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A STUDY ON THE EFFECT OF INFLATION AND TIME VALUE OF MONEY ON LOT SIZING IN SPITE OF REWORKING IN AN INVENTORY CONTROL MODEL

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Original scientific paper
Non production of defective parts during production operations and non-attention to inflation and time value of money are among the hypotheses of economic production quantity model. But the performed studies show that considering defective parts in production size determination models or including the subject of inflation and time value of money in them leads to the change of optimal quantity of production class. Therefore, in production systems with defective parts production, it is necessary to consider these two factors in order to determine production class size. This article studies the effect of time value of money on economic production quantity model in spite of repetition. Due to the complexity of cost function, it is not possible to find an optimal answer easily; therefore, this article uses an algorithm based on combination of two search methods, accelerating and Dico Thomas, in order to solve the problem. Finally, sensitivity analysis has been done on the basis of interest rate, inflation rate and joint rate. Numerical calculation shows that failure to consider inflation and time value of money causes relatively high error in cost.

Keywords: *determining size of production class, repetition, inflation, time value of money*

Proučavanje utjecaja inflacije i promjene vrijednosti novca na veličinu proizvodne serije usprkos preradi na modelu upravljanja skladištem

Izvorni znanstveni članak
U hipotezama modela isplative ekonomske proizvodnje su ne proizvodnja neispravnih dijelova tijekom proizvodnih operacija i ne obraćanje pažnje na inflaciju i promjenu vrijednosti novca. No izvršena proučavanja pokazuju da uzimanje u obzir neispravnih dijelova pri određivanju opsega proizvodnje ili uključivanje pitanja inflacije i promjene vrijednosti novca u vremenu vodi ka promjeni optimalne količine proizvodnje. Stoga je u proizvodnim sustavima koji uključuju proizvodnju neispravnih dijelova potrebno razmotriti ova dva faktora kako bi se odredio opseg proizvodnje. U ovom se članku razmatra učinak promjene vrijednosti novca na model ekonomične proizvodnje unatoč ponavljanju. Zbog složenosti funkcije cijene nije moguće lako pronaći optimalno rješenje te se stoga u članku koristi algoritam zasnovan na kombinaciji dviju istraživačkih metoda – akcelerirajuće i Dico Thomas, kako bi se riješio problem. Na kraju je provedena analiza osjetljivosti na osnovi kamatne stope, stope inflacije i zajedničke stope. Numerički proračun pokazuje da neuzimanje u obzir inflacije i promjene u vrijednosti novca rezultira relativno velikim greškama u cijeni.

Ključne riječi: *određivanje opsega proizvodnje, ponavljanje, inflacija, promjena vrijednosti novca*

1 Introduction

One of the important subjects which all kinds of organizations face is production planning and inventory management. Some issues such as rate and time of raw material orders or semi-finished parts, determination of inventory management system, determining capacity of stores and planning for timely and economic delivery of orders are included in this discussion. Main issue of inventory management and production planning is determining optimal quantity of economic order or determining production class size. This quantity is determined with regard to capabilities and limitations in order to minimize total expenses relating to order, purchase, maintenance and delivery or to maximize total profit relating to inventory management system [1].

For this purpose, this economic quantity order model is used for determining order class size and/or purchase of parts in any producing systems. By considering the production rate, this model has been generalized fixedly into a product economic quantity one. Two cases of the most important generalizations in inventory management models are adding issues of quality control and time value of money to those models. Some works performed on combination of quality control and inventory management and considering time value of money in inventory models are consulted.

The pioneer of quality control and inventory management is Partos. He entered the concept of quality control in a production system in 1986 [2]. Quality of the product in inventory control issues was considered by the researchers. Rosenblatt and Lee considered production model and defective products and concluded that considering defective products caused the decrease of prosecution class size [3]. Rosenblatt and Lee discussed inspection of the parts with the hypothesis that parts can enter border out of the control scope [4].

