NEW DOCTORAL DEGREES

IN THE DEPARTMENT OF MATHEMATICS
UNIVERSITY OF OSIJEK

Dr. Ivan Soldo received his PhD in Mathematics from the Department of Mathematics of the University of Zagreb on 2th July 2012 with the dissertation entitled "Some diophantine problems over the imaginary quadratic fields" (Mentor: Prof. Andrej Dujella).

Abstract

Let z be an element of a commutative ring R. A Diophantine quadruple with the property $D(z)$, or a $D(z)$-quadruple, is a set of four different non-zero elements of R with the property that the product of any two distinct elements of this set increased by z is a square of some element in R.

In the first chapter of this dissertation we considered the existence of a $D(z)$-quadruple in the ring $\mathbb{Z}[\sqrt{-2}]$. We tried to extend previous results on this subject by Abu Muriefah and Al-Rashed, from 2004. We obtained several new polynomial formulas for Diophantine quadruples with the property $D(a+b\sqrt{-2}\,)$, for integers a and b satisfying certain congruence conditions. Also, there appeared some cases where sets cannot contain elements of small norm, so it was necessary to consider the coefficients of z concerning modulus greater than the usual one. Thus, this made those cases harder to handle. However, we obtained some partial results involving some congruence conditions modulo 11 on a and b. During our examination, there appeared three sporadic possible exceptions i.e. $z \in \{-1, 1 \pm 2\sqrt{-2}\}$. Note that $1 \pm 2\sqrt{-2} = -1 \cdot (1 \mp \sqrt{-2})^2$, so the existence of $D(-1)$-quadruples would imply the existence of $D(1+2\sqrt{-2})$ and $D(1-2\sqrt{-2})$-quadruples.

Therefore, in the second part of the dissertation it was reasonable to consider the problem of the existence of $D(-1)$-quadruples in $\mathbb{Z}[\sqrt{-2}\,]$ and in the ring of integers in other quadratic fields. Using some known results about the extendibility of some families of $D(-1)$-pairs over integers, we obtained similar results over the imaginary quadratic fields. Actually, if $t > 1$, we proved the following statements:

i) In $\mathbb{Z}[\sqrt{-t}]$, a $D(-1)$-quadruple of the form $\{1, 2, c, d\}$ does not exist.

ii) If $b \in \{5, 10, 26, 50\}$ and $t \neq b-1$, in $\mathbb{Z}[\sqrt{-t}]$ $D(-1)$-quadruple of the form $\{1, b, c, d\}$ does not exist.

iii) If $t \notin \{4, 16\}$, in $\mathbb{Z}[\sqrt{-t}]$ a $D(-1)$-quadruple of the form $\{1, 17, c, d\}$ does not exist.

iv) If $t \notin \{4, 9, 36\}$, in $\mathbb{Z}[\sqrt{-t}]$ a $D(-1)$-quadruple of the form $\{1, 37, c, d\}$ does not exist.

For $t = 1$ and other exceptions of (ii), (iii) and (iv), we also proved that there exist infinitely many $D(-1)$-quadruples of the form $\{1, b, -c, d\}$, $c, d > 0$ in $\mathbb{Z}[\sqrt{-t}]$.

By considering the extendibility of a $D(-1)$-pair $\{1, 17\}$ in $\mathbb{Z}[\sqrt{-2}]$ and a $D(-1)$-pair $\{1, 37\}$ in $Z[\sqrt{-3}]$, we observed that there are three possibilities for positivity of elements c and d. This led us to form the system of simultaneous Pellian equations and our attempt to find all solutions over the integers. In that way, we used results from simultaneous diophantine approximations, linear forms in logarithm of algebraic numbers and Baker-Davenport reduction.

Published papers

[1] A. DUJELLA, I. SOLDO, *Diophantine quadruples in* $\mathbb{Z}[\sqrt{-2}\,]$, An. Ştiinţ. Univ. "Ovidius" Constanţa Ser. Mat. **18**(2010), 81–98.

[2] I. SOLDO, *On the existence of Diophantine quadruples in* $\mathbb{Z}[\sqrt{-2}\,]$, submitted.

[3] I. SOLDO, *On the extensibility of* $D(-1)$*-triples* $\{1, b, c\}$ *in the ring* $\mathbb{Z}[\sqrt{-t}], t > 0$, submitted.