0.9917 h and P = 108.1 kW [14]. Based on them, the total costs of energy not supplied and switching devices are calculated.

It is assumed that the load-breaking disconnectors and circuit breakers can be installed on the feeder. The observed period is t= 20 years and a discount rate is p = 10%.

With the price of energy not supplied c_e=4.85 ($/kWh) the result is c'_e= 41.3 ($/kWh) [14]. Assuming that the installation costs of manually operated load-break disconnector and circuit breaker are 1700 $ and 8500 $, respectively, and annual maintenance costs are 5 %, the procedure results in C = 2423 $ and f_p=5.

In the case without backup supply capacity, according to (36) and (37), we get the optimal number of switching devices k_{nrOPT} = 2.32 ≈ 2, of which $b_{nrOPT} = 0.73 \approx 1$ is a circuit breaker. The related minimum total cost, according to (42), is T_{nrOPT} = 93 554.2 $.

In the case with backup supply, according to (40) and (41), it follows that k_{rOPT} = 3.70 ≈ 4 and $b_{rOPT} = b_{nrOPT} = 0.73 \approx 1$. The related minimum total cost, according to (43), is T_{rOPT} = 73 442.0 $.

Optimal total costs, consisting of energy not supplied costs and switching devices costs, T_{nrOPT} and T_{rOPT} are lower than in the situation without considering the circuit breaker installation on the feeder [14]. In the case without and with backup supply, total costs are 5.3% and 6.6% lower, respectively. But the difference between the total minimum costs, consisting of energy not supplied costs and switching devices costs, for cases without and with backup supply is the same, because it does not depend on the circuit breaker installation, and equals to Δ = 20 112.2 $.

The values for T_{brOPT}, T_{rOPT} and Δ are obtained for non-integer numbers of switching devices. If we insert integer values into (34) and (39), i.e. k_{nr}=2, k_r=4 i b_{nr}=b_r=1, we will get T_{nrOPT} = 93 981.5 $, T_{rOPT} = 73 827.0 $ and Δ= 20154.5 $, which indicates that the deviation is negligible.

Similarly, by using (44), the savings can be calculated for another feeder which is connected to the considered one with the aim to provide the backup supply. In this case, savings are assumed to be the same for both feeders so that total cost reduction comes to Δ_{tot} = 40224.4 $.

If investment costs of the tie line are 43700 ($/km) and annual maintenance 5%, respectively, then, T_V'= 62294 ($/km). It means that the construction would be cost-effective if the connecting line length is l_{CL} < 0.646 km, as it has already been shown in [14].

6 Conclusion

The explicit inclusion of reliability in optimal MV distribution network planning implies that additional investments in the network should be considered in order to enhance the reliability value and reduce the energy not supplied costs. The most important impact of reliability on the optimal MV feeder routing, with assumed automation level of the distribution network, is reflected through the construction of backup supply lines and decreasing energy not supplied costs. Consequently, the possibility to evaluate these costs efficiently is very important within the whole optimization procedure.

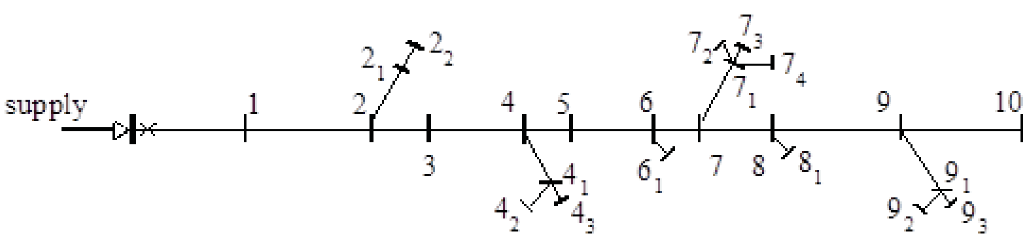

Figure 5. 10 kV MV feeder

Such an approximate method is presented in this paper. It enables a simple calculation of the difference between total optimal costs, consisting of energy not supplied costs and switching (device) costs, for cases without and with backup supply. This difference in costs is a criterion of tie feeder construction profitability. The method is an improvement with regards to [15] in the sense that simultaneous installation of load-breaking disconnectors and circuit breakers can be considered.

This paper considers how the installation of circuit breakers influences the switching time and energy not supplied costs. It is shown that backup supply, for any installation combination of the load-breaking disconnector and circuit breakers, does not have any impact on an optimal number of the circuit breakers in the feeder. This is the consequence of the fact that the circuit breaker realizes its specific function always before the backup supply activation.

Consequently, the same expression for the difference between minimum total costs, for cases without and with backup supply, is obtained as in the situation without considering the circuit breaker installation.

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