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ESTIMATION OF BRIDGES THROUGH IMPLEMENTATION OF ROUGH SETS THEORY

Miroslav Kuburić, Goran Ćirović, Zdravko Kapović

Original scientific paper
The estimation of large infrastructural structures, such as bridges among others, has mainly been confined to estimation of their construction value. The estimation procedure of a real, i.e. objective value implies estimation of a larger number of attributes except the ones that represent construction value of a structure, such as: location, namely location potential, traffic, sociological, ecological and other parameters that represent a set of attributes affecting market value, i.e. usability of infrastructural structures. This paper deals with devising decision rules that define this market value, i.e. usability, as well as with reducing core of attributes, based on which actual value estimation of bridges is carried out through implementation of rough sets theory. The implementation of this theory facilitates the objective formulation of artificial intelligence rules that define parameters referring to determining the value of bridges, as well as to reducing the decision rules that lead to the estimation of their optimal value.

Keywords: *bridges, estimation, rough sets, core of attributes, decision rules reduction*

Procjenjivanje mostova primjenom teorije grubih skupova

Izvorni znanstveni članak
Procjenjivanje velikih infrastrukturalnih objekata, kao što su mostovi između ostalog, u biti se ograničavalo na procjenu njihove građevinske vrijednosti. Postupak procjene stvarne tj. objektivne vrijednosti podrazumijeva procjenu većeg broja atributa osim onih koji predstavljaju građevinsku vrijednost objekta, kao što su: položaj, to jest položajni potencijal, prometni, sociološki, ekološki i drugi parametri koji predstavljaju skup atributa koji utječu na tržišnu vrijednost, tj. upotrebljivost građevinskih objekata. Ovaj se rad bavi osmišljavanjem pravila za donošenje odluka kako definirati tu tržišnu vrijednost, tj. upotrebljivost, kao i smanjenjem atributne jezgre, na temelju koje se provodi stvarna procjena vrijednosti mostova primjenom teorije grubih skupova. Primjena ove teorije omogućava nepristrano utvrđivanje pravila umjetne inteligencije kojima se definiraju parametri za određivanje vrijednosti mostova kao i smanjenje pravila za donošenje odluka koja vode procjeni njihove optimalne vrijednosti.

Ključne riječi: *mostovi, procjena, grubi skupovi, atributna jezgra, smanjenje pravila za donošenje odluka*

1 Introduction

Bridges are inexhaustible inspiration of artists, they often mean life for the inhabitants of the banks that they are connecting, and as a rule, they represent the pride of the engineers and architects and not rarely of the state itself. Frequently they are architectural, cultural and artistic symbols of the cities and as such should deserve attention of their owners or users.

One of the themes around which various dilemmas are generated is the sheer valuation of such structures. As it is in the nature of these structures that they do not belong to a group of real estate which can be put on sale, i.e. which are the subject of circulation or real estate market, we can estimate their value based only on the attempts of objective assessment criteria that influence it. The most common approach to this kind of assessment is reduced only on the estimation of the initial investment, possibly decreased with the function of time and depending on the predicted lifespan of the structure [1]. However, we have witnessed many examples in which these buildings are worth much more than just the construction price, and often this is not directly related to their age.

A conflict appears in the basis of almost any decision to be made in the process of deciding in the big field of construction activities. A conflict situation appears both at the wide, social level – strategic level (ministries, corresponding financial institutions…) and at the level of decision makers in municipalities, companies, production plants, as well as at the level of deciding by their strategic partners or interested parties. There are lots of examples: urban plan adoption, construction production planning, adoption of construction technology or the organization type of company or construction site, the estimation of large infrastructural structures (bridges, railways, motorways) [2]. Many of these decisions depend on the possibilities of decision maker or his preparedness or capability, of environment conditions, social moment, etc. Also some of these decisions are optimal, close to optimal, or satisfactory in relation to some requirements or criteria, as well as in relation to restriction or environment conditions. It is especially noted that during a long period, various management systems have used different methods to rank major repair projects, and many administrations are using poor tools and methods for spending their funds [3]. The gap between the ministries, the top administration of infrastructure, bridge management, and the final decision maker is too big, and understanding among the parties is limited. It is very important in the future to consider bridges as socioeconomic assets and to quantitatively calculate the effective utilization of expenses using the prediction of future expenses necessary for maintenance and life cycle cost, referred by Toshiyuki [4].

This paper is aimed at the attempt to perceive and define the criteria that are relevant in an attempt to assess their values, i.e. it is aimed at the attempt to select core criteria that best represent multicriteria decision-making in the procedure of their value assessment, using the application of the theory of rough sets.

Some cases in the construction business and investments can be expressed, measured and analyzed using the theory of rough sets as one of the mathematical apparatus for the treatment of imprecision, ambiguity and uncertainty. The logic of decision-making based on the theory of rough sets and the decision rules has the advantage that it does not require any prior or additional information about the data, but also the disadvantage that in the case of inconsistent rules the final answer is not possible.

Rough sets are a relatively new mathematical approach to definition and analysis of the inaccuracy, ambiguity and uncertainty. This theory was suggested by a Polish scientist Zdislaw Pawlak [5]. Since then many papers have been published, which develop further the theory of rough sets, mathematical rules and connection with other theories, decision rules [6], as well as the implementation of this theory in the decision analysis [7, 8]. The theory of rough sets is a convenient tool for the support systems in decision-making, especially when vague terms and uncertain data are involved into the process of decision-making.

The basic assumptions that rough sets theory is based on are: 1) that the objects of the universe are related to certain information (knowledge), and 2) that the objects that are characterized by the same information do not differ in relation to the available information. Vague terms, contrary to the precise terms, cannot be characterized as the information about their elements. This relationship of not distinguishing is a mathematical basis for the theory of rough sets. Accordingly, an elementary set is any set of objects that are not different, a sharp (precise) set - any union of some elementary sets, otherwise the set is rough (imprecise, vague).

The starting points in the rough set philosophy which Pawlak [5, 8] had proposed are data tables or attribute-values (information tables) or decision tables in which attributes of conditions and attributes of decision are presented. More formally, a data table is a 4 tuple:

$$S = (U, A, V, \rho), \tag{1}$$

where:
U is a set of objects (called universe);
A is a set of attributes;
V is a set of values for the attribute a (with every $a \in A$);
$\rho : U \to V_a$ is the information function.
Any subset B of A determines a binary relation $IND(B)$ on U, which will be called an *indiscernibility relation* and is defined (for $x, y \in U$) as follows:

$$(x, y) \in IND(B) \text{ if } \rho(x, a) = \rho(y, a), \forall a \in B. \tag{2}$$

Indiscernibility relation is relation of equivalence.
Let $X \subseteq U$ and $B \subseteq A$. Two basic operations in rough set theory are defined by assigning every $X \subseteq U$, two sets $B_*(X)$ i $B^*(X)$, called the *B-lower* and the *B-upper approximation* of X (Pawlak 1991):

$$B_*(X) = \{x \in U : B(x) \subseteq U\}, \tag{3}$$

$$B^*(X) = \{x \in U : B(x) \cap X \neq 0\}. \tag{4}$$

Boundary region of X is the set:

$$BN_B(X) = B^*(X) - B_*(X). \tag{5}$$

From the very definition of approximations it can be seen that they are meaning conditions (notions) of knowledge's granules. Lower approximation of rough set is union of all granules which are in whole implied in the set. Upper approximation is union of all granules which possess non-empty cut with the set. Boundary region of the set is the difference between lower and upper approximation.

The main problems that can be solved using the approach of rough sets are: 1) describing objects through attribute values, 2) describing full or partial dependency among attributes, 3) reduction of attributes, 4) meaning of attributes, and 5) generating decision rules.

From the total population of bridges, only bridges that are a part of some road direction, i.e. a part of the section within road network are considered for the purposes of research in this paper.

