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Borderline Cases and Definiteness

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ABSTRACT: Borderline cases of vague predicates are often characterized with the help of a definiteness operator. Although such operators can certainly contribute to the solution of the problem of vagueness, they may also generate unexpected consequences. Either borderlineness is identified implicitly with a well-defined range of cases, or borderline cases are seen as being definitely borderline. In this paper I argue that we can avoid these consequences by providing an asymmetric definition of borderline cases.

KEY WORDS: Asymmetric definition, borderline cases, definiteness, vagueness.

I. Introduction

Recent theories of vagueness tend to assume, almost routinely, that the most efficient method for providing a suitable formal characterization of borderline cases is the method of definitization. The primary purpose in introducing a definiteness operator, *D*, into the object language, is to make possible a classificatory distinction between different application cases of vague predicates. In contrast to obvious cases where *F* definitely applies or definitely does not apply to *a*, borderline cases are usually thought of as those in which *F* exhibits a certain kind of classificatory uncertainty with respect to *a*. The method of definitization is indeed quite general. It is applied more or less invariantly in otherwise incompatible approaches to vagueness.[1]

[1] In this paper, I will focus only on the most general theoretical consequences of definitization. Though followers of supervaluationism, epistemicism, contextualism and other approaches differ considerably in the strategies they introduce the definiteness operator, I will not analyse the individual differences. On my view, the question of the relation between borderline cases and definiteness is independent enough from particular theories to constitute an autonomous methodological problem.

One can easily explain why the presence of a formal tool like *D* is supposed to be essential for the characterization of borderlineness. Just imagine someone trying to say what it is to be a borderline case without exploiting the benefits of the definiteness operator. The most natural option for her would be to claim that a borderline case is an object *a* to which neither *F* nor ¬*F* applies.[2] That claim would be inappropriate for at least two reasons. First, if *F* is a partially defined predicate, then, *per definitionem*, there must be an *a* to which neither *F* nor ¬*F* applies. As an illustration, let us introduce the predicate nice* in the following way: (i) a whole number *n* is nice*, if *n* is greater than 15, and (ii) a whole number *n* is not nice*, if *n* is less than 15.[3] Since nice* remains undefined for the number 15, it is not misleading to conclude that neither nice* nor ¬nice* applies to it. This might suggest that number 15 is a borderline case of nice*. But it is not so. It is merely an undefined case for our sharply bounded artificial predicate nice*. So it can be seen that the proposed account is not specific enough to allow us to distinguish between borderline cases and cases of undefinedness. Second, in claiming that the classificatory uncertainty in the borderline region emerges from the inapplicability of *F* and ¬*F*, one also implicitly claims that borderline cases of *F* are incompatible with both positive and negative application cases of *F* – and that sounds quite implausible. Even if it is uncertain whether *F* applies to *a* or not, there is no in principle reason to reject categorically the classification of *a* either as *F* or as ¬*F*. Uncertainty with respect to the correct application conditions of *F* does not exclude the possibility that *a* could be classified as being either *F* or ¬*F*. To think otherwise would be to put forth a rather idiosyncratic concept of borderlineness.

The arguments above provide at least a partial explanation for why it is folly to try to characterize borderline cases without using the definiteness operator. In order to show how we can accommodate classificatory uncertainties to our systematic reasoning, we need first identify positive and negative cases where the application of vague predicates does not pose any problem. Such a task can be easily accomplished by using standard logical tools. But when we begin to articulate how positive and negative cases relate to borderline cases, the method of definitization proves to be unavoidable.

Although the introduction of a definiteness operator can certainly contribute to the solution of the classificatory problem of vague predi-

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[2] Such a view is rarely advanced in contemporary debates over borderlineness, but one example is to be found in Horwich (2005).

[3] The example originates from Fine (1975). In that paper, Fine holds that predicates like nice* are deficient in meaning and that is enough for them to be vague. Tye (1997) seems to agree, but see Barnett (2010) for an argument against Fine's account.

cates, it may also generate new difficulties. The potential difficulties seem, in general, to be independent from the informal specification of the operator in question. Some hold that ‘definitely’ should be taken as a primitive notion.[4] If this is the case, then we cannot explain its meaning in terms of more fundamental notions. Rather, we come to understand it in such a way that we recognize the logical rules that govern its use. Others argue that ‘definitely’ must be regarded as a semantic-ontic notion.[5] On this view, there are sufficient semantic and factual conditions for the application of the operator. If a is definitely F, then the linguistic conventions determine conditions for the application of F and the facts about a determine that these conditions are met. A further idea is that ‘definitely’ requires an epistemic explanation. According to this explanation, the effects of the definiteness operator are epistemically similar to the effects of the knowledge operator: F may be thought to apply definitely to a when there are no specific obstacles to knowing that a is F.[6]

One claim of the present paper is that these informal specifications are much less significant to the vexed problem of borderline cases as it is usually assumed. The definiteness operator generates difficulties largely independently from the way in which it is introduced into the object language. That is why it appears to give rise to similar complications in most extant theories of vagueness.