Fine presented a potential dynamic planning model to find optimal policy with regard to inspection [5]. Hang et al. established relationship between process quality and investment (decreasing commissioning time) in determination of production class in dynamic state [6]. Salameh and Jaber developed economic order model in 2000 with the hypothesis that some receivable goods may not have required quality [7]. They made mistake in presentation of final formula which Cardenas corrected [8]. Salameh and Jaber did not refer to defective goods sale time clearly. Papcheritos and Contentras [9] corrected some of the material. Goyal and Cardenas presented a simpler formula for Salameh and Jaber's model [10].

The model presented by them gives a near optimal answer. Hike and Salameh introduced an optimal point for it by assuming that all defective products have the ability of reworking [11]. Chu presented optimal answer to a minimum cost problem in production model with regard to random rate for defective parts, wastes and reworking of defective parts with the ability of reworking and deterred demand of optimal answer [12]. Chan et al. expanded economic production quantity model with regard to the sale of some defective parts at a lower price, some as reworking and some as wastes [13]. They considered three states including sale at the time of identification by inspection operations at the end of the production as a class and at the end of cycle time all at once. Jamal et al. assumed in economic production quantity model that all defective parts have the ability of reworking [14].

They considered reworking process as similar to production process with the difference that a reworking process has no defective output. They considered two policies including reworking in each cycle and reworking at the end of planning horizon for performing reworking process. Haji et al. [15] considered production of defective products in a special state of newspaper seller problem which was presented by Bijari and Haji [16] as one-cycle potential model with random inventory at the beginning of the period

Flopper and Jensen performed a useful review on all works done in the field of reworking [17]. Su studied economic model of order with regard to Taguchi's cost for defective parts [18]. One of the hypotheses of Su is normal distribution of quality specification of commodity. Rasti et al. studied the issue of Su's model for the state that parts quality specification distribution has general distribution and concluded that economic order quantity does not depend on quality specification distribution for that problem [19]. Fathollah Bayati et al. developed an order and marketing economic quantity integrated model in which the products have been divided into four classes of products with high quality, unfinished products, reworked defective products and non-reworked defective products [20].

Time value of money was studied for the first time by Hadley in 1964 [21]. He compared both order quantities calculated by average annual cost and reduced costs with each other. Then he concluded that any differences of costs in both mentioned models are ignorable by numeric examples with quantity parameters in compliance with real world.

He showed that considerable difference may be obtained in limit states. The pioneer who considered inflation in inventory management models is Buzacott who studied economic order quantity with regard to inflation in 1975 [22]. He concluded that economic order quantity model should be corrected with regard to inflation. Many articles have been presented with regard to time value of money and internal and external multiple inflation rates. Most researches on inventory management and money time value between 1964 and 1990 relate to static models and most of the performed works relate to dynamic and potential models. For example, Birman and Thomas studied economic order quantity model in 1977 with regard to time value of money and inflation by assuming discrete cash flow of all costs without regard to the works of Hadley and Bazaket and compared it with order quantity model of Buzacott. Jeas et al. studied economic order quantity model with time value of money by assuming continuous cash flow of cost and concluded that economic order model has equal result with or without inflation [23].

Chandra and Banner [24] studied the effect of inflation and time value of money in two economic order quantity models in permitted shortage and economic production quantity with shortage of two inflation rates for internal and external costs and proposed search methods for both models in order to find the near optimal answer [24]. Sarker and Pan studied the effect of inflation and money time value in economic production model in case of permitted shortage and introduced Hook and Jews search methods due to the complexity of obtaining optimal answer [25, 26]. The concept of time value of money and inflation was used in the model in which demand depended on price and the shortage was permitted in dynamic model in 2001 [27].

San and Korans studied production in multiple product and multistage state with the use of money time value in order to minimize total costs [28]. In 2003, a potential model was presented for multi-product inventory and multistage system with limitation of capacity and with expected value function of the costs' present value and a dynamic planning technique was used in order to find the answer to the problem [29]. Yang et al. developed inventory models in 2005 with regard to variable delivery time (assuming normal density function) and with regard to money time value [30].