2 The selection of criteria

As in most assessments brought every day by an individual, in the value assessment of bridges multicriteria analysis of various criteria is actually hidden behind the overall assessment which is defined either numerically or semantically. Although the individual is not mostly even aware of them, and although he would often find it difficult to define them, the criteria taken into consideration in that occasion are not only numerous, but are also ranked in this process of thinking. However, the attempt to make objective assessments based on more criteria in one process of thinking would often be futile without some of the well-known multicriteria optimization methods.

The number of criteria based on which bridges can be described or evaluated is very high, and they are all intended to represent their construction, traffic, economic, social and other aspects. The need to compare or evaluate bridges in the total population of these structures imposes the need to classify all of them first, depending on their basic construction and traffic characteristics, into at least three subgroups, and then, depending on the sheer traffic profile into three more subgroups. Thus determined groups will be evaluated based on the same set of criteria which can be divided into traffic ones and others in the most general sense.

Therefore, the basic classification of bridges shall be based on the type of road or traffic artery that they belong to. In this sense, we will distinguish three groups: the bridges that are on arteries out of town, the bridges on bypasses that bypass settlements, and the bridges on the city arteries. Depending on the traffic artery profile, each of these groups can be divided into bridges with highway profile, with main road profile or with some other profile.

The representatives of each of these subgroups can be evaluated through many criteria – attributes. For the scientific research in this paper, the attributes are divided into traffic ones and others. Considering traffic attributes, the evaluation is reduced to: functional, ecological, economic and investment. The other attributes are classified as: the structure location selection, the price or investment value, the state of the structure, the planned lifetime of the structure, aesthetic criteria, benefits, indicators of socio-economic development.

2.1 The values of attributes

In the process of evaluation it is necessary to assign specific values to each of these attributes, i.e. it is necessary to define the values or assessments of all the attributes and carry out evaluation based on them. For this purpose, it will be necessary to describe, namely to define its value for each of the attributes particularly.

a) Functional evaluation

In functional terms we can define three types of attribute values which can be replaced by the corresponding numerical values in order to get a consistent review:

i) 3 – satisfactory
ii) 2 – suitable
iii) 1 – not satisfactory.

b) Environmental evaluation

When defining the value of this attribute, a sign "+" will define a positive and a sign "–" a negative impact of a certain phenomenon or activity of the observed structure:

i) 0 – no consequences (impacts) on the environment
ii) ± 1 – less harmful (positive) consequence that can be mitigated by a better project
iii) ± 2 – a significant detrimental (positive) consequence and/or risk (a positive consequence of regional importance)
iv) ± 3 – a very large harmful (positive) consequence of wider national significance.

c) Economic evaluation

In economic terms the most general form of this attribute can be reduced to the evaluation whether there is economic justification of the initial investment at the given time or not, and the value of the attribute can be:

i) 1 – if there is some
ii) 0 – if there is not any.

d) Investment evaluation

Investment evaluation comprises the attribute examined from the standpoint of maintaining the sheer structure, i.e. the projects of its reconstruction related to investment maintenance. The values of this attribute can be:

i) 1 – when the volume of the available financial resources is more than necessary for the planned maintenance
ii) 2 – when really available resources are not sufficient to cover the completion of projects for reconstruction and renovation after the implementation of maintenance programs
iii) 3 – when only projects of high economic feasibility are taken into consideration within the limited funds for reconstruction and new development
iv) 4 – when only projects with the internal rate of return (IRR)[1] greater than the max IRR for the maintenance are included into the programs of funding reconstruction and renovation.

[1] The internal rate of return (IRR) is a common financial valuation metric used by financial analysts to calculate and assess the financial attractiveness / viability of capital intensive projects or investments.

e) Spatial selection of structure location

Assuming that during the design and construction of the bridge special attention has been paid that the selection of structure location has been as qualitative as possible or better to say optimal, the value of attribute is defined through three levels, as follows:

i) 1 – suitable
ii) 2 – good
iii) 3 – excellent,

where the terms themselves are associated with the meaning of each value of this attribute.

f) Price i. e. investment value

The selection of attribute value i.e. investment value is defined based on the fact that, among other things, it significantly differs depending on the type of structure construction. In this sense, the classification is made according to the size of the bridge span into three groups, each of which defines a single value of this attribute as follows:

i) 1 – low price / investment value (for bridges with the span between 5 and 20 meters)
ii) 2 – medium price / investment value (for bridges with the span between 20 and 60 meters)
iii) 3 – high price / investment value (for bridges with the span over 60 meters).

g) Aesthetic criterion

This is the attribute that most certainly has the greatest weight in deciding on the structure value for most of the average users of this type of structure. At the same time this is an attribute that is most difficult to quantify, i.e. the most ungrateful one to be defined. The problem of aesthetics in this attribute is discussed based on the general impression about whether the subject structure fits into the general natural environment by its appearance or not. The attribute values in the most general sense are divided into:

i) 0 – does not satisfy, i.e. does not fit the environment
ii) 1 – satisfies, i.e. its appearance fits into the natural environment that surrounds it.

h) Planned lifetime of a structure

Assuming that the most common planned lifetime of these structures is 50 years and that this period is most frequently arithmetically considered like a criterion in terms of either construction or traffic duration as the planned structure lifetime, the assumption in determining the value of this attribute is that its lower limit should be 50 years, and that its maximum lifetime is 100 years for the same structure. For quite a long lifetime, and not so great opportunities for its testing, this interval is divided into two equal parts; thus two practical values of this attribute are given:

i) 1 – middle lifetime (from 50 to 75 years)
ii) 2 – long lifetime (from 75 to 100 years).

i) Indicators of socio - economic development

The value of this attribute is classified into three groups, i.e. it is defined with three levels, namely:

i) 1 – the structure is less important
ii) 2 – the structure is important

iii) 3 – the building is very important.

j) The benefit

This attribute is often represented as one of the most important or the most essential attributes in final decision-making on the value. For the purposes of this research the values of this attribute are defined as follows:

i) 1 – a small benefit
ii) 2 – a great benefit.

k) The condition of a structure

This attribute is discussed from the standpoint of estimating the state of construction /constructive elements of the structure as well as traffic volume. Through the assessment of more criteria, a characteristic number of structures is obtained, which is inversely proportional to the quality of the structure. For the purposes of this research, the total span, i.e. the interval of this assessment is divided into several smaller characteristic intervals that exactly describe the state of the object itself, primarily in terms of construction. In this paper, the attribute values are defined as follows:

i) 1 – urgent rehabilitation
ii) 2 – rehabilitation planning
iii) 3 – investment maintenance
iv) 4 – intensive regular maintenance
v) 5 – regular maintenance and control
vi) 6 – regular maintenance.

l) The type of the traffic artery on which the structure is located

The values of this attribute are classified into three basic categories as follows:

i) 3 – traffic arteries out of town
ii) 2 – bypasses - traffic arteries that bypass settlements
iii) 1 – municipal traffic arteries.

m) The road profile of a bridge

The values of this attribute are defined as follows:

i) 3 – highway profile
ii) 2 – main road profile
iii) 1 – other road profiles.

n) The attribute of the decision

For the purposes of this research, in the most general terms the decision attribute values are only divided into two classes, namely:

i) 1 – it is a valuable structure
ii) 0 – the structure that does not have great value.

As suggested by Cirovic and Plamenac [9] and Kuburic [10] the evaluation of bridges through implementation of rough sets theory has been proposed.

3 The selection of attribute core through the method of rough sets

The first step in dealing with the issue of attribute core selection by means of rough sets method proves to be the formation of a data chart/table i.e. an attribute - value table or a decision table. Attribute values for each of the selected population representatives are approximately defined by a certain amount of reliability, i.e. a strict procedure of each attribute evaluation has not been conducted for the purposes of this research.

As emphasised above, the problem of selecting the core of attributes from all the attributes that have been treated by applying the theory of rough sets has been run through the following stages:

a. Describing a structure using attribute values
b. Describing dependence (full or partial) among attributes
c. Reduction of attributes
d. The meaning of attributes
e. Generating the decision rules.