II. Unexpected Consequences of *D*-Introduction

One natural-seeming assumption with regard to D-introduction is that in its unconstrained form the rule $\vdash A \rightarrow DA$ is invalid. Borderline application cases appear to provide rather clear counterexamples to this rule. If Fa is an instance of A and F exhibits classificatory uncertainty with respect to a, then, obviously, it is fallacious to infer from A to DA. So presumably the rule $\vdash A \rightarrow DA$ is valid only in cases where F applies obviously to a even in the D-free object language. There may be more than one correct answer to the question of why paradigmatic vague predicates like ‘bald’, ‘heap’, or ‘rich’ must have such obvious application cases. Perhaps the lexical meaning of F would remain essentially incomplete or imperfect without the existence of obvious application cases. Perhaps there are logical principles which require that F be applicable to certain

[4] Field (2001).

[5] McGee and McLaughlin (1994).

[6] See Williamson (2004). It should be mentioned, though, that according to the standard epistemic view not all kinds of epistemic obstacles are removable in vague contexts, so ‘It is known that’ and ‘It is definitely the case that’ are conceived of as distinct operators with partly overlapping functions.

objects without classificatory uncertainty. Or it can be that the existence of obvious cases is a result of a certain set of epistemic processes. Very likely, one of these conceptions is correct, but we do not really need to know which one. As an illustration, let us suppose that poor Harry has zero hairs on his head. Suppose furthermore that he is an adult male who has lost all of his hairs because of the natural process of aging. Then, I think, it would be quite pointless to debate that the predicate 'bald' obviously applies to him. Given this fact, it would be also pointless to debate that he can be called definitely bald. This indicates that no matter what view one holds about the possible sources of the existence of the obvious application cases of 'bald', there will be a strong agreement that Harry is a definitely bald person. The example seems to generalize to many other vague expressions. Predicates like 'heap', 'rich', or 'old' are similar to 'bald' in that they also may be associated with discourse contexts where the correctness of their application seems to be beyond any reasonable doubt. Hence the following principle may be established for this family of predicates:

> CONSTRAINED D-INTRODUCTION. $\vdash A \rightarrow DA$ is valid if and only if D-free instances of A qualify as obvious application cases.[7]

Before the acceptability of CONSTRAINED D-INTRODUCTION can be discussed properly, a short clarification is in order. Nothing has been said so far on the agent-relative conditions of being obvious. What does it mean that F applies obviously to a? Is 'obviously' related to the linguistic/epistemic capacities of a single agent or a particular community? It is advisable here to follow an indirect strategy and say that if there is at least one ideally competent agent in a community c to whom the F-ness of a seems not immediately evident, then in c, F does not apply obviously to a. When there is no such an agent in c, the obviousness condition should be interpreted as automatically satisfied. For our purposes, the notion ideally competent agent' denotes an agent who has full linguistic competence with respect to the meaning of F and has the epistemic capacity to gather and understand all F-relevant information about a. This condition, as it is stated here, is far from being extremely stringent, since the vast majority of adult language users may be regarded as ideally competent with respect to obvious application cases of vague predicates in their own linguistic community.

[7] The proposed constraint on the D-introduction rule should be interpreted again in a theory-neutral way. In spite of the great differences in their conceptual machinery, nearly all extant theories of vagueness admit, in their own way, that there exist obvious application cases of vague predicates.

Of course, it may be doubted that this is the right way to constrain D-introduction.[8] Even the theoretical legitimacy of the notion of obvious application case may be questioned.[9] But we should not forget the theoretical role D-introduction is intended to play in the solution of the classificatory problem of vague predications. If we rejected the constrained version of $\vdash A \rightarrow DA$, borderline cases of application would be characterizable only in terms of F and $\neg F$, and this kind of characterization, as we have seen, is clearly untenable.

It should be added, however, that beyond its theoretical benefits D-introduction has also considerable disadvantages. Borderline application cases are often not only informally characterized, but also *defined*, in a contrastive fashion, with the help of the definiteness operator. The most preferred version of the definition states that a is a borderline case of F if and only if neither F nor $\neg F$ applies definitely to it. By employing semantic descent, a more concise form is attainable:

DEFINITION OF BORDERLINE CASE. An object a is borderline F if and only if it is neither DF nor $D\neg F$.[10]

One apparent disadvantage of this definition is that it implicitly entails a sharp classificatory line between borderline and obvious application cases of F. Borderline cases are directly contrasted with obviously positive and negative application cases, and the resulting tripartite classification does not seem to allow for any kind of overlap between them. In this way, the definition creates the impression that being a borderline F is a sharply delineated property or status which is incompatible both with being DF and with being $D\neg F$. But that clashes with our original intentions. What we were after was a suitable formal characterization of the classificatory uncertainty exhibited by borderline predications. Instead of this, we have now arrived at an eliminative definition that identifies borderlineness with a well-defined range of cases.