Moon et al. studied models for the goods whose value was reduced or added gradually with time dependent demand model and with regard to time value of money and inflation for a specified planning horizon and presented simple search methods for finding near optimal answer [31]. In 2006, Moslehi and Rasti studied the effect of money time value in production class size model with group technology which was introduced by Bucher in order to determine the production class size on the basis of group technology idea and concluded that the error resulting from non-attention to time value of money in Bucher's model was considerable [32, 33]. In 2008, Komar Dey et al. [33] developed an inventory model with regard to limited time horizon for degradable products in which inflation and time value of money were considered. In this model, it was assumed that there were two separate stores and goods maintenance cost in two stores was different. Time dependent demand and permitted shortage were considered. Finally, the obtained model was solved using genetic algorithm [33]. Mirazadeh has developed an inventory model for degradable products in which time dependent inflation and inflation dependent demand have been considered. This model has been developed by assuming permitted shortage and internal and external inflation rates [34]. In 2010, Chive and Su assumed that quality of production process could be improved as exponential function while considering different qualities of products and time value of money i.e. we can decrease products quality fluctuations with necessary investment [35].

One of the generalized models of economic production quantity in spite of defective parts is the model presented by Jamal et al. [14]. Since this model has high conformity to the real world, it has been considered by writers of this article and time value of money has been added to it in order to complete it for more comparability with realities.

Jamal et al. [14] have considered two policies for reworking operations performance time. In the first policy, it is assumed that reworking operations start in any cycle and immediately after completion of production operations of a class. In the second time policy, it has been assumed that all defective parts produced in any cycle are collected and processed at the end of the planning horizon altogether.

This article has studied the effect of money time value on determination of production class size with regard to reworking in any cycle (first type policy). The second part of this article includes the definition of problem, mentioning hypotheses and introducing the symbols applied in the article. In the third part, the mentioned problem is presented and modelled with regard to time value of money. The presented model is cost function in terms of production class size decision variable and it is not possible to find optimal point with the use of derivation technique due to complexity of this function. In the fourth section, an algorithm based on combination of accelerating and Dico Thomas searching methods has been proposed in order to find a near optimal answer. In the fifth section, the importance of considering time value of money has been studied in the mentioned model with an example and sensitivity analysis and numerical calculations. The final section is conclusion with some suggestions.

2 Problem definition

Consider a single product system in which a class with a size of Q unit products is produced in each period. The previous observation shows that θ % of these parts are defective and should be reworked. Therefore, it is assumed that θ % of the parts are defective. Inspection operations are performed at no time and expense and completely. On the other hand, any part is inspected immediately after production n order to find the quality of the parts and identify defective parts. Here, it is assumed that any produced part is either sound or defective. The parts which are recognized to be sound are taken to the store to fulfil needs of the customers. Demand is continuous and with D rate of goods in time unit.

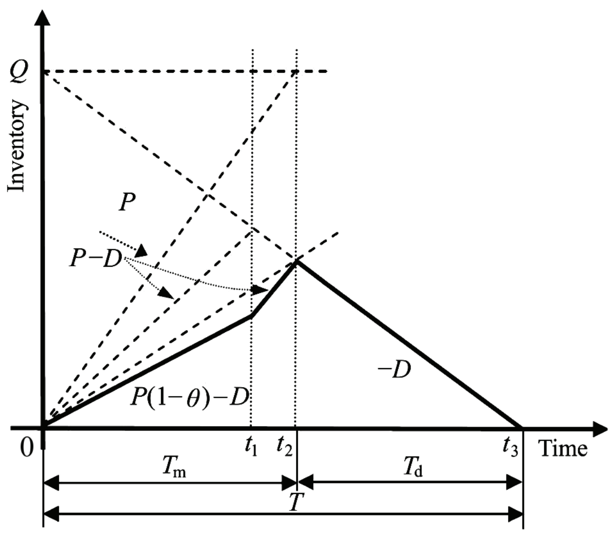

Figure 1 Behaviour of product inventory in store in terms of time

The parts which are defective are maintained beside the system in a defective parts basket to enter reworking process after the end of Q unit production. After reworking operations, all defective parts are converted to the sound parts. Therefore, the defective parts are sent as the sound parts to the store in order to fulfil demand. In Fig. 1, behaviour of the inventory in store is shown in terms of time.