The decision table is a data table which distinguishes two classes of attributes (condition attributes and decision attributes) – Tab. 1. The theory of rough sets includes a decision table with linguistically expressed variable condition attributes. The columns in the table are marked with attributes, and the records within the table are attribute values. Thus, each line can be viewed as the information about a particular bridge.

Output data form the core of attributes, namely attribute reduction is carried out (the reduction of the core volume, i.e. the number of attributes that influence the decision-making is reduced). The aim is to identify those attributes which, according to the requirements of the decision-maker in Tab. 1 significantly affect the decision-making [11]. It is necessary to determine which attribute or value of the bridge and accordingly, which bridge has the priority in financing and the order of rehabilitation.

The first column of this table consists of the representatives of the population of bridges, and other columns contain the values of attributes. The columns that contain attribute values are ranged in the order they are treated in the previous chapter.

With the assistance of defined attributes ten bridges from the whole population of bridges in the scope of road infrastructure are described. For the purpose of the paper their exact names are translated into general: bridge I, bridge II ...., bridge X, and their key features are presented through the values of concerned attributes in decision table.

The decision table generates the following sets:

$$U = \{\text{I, II, III, IV, V, VI, VII, VIII, IX, X}\}, \tag{6}$$

$$A = \{\text{a, b, c, d, e, f, g, h, i, j, k, l, m}\}. \tag{7}$$

Nine elementary sets or A-sets of attributes can be extracted from Table 1. There is one set of objects that are not different {1, 5} and the remaining eight sets of objects that are different.

These are upper and lower boundaries of the decision class "1": {1, 2, 3, 4, 7, 8} and "0": {5, 6, 9, 10}

Lower approximation [8] of the decision class "1": {2, 3, 4, 7, 8}

Upper approximation of the decision class "1": {1, 2, 3, 4, 5, 7, 8}

Boundary region [12] of the decision class "1": BN_A ("1") = {1, 5}

Lower approximation of the decision class "0": {6, 9, 10}

Upper approximation of the decision class "0": {1, 5, 6, 9, 10}

Boundary region (of the decision class "0": {1, 5}

Accuracy of the decision class approximation [7]: αA("1") = 0,71 and αA("0") = 0,60.

Boundary region of the decision in both decision classes consists of two objects – 1 and 5. These objects (bridges) have the identical attribute values of conditions, but different attribute values of the decision attribute. For this reason, Tab. 1 is not consistent or, in other words, the decision of the decision-maker is inconsistent with the description of objects with attributes from the set A.

The next step is made of compiling the minimum subset of independent attributes, which guarantees the same quality of classifications as a whole set A, i. e. a reductor, as the core of attributes. Finding the reductor and core of attributes will be carried out through the discernibility matrix [13]. For this purpose it is necessary to rearrange or regroup the decision table first. Tab. 2 represents a realistic review of the decision table because the decisions inconsistent decision classes are presented as one class.

The discernibility matrix in the data table (Tab. 2 without decision attributes and without objects that do not differ) is a symmetric matrix $\boldsymbol{n} \times \boldsymbol{n}$ with c_{ij} records, and is given in Tab. 3.

The answer to the question whether the objects discern from each other is obtained using the conjunction of all the records in the discernibility matrix. The discernibility function (Boolean function) [14], has the form (8).

The problem of finding a subset of attributes that protects the indiscernibility relationship using the discernibility matrix is reduced to finding implicants of the previous equation (8). The implicant is such that if these variables are true, then the function is correct. Prime implicants which are in fact implicants of minimal size are particularly important [15]. The equation (8) is reduced to:

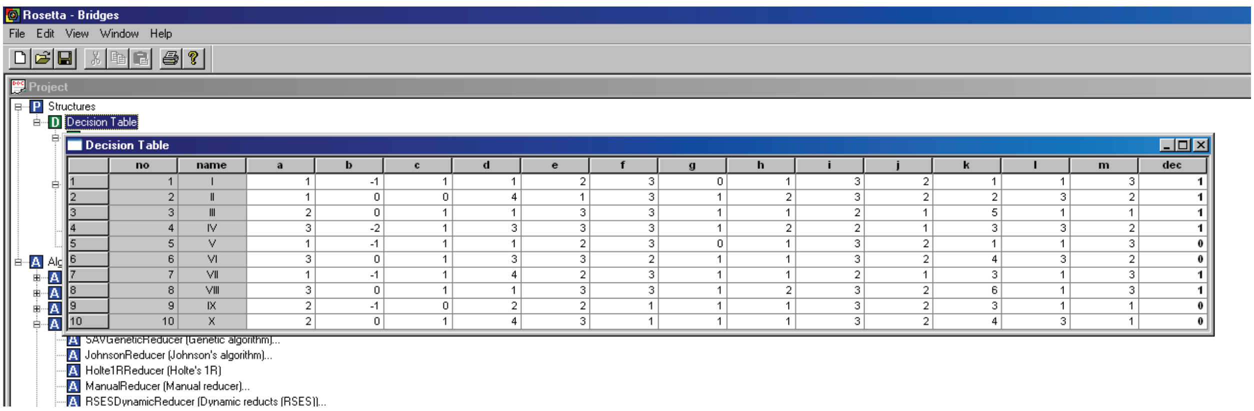
$$F(\mathrm{a, b, c, d, e, f, g, h, i, j, k, l, m}) = F(\mathrm{a, k})\,. \tag{9}$$

This is completely in compliance with the results obtained using software systems in the field of rough sets, which are shown below.

4 Application of the Rosetta software system in bridge estimation

Many software systems based on the theory of rough sets are commercially available and successfully applied in many areas; some are also available free via the Internet. It can be said that rough sets have developed their application in the field of construction and investment projects. The following will show the application of the particular software system – Rosetta [16].

The Rosetta system has been developed as a response to the demands for the formation of tools which should be both flexible and easy to use at the same time. It is a software system aimed at broadening knowledge and extracting data within the theory of rough sets. The authors are Øhrn and Komorowski [17] in the Faculty of Computer and Information Science, Norwegian University of Science and Technology in Trondheim, Skowron and Synak [18] in the Mathematics Institute, The University of Warsaw. The scope of rules considered by Rosetta consists of IF-THEN rules. If the rule is generated and the appropriate attribute/characteristic is discovered, the largest computational effort is the calculation or the approximation of reduction. Reduction is the minimum subset of attributes/characteristics that can preserve the indiscernibility relation. Such a relationship can be formulated for the entire system or in relation to the individual class of objects.



	no	name	a	b	c	d	e	f	g	h	i	j	k	l	m	dec
1	1	I	1	-1	1	1	2	3	0	1	3	2	1	1	3	1
2	2	II	1	0	0	4	1	3	1	2	3	2	2	3	2	1
3	3	III	2	0	1	1	3	3	1	1	2	1	5	1	1	1
4	4	IV	3	-2	1	3	3	3	1	2	2	1	3	3	2	1
5	5	V	1	-1	1	1	2	3	0	1	3	2	1	1	3	0
6	6	VI	3	0	1	3	3	2	1	1	3	2	4	3	2	0
7	7	VII	1	-1	1	4	2	3	1	1	2	1	3	1	3	1
8	8	VIII	3	0	1	1	3	3	1	2	3	2	6	1	3	1
9	9	IX	2	-1	0	2	2	1	1	1	3	2	3	1	1	0
10	10	X	2	0	1	4	3	1	1	1	3	2	4	3	1	0

Figure 1 Decision table

Using the software system calculatingly the optimal estimation of bridges has been calculated through some of the defined conditions attributes. Applying the *Rosetta* program, generating reductions can be obtained in several ways, some of which are illustrated by the reviews of the screen below. The review of the screen in Fig. 1 presents decision table.

The review of the screen in Fig. 2 presents reduction generating through GA RSES application. RSES - Rough Set Exploration System has been developed by Bazan in the Institute of Mathematics, The University of Rzeszow in Warsaw, Syczuka in the Institute of Mathematics, University of Warsaw [19, 20] with the Polish - Japanese Institute of Information Technology in Warsaw, Poland. It is a tool for the analysis of tables, which works in Windows environment, or in Linux. It uses methods and algorithms in the field of rough sets.

