NEW DOCTORAL DEGREES

IN THE DEPARTMENT OF MATHEMATICS
UNIVERSITY OF OSIJEK

Dr. Ivana Kuzmanović received her PhD in Mathematics from the Department of Mathematics of the University of Zagreb on 12th June 2012 with the dissertation entitled "OPTIMIZATION OF THE SOLUTION OF PARAMETER DEPENDING SYLVESTER EQUATION AND APPLICATIONS " (Mentor: Prof. Ninoslav Truhar).

Abstract

In dissertation the problem of solving and optimizing the solution of structured Sylvester (i.e. Lyapunov) and T-Sylvester matrix equations is considered, with special focus on the parameter dependent Sylvester and T-Sylvester equations. It has been shown that the use of a structure can significantly contribute to acceleration of the process of solving the Sylvester as well as the T-Sylvester equation, and especially some related sequences of equations that arise e.g. in the process of optimization of the solution of parameter-dependent Sylvester and T-Sylvester equations.

Due to the appropriate structure, Sylvester equations in which matrices are the sum of simple matrices (e.g. diagonal or block-diagonal) with small-rank matrices are considered in dissertation, i.e. Sylvester equations of the form

$$(A_0 + U_1V_1)X + X(B_0 + U_2V_2) = E,$$

where A_0, B_0 are simple matrices and U_1, V_1, U_2, V_2 are matrices with a low rank r. By using the standard Sherman-Morrison-Woodbury formula it is possible to obtain the so-called Sherman-Morrison-Woodbury formula for the solution of the previous equation. The obtained formula can be used for the construction of an algorithm that solves the equation of the given form much more efficiently than standard algorithms. An algorithm based on the Sherman-Morrison-Woodbury formula is especially efficient for computing the solution of the parameter dependent Sylvester equation

$$(A_0 - vU_1V_1)X(v) + X(v)(B_0 - vU_2V_2) = E$$

for many different values of parameter v. While the standard methods need $O(n^3)$ elementary operations for each different value of parameter v, the method based on the Sherman-Morrison-Woodbury formula has complexity $\mathcal{O}(rkn^2)$, where $r, k \ll n$ for the first value of v, while each of the following solving processes with the other value of v needs only $\mathcal{O}(rn^2)$ operations, where k is the dimension of the corresponding Krylov subspace. In addition, this approach also allows computation of derivatives of $X(v)$ in $O(rn^2)$ elementary operations, which enables efficient optimization of the solution $X(v)$ with respect to parameter v.

A special case, a parameter-dependent Lyapunov equation of the form

$$(A_0 - vUU^T)X(v) + X(v)(A_0 - vUU^T)^T = E,$$

occurs during calculation and optimization of dampers' viscosity in mechanical systems with respect to the criterion of minimizing the average total unit energy.

Similarly to the case of the Sylvester equation, the Sherman-Morrison-Woodbury formula can be applied to the T-Sylvester equation of the form

$$(A_0 + U_1V_1)X + X^T(B_0 + U_2V_2) = E,$$

where A_0, B_0 are simple matrices and U_1, V_1, U_2, V_2 are matrices with a low rank r. The obtained formula is used for the construction of an efficient algorithm for solving the aforementioned T-Sylvester equation, as well as for solving and optimizing the solution of the parameter-dependent T-Sylvester equation.

The aforementioned problem of optimizing dampers' viscosity in mechanical systems is closely related to the Lyapunov equation. The last part of dissertation considers optimal modal damping of dynamical system described by equation $M\ddot{x}+D\dot{x}+Kx = 0$ which will for given mass and stiffness matrices M and K ensure the best (with respect to some criterion) evanescence of the oscillations of the system in time. Criteria of minimum trace, minimum spectral norm and minimum Frobenius norm of the solution of system's corresponding Lyapunov equation are used as optimization criteria for this problem and optimal parameters for some special types of modal damping, such as mass proportional damping, stiffness proportional damping, Rayleigh damping, etc. are derived.

Published papers

[1] I. Kuzmanović, Z. Tomljanović, N. Truhar, *Optimization of material with modal damping*, Applied mathematics and computation, 218 (2012) , no. 13, 7326-7338.

[2] I. Kuzmanović, N. Truhar, *Optimization of the solution of the parameter-dependent Sylvester equation and applications*, Journal of computational and applied mathematics, 237 (2013) , no. 1, 136–144.

[3] I. Kuzmanovi, N. Truhar, *Sherman-Morrison-Woodbury formula for Sylvester and T-Sylvester equation with applications*, International journal of computer mathematics, accepted, published online DOI:10.1080/00207160.2012.716154