It is necessary to note that the reworking process is assessed to be similar to the production process in terms of time and cost. The goal is to determine the production class size so that total reduced expenses relating to inventory system including present value of the commissioning, production (including primary production and reworking costs) and goods maintenance costs can be minimized. Like Jamal et al. [14], defective parts maintenance cost is assumed to be negligible besides the system.

2.1 Hypotheses

The studied model hypotheses in this article are as follows:

- There is one product.
- Shortage is not permitted.
- Inspection time cost is assumed to be negligible.
- Commissioning operation does not need time but a fixed cost is considered for each time of commissioning.
- There is one system and the sound products are sent to the store for fulfilling the demand.
- There is no pause during production operation for a class.
- All parameters such as demand rate, part machining time, commissioning time, defective parts percentage etc. (except for the costs which are inflamed) are specified and fixed (non-random).
- Value is added during production operations and payments are assumed to be continuous and instantaneous.
- Planning is horizontally infinite.

2.2 Symbols

The symbols used in this article are as follows:

A – cost of any commissioning of the system at the beginning of planning horizon (monetary unit by order)

c – any goods processing cost at the beginning of planning horizon (money unit in goods unit)

h – any product unit maintenance cost unit in time unit at the beginning of planning horizon (money unit in goods unit in time unit)

D – demand rate in time unit (goods unit in time unit)

P – production rate in time unit (goods unit in time unit)

θ – defective to total product ratio

α – continuous combined interest rate

β – continuous combined inflation rate

t_1 – end of production period in any cycle (time unit)

t_2 – end of reworking period in any cycle (time unit)

T_m – total processing time in any cycle (time unit)

T_d – production free consumption interval length (time unit)

T – length of each period (time unit)

Q – production class size-decision variable (goods unit)
Q_{Jamal} – optimal production class size of model belonging to Jamal et al. [14]
Q^* – near optimal size of production class with regard to money time value (goods unit)
$H(t)$ – cost unit of goods maintenance in store a time t (money unit in goods unit in time unit)
$I(t)$ – store inventory level function in terms of time (goods unit)
C_s – present value of commissioning cost at any period (money unit)
C_m – present value of production cost at any period (money unit)
C_h – present value of inventory maintenance cost at any period (money unit)
C – present value of total expenses relating to inventory for any period (money unit)
$TC(Q)$ – present value of total expenses relating to the inventory for infinite planning horizon (money unit).

3 Determining production class size with regard to time value of money

It is necessary to study the behaviour of the finished goods inventory in production cell and end product in the store during time in order to calculate all kinds of inventory costs.

Since the demand is final, behaviour of finished goods inventory in store is similar to economic production quantity model. The reason for the inequality of the mentioned model behaviour and economic production quantity model is the reworking process.

Product inventory level process in store is shown in Fig. 1. Due to the necessity of mentioning the points during calculation of all costs and due to the calculation of error resulting from non-attention to time value of money in numerical calculation section, relations obtained by Jamal et al. [14] are given below. Relations (1) and (2) show cost function and optimal point of model presented by Jamal et al. [14]:

$$TC = A\frac{D}{Q} + C\cdot D(1+\theta) + \frac{H\cdot Q\cdot D}{2P^2}\Big[\big(P(1-\theta)-D\big)(1+2\theta)+\theta^2(P-D)\Big] + \frac{H\cdot Q}{2P}\left(1-\frac{D}{P}-\frac{\theta\cdot D}{P}\right)^2. \tag{1}$$

$$Q^*_{\text{Jamal}} = \sqrt{\frac{2A\cdot D}{h\left[1-\frac{D}{P}(1+\theta+\theta)^2\right]}}. \tag{2}$$

The relations which have been obtained by Jamal et al about production costs including primary processing cost and reworking operations (sentence $c\cdot D(1+\theta)$ in relation (1)) show that these two costs do not depend on production class size, therefore, they do not take part in determination of optimal size of production class without considering time value of money. But, as shown later, these two costs are effective on determination of optimal size of the production class with regard to time value of money. Relations (3) to (6) show calculation of time components of each period including t_1, t_2, T_m and T. These relations can be achieved with regard to Fig. 1.