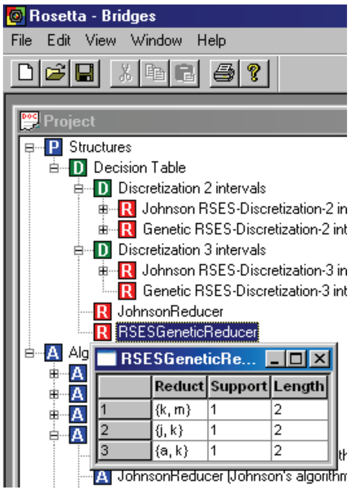
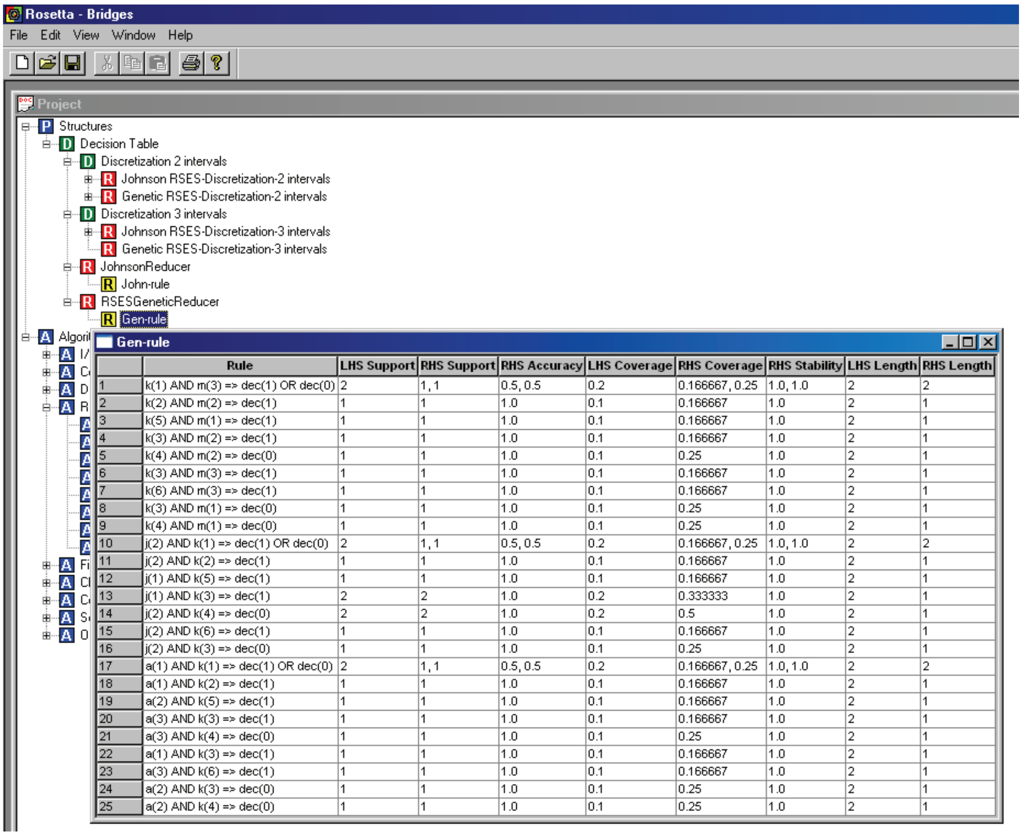


Figure 2 Generating reductions through GA RSES application

Evidently, from the screen in Fig. 2, the core of attributes is great, and a large number decision attributes participates in it. The decision rules are presented in Fig. 3.

The final selection of the optimal results can be reached by the construction of efficient reversible semi-minimal set of intersections based on heuristics. Heuristics that is applied here is based on finding the intersections with maximum number of pairs of objects that differ by this intersection.

The simplest model forming the prior is analogous to Johnson's approximation algorithm – Johnson's greedy ("voracious") algorithm [21], which is presented by the screen review in Fig. 4.



	Rule	LHS Support	RHS Support	RHS Accuracy	LHS Coverage	RHS Coverage	RHS Stability	LHS Length	RHS Length
1	k(1) AND m(3) => dec(1) OR dec(0)	2	1, 1	0.5, 0.5	0.2	0.166667, 0.25	1.0, 1.0	2	2
2	k(2) AND m(2) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
3	k(5) AND m(1) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
4	k(3) AND m(2) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
5	k(4) AND m(2) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1
6	k(3) AND m(3) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
7	k(6) AND m(3) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
8	k(3) AND m(1) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1
9	k(4) AND m(1) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1
10	j(2) AND k(1) => dec(1) OR dec(0)	2	1, 1	0.5, 0.5	0.2	0.166667, 0.25	1.0, 1.0	2	2
11	j(2) AND k(2) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
12	j(1) AND k(5) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
13	j(1) AND k(3) => dec(1)	2	2	1.0	0.2	0.333333	1.0	2	1
14	j(2) AND k(4) => dec(0)	2	2	1.0	0.2	0.5	1.0	2	1
15	j(2) AND k(6) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
16	j(2) AND k(3) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1
17	a(1) AND k(1) => dec(1) OR dec(0)	2	1, 1	0.5, 0.5	0.2	0.166667, 0.25	1.0, 1.0	2	2
18	a(1) AND k(2) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
19	a(2) AND k(5) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
20	a(3) AND k(3) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
21	a(3) AND k(4) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1
22	a(1) AND k(3) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
23	a(3) AND k(6) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
24	a(2) AND k(3) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1
25	a(2) AND k(4) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1

Figure 3 GA RSES application – Decision rules

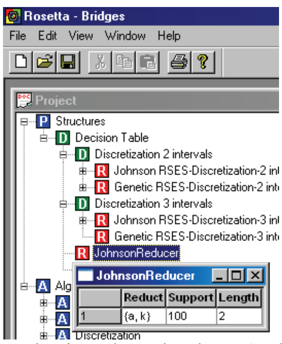


Figure 4 Generating reductions through Johnson's algorithm application

Unlike the previous case, evidently, the attribute core is reduced to only one decision attribute in the screen review in Fig. 4, which is a couple of condition attributes *a* – functionality and *k* – the state of the bridge, the condition rating of structural elements of the bridge and traffic loads.

The decision rules are presented in Fig. 5.

The following are showing the application of discretization to the problem.

Table 1 Decision table

BRIDGE		CONDITION ATTRIBUTES													DECIS.
No	NAME	a	b	c	d	e	f	g	h	i	j	k	l	m	ATTR.
1.	I	1	-1	1	1	2	3	0	1	3	2	1	1	3	1
2.	II	1	0	0	4	1	3	1	2	3	2	2	3	2	1
3.	III	2	0	1	1	3	3	1	1	2	1	5	1	1	1
4.	IV	3	-2	1	3	3	3	1	2	2	1	3	3	2	1
5.	V	1	-1	1	1	2	3	0	1	3	2	1	1	3	0
6.	VI	3	0	1	3	3	2	1	1	3	2	4	3	2	0
7.	VII	1	-1	1	4	2	3	1	1	2	1	3	1	3	1
8.	VIII	3	0	1	1	3	3	1	2	3	2	6	1	3	1
9.	IX	2	-1	0	2	2	1	1	1	3	2	3	1	1	0
10.	X	2	0	1	4	3	1	1	1	3	2	4	3	1	0

Table 2 Regrouped decision table

BRIDGE		CONDITION ATTRIBUTES													DECIS.
No	NAME	a	b	c	d	e	f	g	h	i	j	k	l	m	ATTR.
1.	I	1	-1	1	1	2	3	0	1	3	2	1	1	3	1
5.	V	1	-1	1	1	2	3	0	1	3	2	1	1	3	0
2.	II	1	0	0	4	1	3	1	2	3	2	2	3	2	1
3.	III	2	0	1	1	3	3	1	1	2	1	5	1	1	1
4.	IV	3	-2	1	3	3	3	1	2	2	1	3	3	2	1
6.	V	3	0	1	3	3	2	1	1	3	2	4	3	2	0
7.	VII	1	-1	1	4	2	3	1	1	2	1	3	1	3	1
8.	VIII	3	0	1	1	3	3	1	2	3	2	6	1	3	1
9.	IX	2	-1	0	2	2	1	1	1	3	2	3	1	1	0
10.	X	2	0	1	4	3	1	1	1	3	2	4	3	1	0