The usual and probably the only available response to this unexpected difficulty is to deny that the definition draws a sharp line between borderline and non-borderline predications. One might argue that, for some reason, D-introduction is not an appropriate tool for eliminating entirely the vagueness of F. For example, if in vague predications D functions merely as an operator of contrast intensifier, then it may not be unreasonable to

[8] An alternative account explicates the definiteness operator on the model of the necessity operator. On this account, D-introduction may be constrained in terms of accessible possible worlds. See, for example, Gaifman (2010).

[9] Braun and Sider (2007).

[10] Cf., among others, Gaifman (2010) and Bobzien (2010).

introduce into the formal system a new classificatory set of cases, namely borderline cases of being $DF/D\neg F$. After this step, there remains no room for making a sharp discrimination between borderline and non-borderline cases of being F. But the difficulty reappears in the same form, since the introduction of borderline cases of being $DF/D\neg F$ induces a sharp classificatory line between cases of being $DF/D\neg F$ and borderline cases of being $DF/D\neg F$. Again the usual response is to run an iterative process and permit the existence of borderline cases of being $DF/D\neg F$. Given that the process does not apparently terminate, one obtains infinitely many higher-order borderline cases of being $DF/D\neg F$.

Even if this conception of infinite higher-order vagueness is coherent in itself, one might doubt whether it provides an adequate solution to the difficulty arising from the definition of borderline case.[11] The troubling factor is that each member of the hierarchy of iterated cases involves inevitably a significant weakening of the definitizing effect of D. The first step in the iteration process implies already the presence of some uncertainty in predications containing $DF/D\neg F$. This is, at least, controversial. Recall that in our earlier example the predicate 'is definitely bald' proved to be indisputably applicable to Harry. If it turned out that Harry represents, contrary to our deep convictions, a borderline case of definite baldness, then this would show that 'bald' cannot be applied with certainty even under maximally appropriate application conditions. The standard conception of higher-order vagueness does not exclude this counterintuitive possibility, so it would probably not get us out of our present difficulty.

A further and perhaps even more peculiar difficulty following from the definition above concerns the existence of definite borderlineness. We assumed earlier that D-introduction for A is valid if and only if D-free instances of A qualify as obvious application cases. On that natural assumption, DF applies to a only in cases where there is no reasonable ground for doubting that a is F. The same holds for $D\neg F$. But note that, according to their definition, the classificatory status of borderline cases is not different in *this* respect. If a is neither DF nor $D\neg F$, then we have no reasonable ground for doubting its borderline status. Thus D-introduction must also be valid for borderline predications, and, as a surprising consequence, each borderline case has to be conceived as a definite borderline case. That

[11] It is worth mentioning that Raffman (2010), Wright (2011) and others argue persuasively against the coherence of the notion of non-terminating higher-order vagueness. The main point of their criticism is that, on purely conceptual grounds, we cannot have a stable idea of the phenomenon to which the notion would apply. Wright even goes so far as to state that higher-order vagueness "is a fantasy". Such an objection seems to me relevant and well-founded, but I will not argue that here.

is, it seems that after endorsing CONSTRAINED D-INTRODUCTION and DEFINITION OF BORDERLINE CASE we have no choice but to accept the rule $\vdash (\neg DFa \,\&\, \neg D\neg Fa) \rightarrow D(\neg DFa \,\&\, \neg D\neg Fa)$. With this enforced step we end again in a situation in which all attempts at a formal rendering of the uncertainty of borderline predications seem to be doomed to failure.

III. Is There a Way Out?

The difficulties we have just discussed arise from the combination of two general insights: (i) the uncertainty of borderline predications cannot be properly formalized in a system in which a suitably constrained form of *D*-introduction is not valid, and (ii) borderline cases of F have to be defined as being neither DF nor $D\neg F$. As we have already seen, conceptual considerations strongly suggest that (i) is correct independently of the informal interpretation of the definiteness operator. So if we want to make any progress, we must revise (ii). It is not easy to accomplish this task, however, since (ii) also appears intuitively accurate.

My proposal is that, at the present level of discussion, we should interpret (ii) as a kind of *asymmetric definition*. While there is nothing wrong in saying that a is a borderline case of F if it is neither DF nor $D\neg F$, it may be doubted whether the reverse direction of the condition relation holds.