$$t_1 = \frac{Q}{P}, \tag{3}$$

$$t_2 = t_1 + \frac{\theta\cdot Q}{P} = \frac{(1+\theta)Q}{P}, \tag{4}$$

$$T_m = \frac{(1+\theta)Q}{P}, \tag{5}$$

$$T = \frac{Q}{D}. \tag{6}$$

As referred above, inventory systems costs include commissioning, production (primary production cost and reworking cost) and goods maintenance cost. In order to calculate present value of total costs of planning horizon, present value of a period is calculated. Then, with regard to the fact that periods are similar, present value of costs can be calculated. Present value of total costs in each period equals to:

$$C = C_s + C_m + C_h. \tag{7}$$

With regard to the above remarks and Fig. 1, we calculate present value of each cost mentioned during a period.

3.1 Calculation of present value of commissioning cost in each period

With regard to the defined symbols, fixed cost of each commissioning of machine is A and it is specified that in Fig. 1, this cost occurs at the beginning of each period. Therefore, present value of commissioning cost in each period equals to:

$$C_s = A. \tag{8}$$

3.2 Calculation of present value of production cost in each period

With regard to the defined symbols, production cost of each product unit, c is money unit and this product may be sound or defective.

With regard to the fact that production and reworking rate in time unit is goods unit P, production or reworking cost in time unit equals $c\cdot P$ as long as production is done. With regard to engineering economic relations, present value of cash flow f at time t equals $f\mathrm{e}^{-\alpha t}$ and in case that the inflation is considered, we can say that if g is goods value at time zero, its value equals $g\mathrm{e}^{-\beta t}$ with continuous application of inflation at time t in which β is inflation rate. With regard to the fact that present value of production and reworking cost in each period is $\rho = \beta - \alpha$, we have:

$$C_\mathrm{m}=\int_0^{t_2}(cP\mathrm{e}^{\beta t})\mathrm{e}^{-\alpha t}\,\mathrm{d}t=\frac{cP}{\beta-\alpha}\left(\mathrm{e}^{(\beta-\alpha)t_2}-1\right)=\frac{cP}{\rho}\left(\mathrm{e}^{\frac{\rho(1+\theta)Q}{P}}-1\right). \quad (9)$$

3.3 Calculation of present value of maintenance cost in each period

With regard to the defined symbols, $H(t)$ and $I(t)$ are goods unit maintenance cost and inventory in store in time t respectively, inventory maintenance cost during each period is equal to:

$$C_\mathrm{h}=\int_0^T H(t)I(t)\mathrm{d}t. \quad (10)$$

If h is goods maintenance cost at time unit and time origin (zero time), in this case, $H(t)$ is equal to:

$$H(t)=(h\mathrm{e}^{\beta\cdot t})\,\mathrm{e}^{-\alpha\cdot t}=h\mathrm{e}^{\rho\cdot t}. \quad (11)$$

With regard to relation (10), it is necessary to know inventory quantity function in terms of time in order to calculate inventory maintenance cost. We can observe in Fig. 1 that behaviour of inventory in total term can be shown as time function by dividing the period into three components. With regard to Fig. 1, the behaviour of inventory as time function is equal to:

$$I(t)=\begin{cases}[P(1-\beta)-D]^t & t\in[0,t_1]\\ [P(1-\beta)-D]_1^t+(P-D)(t-t_1) & t\in[t_1,t_2]\\ [P(1-\beta)-D]_1^t+(P-D)(t_2-t_1)-D(t-t_2) & t\in[t_2,t_3]\end{cases}. \quad (12)$$

After substituting relations (3), (4) and (5) in relation (12) and simplifying them, we have:

$$I(t)=\begin{cases}[P(1-\theta)-D]t & t\in[0,t_1]\\ -\theta\cdot Q+(P-D)t & t\in[t_1,t_2]\\ Q-Dt & t\in[t_2,t_3]\end{cases}. \quad (13)$$