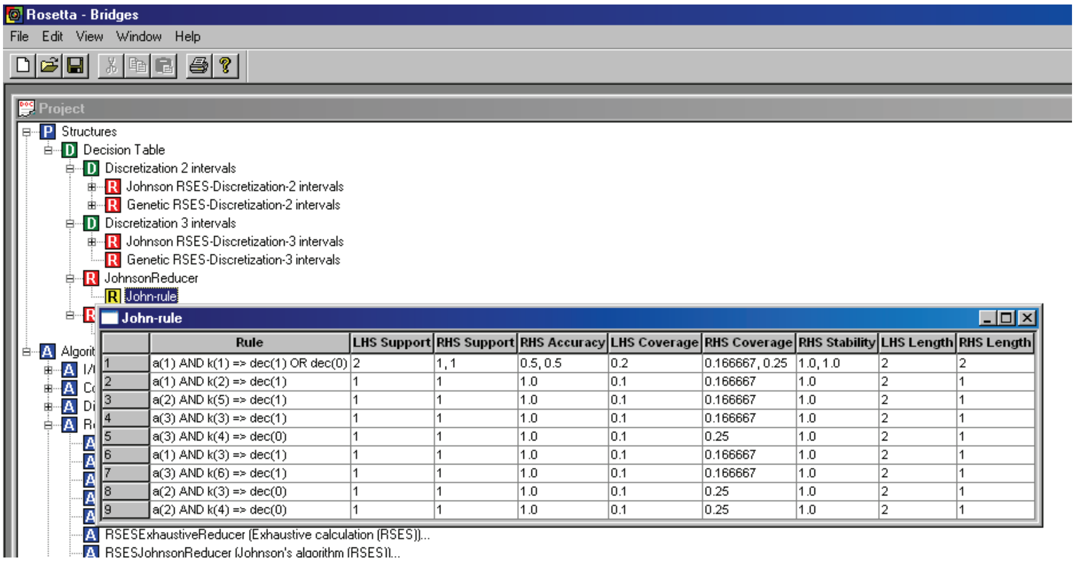
Table 3 Discernibility matrix

	1	2	3	4	6	7	8	9	10
1	-	b, c, d, e, g, h, k, l, m	a, b, e, g, i, j, k, m	a, b, d, e, g, h, j, k, l, m	a, b, d, e, f, g, k, l, m	d, g, i, j, k	a, b, e, g, h, k	a, c, d, f, g, k, m	a, b, d, e, f, g, k, l, m
2	-	-	a, c, d, e, h, i, j, k, m	a, b, c, d, e, i, j, k	a, c, d, e, f, h, k	b, c, e, h, j, k, l, m	a, c, d, e, k, l, m	a, b, d, e, f, h, k, l, m	a, c, e, f, h, k, m
3	-	-	-	a, b, d, h, k, l, m	a, d, f, i, j, k, l, m	a, b, d, e, k, m	a, h, i, j, k, m	b, c, d, e, f, i, j, k	d, f, i, j, k, l
4	-	-	-	-	b, f, h, i, j, k	a, b, d, e, h, l, m	b, d, i, j, k, l, m	a, b, c, d, e, f, h, i, j, l, m	a, b, d, f, h, i, j, k, m
6	-	-	-	-	-	a, b, d, e, f, i, j, k, l, m	d, f, h, k, l, m	a, b, c, d, e, f, k, l, m	a, d, f, m
7	-	-	-	-	-	-	a, b, d, e, h, i, j, k	a, c, d, f, i, j, m	a, b, e, f, i, j, k, l, m
8	-	-	-	-	-	-	-	a, b, c, d, e, f, h, k, m	a, d, f, h, k, l, m
9	-	-	-	-	-	-	-	-	b, c, d, e, k, l
10	-	-	-	-	-	-	-	-	-

F(a, b, c, d, e, f, g, h, i, j, k, l, m) =
(bUcUdUeUgUhUkUlUm)(aUbUeUgUiUjUkUm)(aUbUdUeUgUhUjUkUlUm)(aUbUdUeUfUgUkUlUm)(dUgUiUjUk)(aUbUeUgUhUk)(aUcUdUfUgUkUm)(aUbUdUeUfUgUkUlUm)(aUcUdUeUhUiUjUkUm)(aUbUcUdUeUiUjUk)(aUcUdUeUfUhUk)(bUcUeUhUjUkUlUm)(aUcUdUeUkUlUm)(aUbUdUeUfUhUkUlUm)(aUcUeUfUhUkUm)(aUbUdUhUkUlU)(aUdUfUiUjUkUlUm)(aUbUdUeUkUm)(aUhUiUjUkUm)(bUcUdUeUfUiUjUk)(dUfUiUjUkUl)(bUfUhUiUjUk)(aUbUdUeUhUlUm)(bUdUiUjUkUlUm)(aUbUcUdUeUfUhUiUjUlUm)(aUbUdUfUhUiUjUkUm)(aUbUdUeUfUiUjUkUlUm)(dUfUhUkUlUm)(aUbUcUdUeUfUkUlUm)(aUdUfUm)(aUbUdUeUhUiUjUk)(aUcUdUfUiUjUm)(aUbUeUfUiUjUkUlUm)(bUcUdUeUfUhUkUm)(aUdUfUhUkUlUm)(bUcUdUeUkUl). (8)

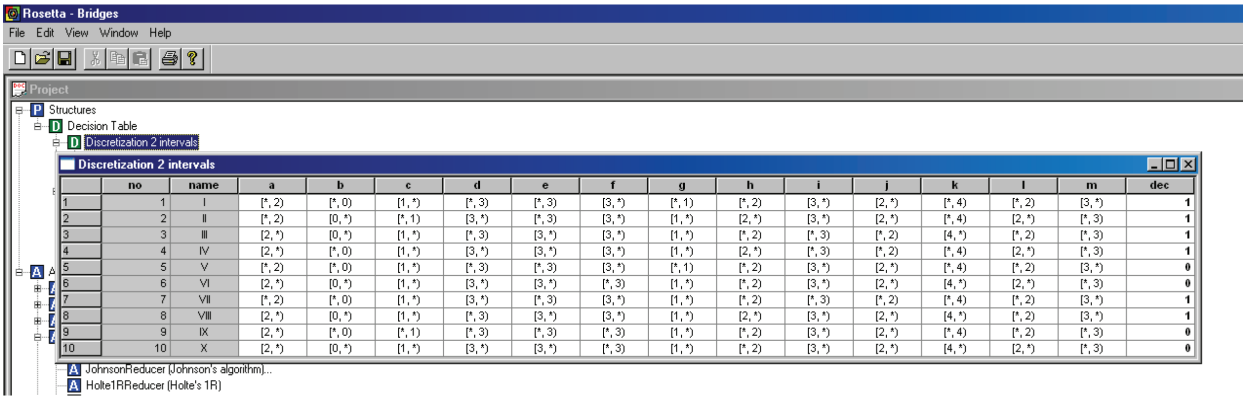
The discretization is a hybrid method (system) of rough sets and Boolenian reasoning. The discretization method is emanated on the level of development of rough sets theory on which level the condition attributes with real numbers has appeared in the decision tables which compose their starting base. Until then, the condition attributes mostly have been linguistic variables, but operations with real values required discretization determined strategy, in order to obtain higher level of classification rules, i. e. decision tables with enough small value of attributes numbers. These problems are intensively studied by Nguyen, H. S. [22], Nguyen, S. H. [23], Nguyen and Skowron [24], Nguyen et al. [25], Wakulicz – Deja, Boryczka and Paszek [26].