Let us divide the diagnosis into two parts. Notice, first, that from the fact that a is not DF, one cannot infer to *a*s object-language status with respect to being F, because being not DF is not a firm basis to support any such conclusion. Perhaps a can be classified as a borderline case of F. Perhaps it can be acceptably classified as F. It may even turn out that it can be considered to be $\neg F$. Unfortunately, there is no known principled way to decide which of the candidate classifications is the ultimately correct one. The situation is analogical when a is not $D\neg F$: on this basis alone, one cannot ascertain the correct classificatory status of a. If this is so, no stable classificatory judgment can be derived from the fact that a is neither DF nor $D\neg F$. This means, in a more formal way, that $BFa \rightarrow (\neg DFa \,\&\, \neg D\neg Fa)$ but $(\neg DFa \,\&\, \neg D\neg Fa) \not\rightarrow BFa$, where BFa stands for a borderline case of F.[12]

Notice, second, that we must not remain entirely silent about the formal properties of borderline cases even when we read the asymmetric definition from right to left. Although the classificatory status of a proves

[12] Perhaps $(\neg DFa \,\&\, \neg D\neg Fa) \rightarrow (Fa \vee \neg Fa \vee BFa)$ is a valid derivation. That would provide further support to our observation that from $\neg DFa \,\&\, \neg D\neg Fa$ no *stable* judgment can be derived concerning the classificatory status of a.

to be unascertainable on this reading, one is still allowed to adopt a modal perspective. The core idea here is that in the borderline region only non-factual modal judgments can be regarded as entirely unobjectionable. In particular, in cases where a is neither DF nor $D\neg F$ all that can reasonably be asserted positively about a is that it might be F, or that it might be $\neg F$.[13] Such non-factual judgements can even be made simultaneously in the same discourse situation. Let us take the example of Jerry, who is claimed to be neither definitely bald nor definitely not bald. According to the present proposal, we can then confidently and simultaneously assert that Jerry might be bald and that Jerry might be not bald. The emphasis lies on the non-factuality of these assertions: in making might-claims we leave open the question concerning the factual relation between Jerry and the property of baldness. Speaking in this manner seems unavoidable if we do not wish to assign a sharply delineated factual status to Jerry between baldness and non-baldness.

This latter point is of considerable importance. A quick comparison with close alternatives may help to reveal the tenability of the present approach.

Absolute agnosticism maintains that in the borderline region judgments about baldness are impossible. According to this radical view, even ideally competent agents are unable to tell whether Jerry is bald or not bald.[14] The scope of the ignorance is assumed to extend also to the property of being a borderline case of baldness. Thus, it is not only the case that we cannot tell whether Jerry is bald or not bald, we cannot even tell whether Jerry is borderline bald or not borderline bald. This type of absolute agnosticism might be criticized on the ground that it sees borderline cases as a sharply separable kind: borderline cases are thought to be precisely those cases in which competent agents cannot make any judgment.

Moderate versions of agnosticism claim only that in the borderline region agents are not in a position to make stable judgments about hardly classifiable objects. Moderate agnosticists think the characteristic reaction to the case of Jerry is something like “a drying of the springs of opinion”.[15] If this is intended to mean that there is a systematic correlation between borderline cases and the reaction of withdrawal from judging, then we are dangerously close again to positing sharply separated borderline cases. If it means something significantly weaker, then moderate agnosticism may be very similar, in one respect, to the view presented here.

[13] Some theories of modality consider ‘might’ statements factual. So I must admit that my proposal is not compatible with all extant views in this research area.

[14] The best elaboration of this position is to be found in Bobzien (2010).

[15] The metaphor is taken from Wright (2011).

An important lesson is that DEFINITION OF BORDERLINE CASE should not be understood as a concealed procedure for establishing the existence of a third type status between being definitely *F* and being definitely not *F*. Recognizing the asymmetric structure of this definition may be a decisive step in that direction. It should be realized that, because of the validity of CONSTRAINED D-INTRODUCTION, the left-right direction holds without any restriction, but given the irremediable lack of appropriate logical and conceptual tools, the right-left direction allows only a modally weakened reading.

For the purposes of a formal analysis, this may seem a rather disappointing result. To contend that borderline cases have to be identified in part with classificatory possibilities is, after all, to acknowledge that they are not fully systematizable. At the same time, the account offered by the present proposal is not wholly pessimistic. It shows that an asymmetric definition does not eliminate the uncertainty characteristic of borderline cases. It also makes clear that a suitably constrained *D*-introduction need not necessarily involve a sharp classificatory line between different application cases of vague predicates. A fortunate side-effect of this is that one should no longer have to worry about the difficulties associated with higher-order vagueness and definite borderline cases.[16,17]

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[16] For a similar approach which focuses on the epistemic dimension of the problem surrounding borderline cases, see Vecsey (2011).

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