With the use of relations (10), (11) and (13), the present value of total cost of finished goods maintenance is equal to:

$$\begin{aligned}C_\mathrm{h}&=\int_0^T I(t)H(t)\mathrm{d}t=\int_0^{t_1}\left[\left(P(1-\theta)-D\right)t\right]h\mathrm{e}^{\rho\cdot t}\mathrm{d}t+\\&+\int_{t_1}^{t_2}\left[-\theta\cdot Q+(P-D)t\right]h\mathrm{e}^{\rho\cdot t}\mathrm{d}t+\int_{t_2}^{t_3}\left(Q-D\cdot t\right)h\mathrm{e}^{\rho\cdot t}\mathrm{d}t=\\&=\frac{Ph}{\rho^2}\left[\left(1-\mathrm{e}^{\frac{Q\cdot\rho(1+\theta)}{P}}+\theta(\mathrm{e}^{\frac{Q\cdot\rho}{P}}-1)\right)+\frac{D}{p}(\mathrm{e}^{\frac{Q\cdot\rho}{D}}-1)\right].\end{aligned} \quad (14)$$

3.4 Calculation of present value of total costs

In the previous section, each one of the sentences relating to the cost of a period was calculated separately, therefore total costs of the inventory for a period can be calculated with regard to relation (7). In addition, it is clear that this cost is repeated in each period.

In order to obtain present value of total costs, the present value of the cost of each period for planning horizon (infinite time) is added with regard to time value of money. With regard to the fact that the distance of the periods from each other is equal to the length of each period T, present value of costs is equal to:

$$\begin{aligned}TC(Q)&=C+C\mathrm{e}^{\rho T}+C\mathrm{e}^{2\rho T}+\ldots=\\&=C\left(1+\mathrm{e}^{-(\alpha-\beta)\frac{Q}{D}}+\mathrm{e}^{-2(\alpha-\beta)\frac{Q}{D}}+\ldots\right).\end{aligned} \quad (15)$$

If $\rho=\beta-\alpha>0$, in case that the planning horizon is assumed to be infinite, sentence inside the parenthesis of relation (15) will be divergent , then, no specified term is obtained for $TC(Q)$. Therefore, in case that $\beta-\alpha>0$, a finite planning horizon should be considered (for one year) so that the total cost function can be convergent. But in case that $\alpha-\beta\geq 0$ (which is true because interest rate equals inflation rate), the sentence inside the parenthesis on the right side of relation (15) is equal to $1/(1-\mathrm{e}^{Pq/D})$ and in this case, $TC(Q)$ is divergent. In this case, present value of total inventory costs is obtained as follows by substituting present value of total costs of each cycle (C) which is obtained with the use of relations (8), (9) and (14), the present value of total inventory costs is obtained as follows in relation (15):

$$\begin{aligned}TC(Q)&=\left\{A+\frac{c\cdot P}{\rho}\left(\mathrm{e}^{\rho(1+\theta)\frac{Q}{P}}-1\right)+\right.\\&\left.+\frac{P\cdot h}{(\alpha-\beta)^2}\left[\left(1-\mathrm{e}^{\rho(1+\theta)\frac{Q}{P}}+\theta\left(\mathrm{e}^{\rho\frac{Q}{P}}-1\right)\right)+\frac{D}{P}\left(\mathrm{e}^{\rho\frac{Q}{D}}-1\right)\right]\right\}\times\\&\times\left(\frac{1}{1-\mathrm{e}^{\rho\frac{Q}{D}}}\right).\end{aligned} \quad (16)$$

4 Problem solving method

As observed above, relation (16) is a one-variable function in terms of Q. In case that the second derivative of function is positive, this function will be convex and have minimum absolute point. In order to find optimal point, we should take the derivative of the function relative to Q and find its root. Calculations show that the result of the first and second derivative is complex and long. The performed studies show that it is not possible to obtain the root of the first derivative as the optimal point and show positive second derivative. The use of linear approximation of $\mathrm{e}^{-\delta}=1-\delta+\frac{\delta\hat{}2}{2}$ does not solve the problem because the calculations show that the complexity of target function derivative continues with the use of this approximation. With regard to inefficiency of derivative technique to obtain root of the function, we should go to searching methods. Most numerical optimization methods are based on sequential production of approximate values so that target function is optimized with regard to those values. In this article, combination of searching algorithm with accelerating motion (in order to search the interval in which optimal point is located) and Dico Thomas method (in order to reduce the selected interval and introduce a near optimal point) have been used for problem solving. This combined method is able to get close to optimal answer with any desirable accuracy. In the said method, Q_{Jamal} (relation (2)) has been used as starting point. Stages of this algorithm are as follows (in this algorithm, δ is the desirable accuracy and ε is a small number).