The interval of discretization determinates how roughly the system is observed. The choice of appropriate intervals and division of attribute values sets are very composite problems and their complexity arise with the number of attributes to be discretized. The application of discretization to resolve this problem is adequate and reasonable future as to the problem elections addressed effectively with discretized data, which in a large extent retains their originality.



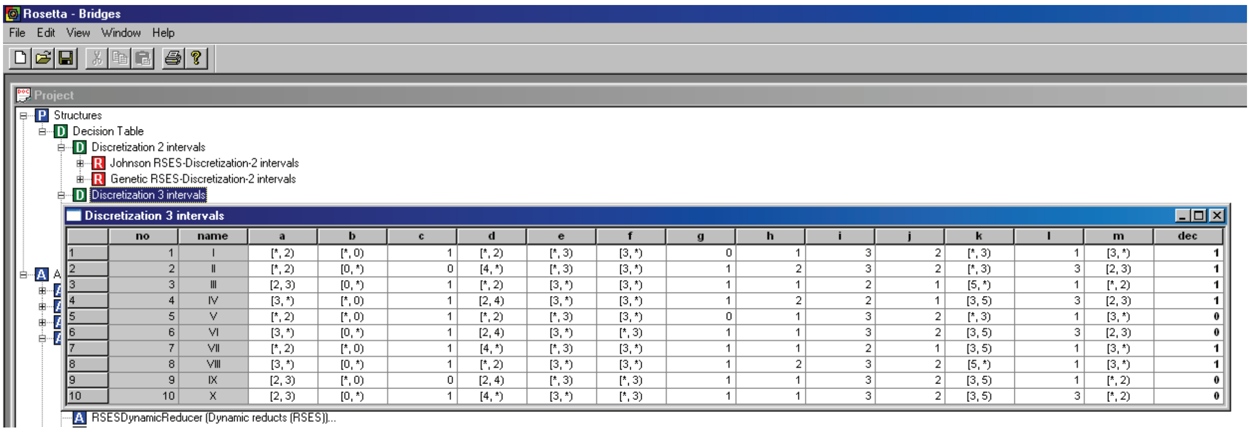
	Rule	LHS Support	RHS Support	RHS Accuracy	LHS Coverage	RHS Coverage	RHS Stability	LHS Length	RHS Length
1	a(1) AND k(1) => dec(1) OR dec(0)	2	1, 1	0.5, 0.5	0.2	0.166667, 0.25	1.0, 1.0	2	2
2	a(1) AND k(2) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
3	a(2) AND k(5) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
4	a(3) AND k(3) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
5	a(3) AND k(4) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1
6	a(1) AND k(3) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
7	a(3) AND k(6) => dec(1)	1	1	1.0	0.1	0.166667	1.0	2	1
8	a(2) AND k(3) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1
9	a(2) AND k(4) => dec(0)	1	1	1.0	0.1	0.25	1.0	2	1

Figure 5 Johnson's algorithm application – Decision rules



	no	name	a	b	c	d	e	f	g	h	i	j	k	l	m	dec
1	1	I	[*, 2)	[*, 0)	[1, *)	[*, 3)	[*, 3)	[3, *)	[*, 1)	[*, 2)	[3, *)	[2, *)	[*, 4)	[*, 2)	[3, *)	1
2	2	II	[*, 2)	[0, *)	[*, 1)	[3, *)	[*, 3)	[3, *)	[1, *)	[2, *)	[3, *)	[2, *)	[*, 4)	[2, *)	[*, 3)	1
3	3	III	[2, *)	[0, *)	[1, *)	[*, 3)	[3, *)	[3, *)	[1, *)	[*, 2)	[*, 3)	[*, 2)	[4, *)	[*, 2)	[*, 3)	1
4	4	IV	[2, *)	[*, 0)	[1, *)	[3, *)	[3, *)	[3, *)	[1, *)	[2, *)	[*, 3)	[*, 2)	[*, 4)	[2, *)	[*, 3)	1
5	5	V	[*, 2)	[*, 0)	[1, *)	[*, 3)	[*, 3)	[3, *)	[*, 1)	[*, 2)	[3, *)	[2, *)	[*, 4)	[*, 2)	[3, *)	0
6	6	VI	[2, *)	[0, *)	[1, *)	[3, *)	[3, *)	[*, 3)	[1, *)	[*, 2)	[3, *)	[2, *)	[4, *)	[2, *)	[*, 3)	0
7	7	VII	[*, 2)	[*, 0)	[1, *)	[3, *)	[*, 3)	[3, *)	[1, *)	[*, 2)	[*, 3)	[*, 2)	[*, 4)	[*, 2)	[3, *)	1
8	8	VIII	[2, *)	[0, *)	[1, *)	[*, 3)	[3, *)	[3, *)	[1, *)	[2, *)	[3, *)	[2, *)	[4, *)	[*, 2)	[3, *)	1
9	9	IX	[2, *)	[*, 0)	[*, 1)	[*, 3)	[*, 3)	[*, 3)	[1, *)	[*, 2)	[3, *)	[2, *)	[*, 4)	[*, 2)	[*, 3)	0
10	10	X	[2, *)	[0, *)	[1, *)	[3, *)	[3, *)	[*, 3)	[1, *)	[*, 2)	[3, *)	[2, *)	[4, *)	[2, *)	[*, 3)	0

Figure 6 GA RSES application – Decision table with discretization into 2 intervals



	no	name	a	b	c	d	e	f	g	h	i	j	k	l	m	dec
1	1	I	[*, 2)	[*, 0)	1	[*, 2)	[*, 3)	[3, *)	0	1	3	2	[*, 3)	1	[3, *)	1
2	2	II	[*, 2)	[0, *)	0	[4, *)	[*, 3)	[3, *)	1	2	3	2	[*, 3)	3	[2, 3)	1
3	3	III	[2, 3)	[0, *)	1	[*, 2)	[3, *)	[3, *)	1	1	2	1	[5, *)	1	[*, 2)	1
4	4	IV	[3, *)	[*, 0)	1	[2, 4)	[3, *)	[3, *)	1	2	2	1	[3, 5)	3	[2, 3)	1
5	5	V	[*, 2)	[*, 0)	1	[*, 2)	[*, 3)	[3, *)	0	1	3	2	[*, 3)	1	[3, *)	0
6	6	VI	[3, *)	[0, *)	1	[2, 4)	[3, *)	[*, 3)	1	1	3	2	[3, 5)	3	[2, 3)	0
7	7	VII	[*, 2)	[*, 0)	1	[4, *)	[*, 3)	[3, *)	1	1	2	1	[3, 5)	1	[3, *)	1
8	8	VIII	[3, *)	[0, *)	1	[*, 2)	[3, *)	[3, *)	1	2	3	2	[5, *)	1	[3, *)	1
9	9	IX	[2, 3)	[*, 0)	0	[2, 4)	[*, 3)	[*, 3)	1	1	3	2	[3, 5)	1	[*, 2)	0
10	10	X	[2, 3)	[0, *)	1	[4, *)	[3, *)	[*, 3)	1	1	3	2	[3, 5)	3	[*, 2)	0

Figure 7 GA RSES application – Decision table with discretization into 3 intervals

The reviews of the screen in Fig. 6 and Fig. 7 present decision tables with discretization into 2, i. e. 3 intervals.

The review of the screen in Fig. 8 presents reduction generating through GA RSES application with discretization into 2 intervals, and in Fig. 9 with discretization into 3 intervals. Obviously in both cases the same solution has been obtained.

The review of the screen in Fig. 10 presents reduction generating through Johnson's algorithm application with discretization into 2 intervals, and in Fig. 11 with discretization into 3 intervals. Obviously in both cases has been obtained the same solution. In Fig. 12 the decision rules with discretization into 3 intervals are presented.

The results of optimal cuts point to the high sensitivity requesting an optimal solution by applying heuristical models.