Step 0: consider function $TC(Q)$ which is a one-variable cost function.
Step 1: determine the starting point (Q_0), ($Q_0 = Q_{\text{Jamal}}$). In case that $TC(Q_0+\varepsilon)>TC(Q_0)$ put: $s=-0{,}02\cdot Q_0$, otherwise, $s=0{,}02\cdot Q_0$
Step 2: put $Q_1=Q_0$ until $TC(Q_1)<TC(Q_0)$, put $Q_1=Q_1+s$
Step 3: if $Q_0 > Q_1$, replace Q_0 and Q_1. $Q^*_1=Q_0+Q_1-Q_0)/2+\delta/2$, $Q^*_0=Q_0+Q_1-Q_0)/2+\delta/2$
Step 4: put
If $TC(Q^*_0)>TC(Q^*_1)$ then, exclude values of $Q>Q^*_1$ and put: $Q=Q^*_1$

- If $TC(Q^*_0)>TC(Q^*_1)$ then, exclude values of $Q>Q^*_1$ and put: $Q_0=Q^*_1$
- If $TC(Q^*_0)=TC(Q^*_1)$ then, select two other points as Q_0 and Q_1 (with change in δ)

Step 5: if $TC(Q^*_0)-TC(Q^*_1)<\delta$, put $Q^*=(Q^*_0+Q^*_1)/2$ Q^* is near optimal answer of relation 16). Otherwise, go to step 4.

5 Numerical example and sensitivity analysis

In this section, results of the proposed model are compared with results of the model presented by Jamal et al. and then the proposed model sensitivity is analysed.

D = 100 Year/goods unit
P = 1100 Year/goods unit
θ = 17 %
A = 1900 Order time/Rial
c = 120 Goods unit/Rial
h = 6 Goods unit, year/Rial.

Tab. 1 shows near optimal answer with present value of total costs for near optimal answer and optimal point which is obtained with relation (2). In this Tab. 9 different values have been considered for interest rate and inflation rate.

Column 4 of this table shows optimal value of total cost for different rates. This value has been obtained with the use of the mentioned search algorithm about present value of costs. Column 5 shows minimum costs which have been obtained from relation 16 and for production class equivalent to column 4. Columns 6 and 7 are optimal values and minimum cost of model presented by Jamal et al. column 8 shows percentage of error resulting from non-attention to inflation and time value of money in determination of production class size.

Table 1 Solving a numerical example for different values of interest rate and inflation rate and comparison with the model presented by Jamal et al.

1	2	3	4	5	6	7	8
Row	α / %	β / %	α / %	$TC(Q^*)$	Q^*_{Jamal}	$TC(Q^*_{\text{Jamal}})$	$\frac{TC(Q^*_{\text{Jamal}})-TC(Q^*)}{TC(Q^*)}\times 100$
1	50	4	239	1 563 600	267	1 564 500	0,06
2	10	5	179	324 030	267	327 540	1,08
3	15	6	148	185 150	267	190 310	2,79
4	20	7	130	131 280	267	137 680	4,87
5	25	8	116	102 540	267	109 920	7,20
6	30	9	106	84 607	267	92 824	9,71
7	35	10	98	72 327	267	81 267	12,36
8	40	11	92	63 374	267	72 955	15,12
9	45	12	87	56 549	267	66 706	17,96

Table 2 Changing trend of total costs for changes of interest rate, inflation rate and joint rate