The first pair of attributes (a, k) – functional evaluation and the condition of a structure, is more acceptable from the engineering point of view, responds to the structures elements condition, but the second one (f, g) – price / investment value and aesthetic criterion, i.e. what is the cost and how it looks like, is much more convenient for the others, it implies that the most of all decide owing to price and appearance. Namely, functionality and the conditions of the bridge structure characteristics as the preference in the evaluation procedure are compatible with expert opinion, as the investment value and aesthetic criterion closely to the opinion of common users.

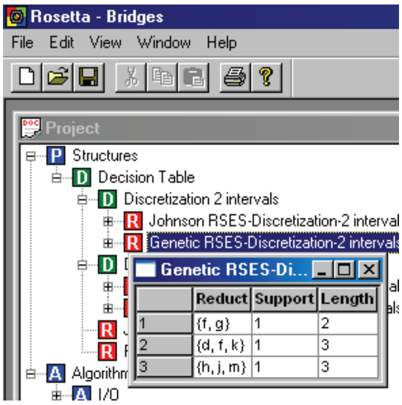


Figure 8 Generating reductions through GA RSES application – discretization into 2 intervals

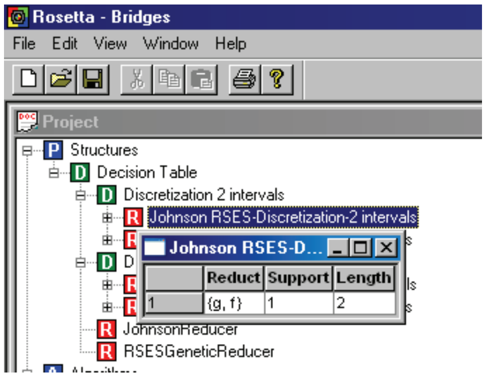


Figure 10 Generating reductions through Johnson's algorithm application – discretization into 2 intervals

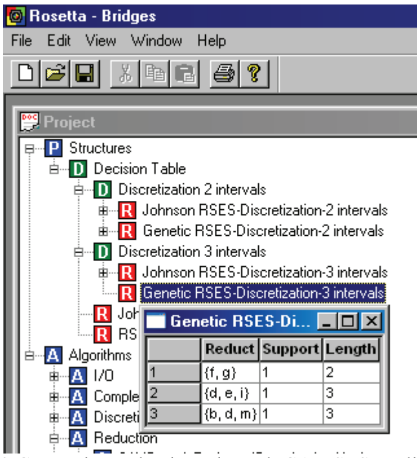


Figure 9 Generating reductions through GA RSES application – discretization into 3 intervals

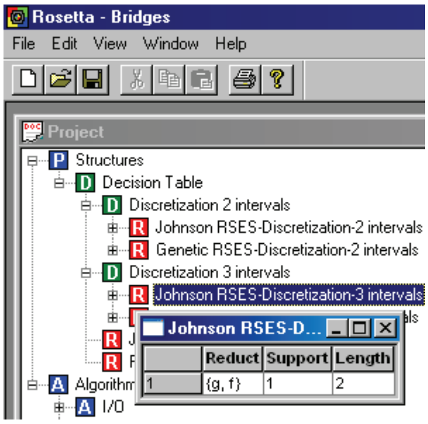


Figure 11 Generating reductions through Johnson's algorithm application – discretization into 3 intervals

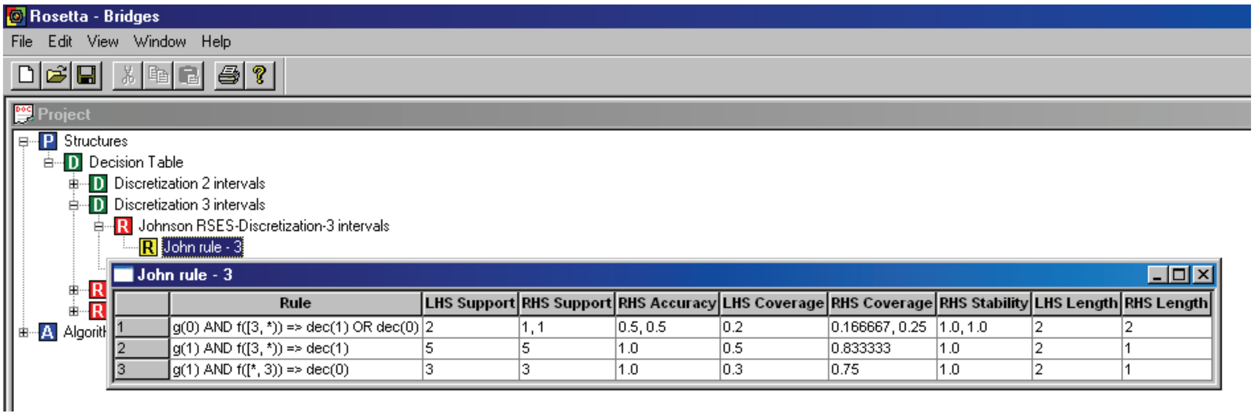


Figure 12 Johnson's algorithm – Decision rules with discretization into 3 intervals

5 The discussion about the obtained results

The functionality and condition of construction characteristics of the bridge and its traffic load, as the decision attributes, obtained as a preference in the estimation of bridges, make sense given that the bridges that have the greatest value of the works are usually a priority - either at length, because it is necessary to establish an important new road direction, or because the value contains the works which include the rehabilitation or works of greater scope, due to the wearing of the mentioned bridge.

During the decision-making process, when the estimation of bridges is carried out, a large number of attributes is used, the decision cannot be made with certainty unless some of the methods of operational research that eliminate the subjectivity of the decision-maker are applied, and by the presented method of generating reduction the goal has been achieved, i.e. decision attributes are identified. In the particular case, this attribute core "intensified" by generating the core of attributes is reduced to two attributes containing the state of structural elements as the most important factor for proposing and deciding on the priority of the concerned bridge rehabilitation.

The condition of a bridge depends on several factors, which can often be descriptively designated. Precisely these characteristics also initiate a model for making certain decisions regarding the order of selecting priority roads and bridges for construction or rehabilitation, as well as the possibility of comparing different bridges in particular time sections.

It can be especially noted that in the world many methods for the bridges' evaluation are developed, based on inspection / construction stability. Many papers and many conventions are committed to this subject. Thus, Prine [27] has mentioned that analytical techniques frequently have not provided sufficient accuracy when applied to complex structural details. It is necessary to incorporate a series of risk management tools and reports to help deliver the longer term strategic management objectives, assigned by Prasad and Coe [28]. Bridge Evaluation Quality Assurance (2007) in U.S. is based on National Bridge Inspection Standards (NBIS). These standards were established to set minimum standards for a nationwide bridge. There is high interest in making sure that the quality of a bridge inspection program is maintained at the highest level, and that funds are utilized as effectively as possible. This program has become very successful at preventing failures and assuring the public that the bridges they cross remain safe. It is very important in the future to consider bridges as socioeconomic assets and to quantitatively calculate the effective utilization of expenses using the prediction of future expenses necessary for maintenance and life cycle cost, referred by Toshiyuki [4].

However, in this paper a special form evaluation bridges which is not only in the assessment situation, but reviews and other factors, necessary for complex assessment and ranking priorities for rehabilitation.

6
The directions of further research

The previous considerations represent one of the possible approaches for estimating bridges. The basic idea is that when applying multicriteria classification to the data provided in the form of tables, the user can obtain general decision rules and derive conclusions. Considering the system analysis in which there is a large number of attributes relevant for decision-making, it is very difficult to create a methodology that will successfully be able to predict all the possible cases.

A large number of different types of bridges and an array of their characteristics stipulate the *complexity* of the observed system. Also, social conditions (demographic factors, population density, measures of economic policy) and external - natural conditions (topographic - morphological, communications, climate - meteorological, geological - geotechnical and hydrological factors) cause the *uncertainty*. Characteristics fixed for a bridge (project) involve type, size and configuration, the complexity of the construction, relationship between construction and craft works, the way of making construction (technology level) and the state of project documentation, quality and price of works and construction costs. All this may also lead to the *imprecision* in their description.