Changes in inflation with Fixed interest rate (20 %)		Changes in interest rate with Fixed inflation rate (7 %)		Changes in joint rate	
β / %	$TC(Q^*)$	α / %	$TC(Q^*)$	Joint rate / %	$TC(Q^*)$
4	108 620	10	789 990	19	789 990
5	115 220	14	272 470	22	660 870
6	122 880	18	168 010	27	658 630
7	131 570	22	122 880	30	499 270
8	141 700	26	97 510	35	445 450
9	153 670	30	81 200	40	402 240
10	168 010	34	69 870	43	366 870
11	185 520	38	61 520	48	337 550
12	207 380	42	55 100	52	312 590

This example shows that the error of neglecting time value of money and inflation in the model presented by Jamal et al. [14] is considerable. For example, as the table shows, this error is 25 % for interest rate and 7,2 % for inflation rate of 8 %. It is clear that this error is important in costs. We analyse sensitivity of the proposed model compared to parameters of interest rate, inflation rate and joint rate. Changing trend of optimal value of total costs for changes of interest rate, inflation rate and joint rate is given in Tab. 2. Changing trend is shown in Figs. 2 to 4.

As observed in Fig. 2, total costs are increased with increase of inflation rate. Fig. 3 shows that the increase of interest rate causes the decrease of total costs. On the other hand, Fig. 4 shows that joint rate has a reverse effect on total costs like interest rate and its increase decreases total costs.

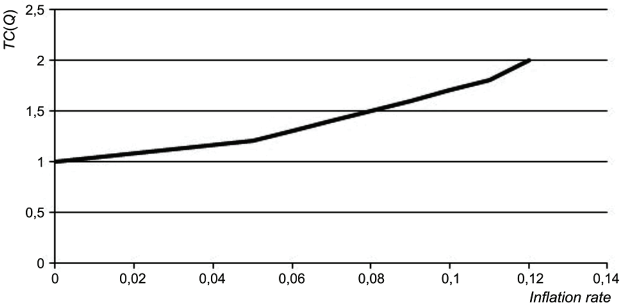

Figure 2 Changing trend of total costs compared to inflation rate (for fixed interest rate 20 %)

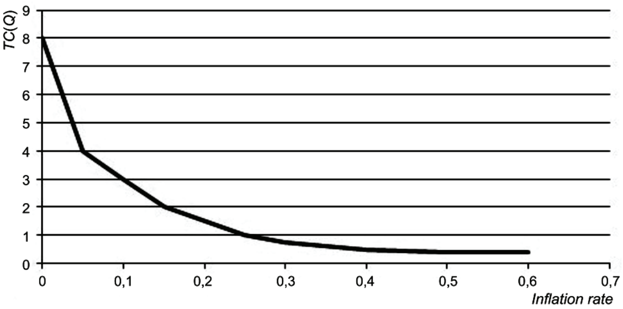

Figure 3 Changing trend of total costs compared to interest rate (for fixed inflation rate 7 %)

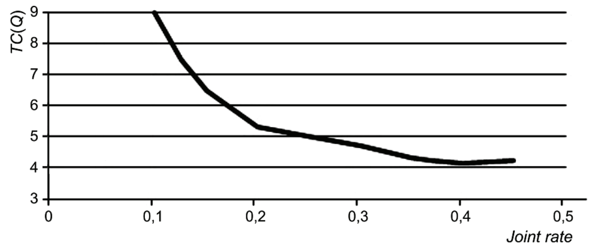

Figure 4 Changing trend of total costs compared to joint rate

6 Conclusion and suggestions

Recently, combination of classic models of inventory management and quality control has been considered by many researchers. Including time value of money and inflation in inventory management models is another aspect for expansion of these models. Considering time value of money and inflation in combined quality control and inventory management models has not been observed in the performed studies. In this article, we study this issue i.e. the effect of time value of money on production class size model with regard to reworking for a single product state. In order to solve the problem, an algorithm based on combination of two accelerating and Dico Thomas search methods has been used and effectiveness of the model was shown with the use of a numerical example. As the future researchers, we can refer to generalization of the problem for multiple product state, generalization of the model with regard to failure of the machine and considering time value of money and inflation in similar models. Like works done by Chung et al. Chung and Hung, finding upper and lower limits for near optimal point of the developed model in this article to be used in middle value method can be one of the future works in order to perform numerical calculation.

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