Measuring effectiveness is quantitative measuring aimed at comparing the results taken in various external conditions and circumstances of different decisions [29]. Measures of effectiveness are, for example, costs, quality, employee satisfaction with working conditions, overall satisfaction of social demands, etc. The measure of success is the numerical value that summarizes the whole system of decision-making. Sometimes, if the units are measurable, the measure of effectiveness can easily be determined, e.g. cost, income, etc. However, in other cases, including when the quality level of requirement satisfaction or social responsibility are included, it is much harder to define and measure effectiveness. When the measure of effectiveness is determined, it becomes a key parameter of the assessment system for decision-making and it presents the output of the system, which reflects the combined effects of all the elements involved in the system.

Often, the right optimization is not implemented in project managing, but a certain number of criteria are satisfied. Rather than to minimize the cost of the system, the system is projected in such a way not to exceed certain value of particular parameters, while ensuring that cost does not exceed certain limit. In this way the system seems to be satisfactory ("satisfice") [30] rather than optimal, which is closely related but not the same. "Satisficing" is the basis for optimization. It is often possible to set the value of variables relevant to the decision so that certain conditions are satisfied in project managing.

Obviously, the crucial issue is whether the rules of artificial intelligence shown in this paper are a good indicator. For the purpose of success estimation it is essential how to move from the grade "bad" to the grade "good" or "satisfactory".

What are all the possible directions of further research in this area that could contribute to a satisfactory concluding? One of the possible directions is modelling estimation of bridges that is based on recognisable subjective preferences (by the decision-maker, representative of public administration, stakeholders). Also, a number of studies are based on neural networks.

Perhaps the proper direction of the assessment of bridges (which could be also generally applied to other structures of infrastructure, or to residential and business facilities), is the one based on the concept of deduction on the case-based reasoning (CBR), which for the given input attribute values of the bridge finds the most similar value from the knowledge base – the prototypes and it estimates the value of the bridge in compliance with that. Case-based reasoning model has appeared as an answer to a growing need for Industry Knowledge Base in construction industry [31]. CBR is the concept of business intelligence that uses data from the past (memory) for decision-making – it solves problems by using or adapting solutions of old problems.

This is perhaps the model most appropriate to the real demands of business – predicting the value of structures / bridges is important for owners, contractors, investors, appraisers, tax collectors, insurance companies, etc. In this sense, the benefit from the model would be that an adequate value could be determined for a particular bridge (depending on the type, construction type, road profile, condition, etc.) in a particular region (with a certain degree of density, development of local industry and economic development, environmental characteristics, traffic development, etc.). This is especially important when dealing with mass estimation required for the work of administrative authority (e.g. register, data processing for different purposes).

In this procedure of multicriteria estimation based on the concept of deduction carried out on the CBR, first of all it is necessary to define the criteria on which it will be conducted, as it has been noted previously in this paper. Each criterion can have appropriate values no matter if they are expressed numerically or semantically. In the concrete case, the value of each criterion represents a grade of each spatial unit from the aspect of criteria by which it is evaluated.

In order to carry out the process of estimating, i.e. assessing the value of each particular criterion, it is necessary to describe each of the attributes, namely to define the criteria for attribute values.

The criteria should be selected in such a way that they in the best way describe a territorial unit evaluated from the standpoint of the value of the bridge in it, and that data for attributes evaluation are available from reliable sources so that the same source could be used when applying this model in the future in the function of the proposed model actualization and modification.

Also, after describing and defining the values of particular attributes, it is necessary to normalize each of them - to bring data into the form that allows efficient manipulation with these data (providing some adequate values, e.g. from 0 to 1). At normalization it is necessary to arrange all the cases so that all the values are comparable. After the analysis and normalization have been carried out, it is possible to make data synthesis – aggregation, as well as determining the actual value of the concrete bridge in the particular area from the formed knowledge base.

In the paper of Radojevic [32] it has been noted that explicit inclusion of logic in the process of aggregation (information fusion) is very important in real problems from many points of view such as adequacy and transparency. The aggregation of different attributes, aspects, partial goals – into one representative global criterion is a very important task in many fields of real applications as well as for theoretical purposes.

The variety of bridge constructions and complex and changeable environmental, operational and economical conditions require advanced tools making the systems intelligent by equipping them with the ability of learning, recognizing, concluding, and even choosing and achieving goals [33].

Evidently, due to the unavailability of complete information that would lead to high quality sets in the knowledge base, these models have limitations in terms of complete accuracy, but also the advantage related to the others because they are comprehensive.

7 Conclusions

Here is a set of possible decision-making systems in an uncertain environment of problem-solving estimation of bridges. The system is based on a stochastic approach and, due to uncertain and imprecise conditions of the environment, it is based on a rough approach when dealing with assessing the condition of bridges, or with ranking the alternatives – for those bridge structures that have the priority for rehabilitation. The system has been set and the directions for solutions have been pointed out, and the concrete calculation has been presented.

It has been shown that the methods of artificial intelligence, particularly the theory of rough sets, can be successfully applied for the jobs of investment procedures management, such as determining the state of bridge infrastructure, as well as determining the order which bridges are a priority for the construction or rehabilitation.

The method illustrated in the paper is a contribution to interpretation, analysis and finding of the optimal solution to the practical problems which occur in different fields, among them in bridges estimation. The rough set theory is applied as suitable both for researching qualitative and quantitative attributes, and in the case when sets of empirical or experimental data are too small for conventional statistical methods to be used. The paper shows to researchers and practitioners that rough sets, as one of the fundamental soft-computing techniques, can be successfully applied in daily practice without previous cognition or additional knowledge about empirical data. The subjectivity in knowledge analysis and in evaluation of decision-making process is excluded by usage of the presented method.

The issues arising in modelling and analysing a real system due to the functional, financial, aesthetic criteria of estimation, spatial selection of structure location, prices i.e. investment values of the planned lifetime of a structure, state of the structure, type of the traffic artery on which the structure is located, the road profile of the bridge, etc., can be treated through rough sets. Thus, the engineering system can be regarded as a rough system, which is essentially important because of the lack of precise system description. The main advantage of these algorithms is that they are conceptually simple, and that they imply the possession of knowledge, intuition, creativity and experience. So the optimal strategy can be reached by selecting the optimal variant in a data and decision table. The decision table shows the condition attributes and decision attributes, but without affecting the probability of occurrence and the application of technological variations, because they do not require previous knowledge of certain characteristics, which is the main advantage of the analysis and decision-making through rough sets.

It has been proven that it is possible to identify key elements of influence, although there is an array of variable parameters that affect the successful estimation of bridges.

Financing the construction and modernization of traffic infrastructure and structures such as bridges is the most complex task in the development of traffic. This issue is actual, from both theoretical and methodological aspect of defining the systems and methods of financing, and from the aspect of practical needs for capital engagement.

The conditions attributes, such as the function and state of the bridge, appear to be the most important attributes in the multi-criteria problem for estimating bridges, as it is obtained by the methodology presented in the paper. Evidently, in terms of defining models and formulating decision rules in another way, a different core of attributes intersection that the calculation depends on could possibly be obtained.

The obtained results of optimal intersections indicate the high sensitivity of seeking optimal solutions using heuristic models.

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Authors' addresses

Miroslav Kuburić, Ph.D., assistant professor
University of Novi Sad
Faculty of Civil Engineering
Kozaračka 2a, 24000 Subotica, Serbia
E-mail: mkuburic@eunet.rs

Goran Ćirović, Ph.D., associate professor
Belgrade University College of Applied Studies in Civil Engineering and Geodesy
Hajduk Stankova 2, 11000 Belgrade, Serbia
E-mail: cirovic@sezampro.rs

Zdravko Kapović, Ph.D., full professor
University of Zagreb, Faculty of Geodesy,
Kačićeva 26, 10 000 Zagreb, Croatia
E-mail: zkapovic@geof.hr