CODEN STJSAO ISSN 0562-1887
ZX470/1560 UDK 621.888.1:519.6

Modelling of Dynamical Behavior of a Spindle-Holder-Tool Assembly

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Original scientific paper

This paper describes the complete procedure for mathematical modeling of dynamic behavior of a spindle - holder – tool assembly. The developed model, in addition to translational, takes into account the rotational degrees of freedom, and therefore can be used for calculation and prediction of frequency response function of a spindle - tool holder – tool assembly. In order to properly describe behavior of a dynamic system with correspondent mathematical model, including a spindle assembly, it is necessary, in addition to the exact mathematical model, to define unknown model parameters, i.e. different types of connections, which are very difficult, often impossible, to determine in the experimental way. Accordingly, this paper describes the mathematical formulation of the Levenberg-Marquardt method which was applied to identify the unknown parameters of a spindle assembly. In order to verify the proposed mathematical model of the spindle – holder – tool assembly and the principles for identification of unknown parameters, the numerical analysis of the above systems was carried out. Furthermore, the model was experimentally verified on a free-free spindle – holder – tool system.

Keywords
Spindle assembly
Dynamics
Parameter identification

Modeliranja dinamičkog ponašanja sustava glavno vreteno – držač alata – alat

Izvornoznanstveni članak

U radu se opisuje kompletna procedura matematičkog modeliranja dinamičkog ponašanja sustava glavno vreteno – držač alata – alat. Razvijeni model, pored translatornih, uzima u obzir i rotacijske stupnjeve slobode, a može poslužiti za proračun i predviđanje funkcije frekvencijskog odaziva sustava glavno vreteno – držač alata – alat. Ukoliko se želi pravilno procijeniti ponašanje nekog dinamičkog sustava opisanog odgovarajućim matematičkim modelom, pa tako i sklopa glavnog vretena, neophodno je, pored točnog matematičkog modela, definirati i parametre modela koji nisu poznati, tj. različite tipove veza, koje je veoma teško, najčešće i nemoguće, odrediti eksperimentalnim putem. U tom smislu, u radu se detaljno opisuje matematička formulacija Levenberg-Marquardt-ove metode koja je primijenjena za identifikaciju nepoznatih parametara veza sklopa glavnog vretena. U cilju verifikacije predloženog matematičkog modela, kao i opisanih principa identifikacije parametara, izvršena je numerička simulacija sklopa glavnog vretena. Nadalje, model je eksperimentalno verificiran na slobodno oslonjenom sustavu glavno vreteno – držač alata – alat.

Ključne riječi
Sklop glavnog vretena
Dinamika
Identifikacija parametara

Primljeno (Received): 2011-10-10
Prihvaćeno (Accepted): 2012-01-26

1. Introduction

Most of the research related to machine tools is connected to the machine tools spindle, since the characteristics of the spindle, such as static and dynamic behavior, strength, speed, among many others, have a significant impact on machine tools performance. Emphasized the importance of spindle assembly is based on the fact that the essence of the machining process is reduced to relative motion of the tool in relation to a workpiece, so the accuracy of the spindle movement directly reflects the accuracy of the tool motion relative to the workpiece, and thus the accuracy of the final product. In order to ensure proper performance during the operation of machine tools, a spindle assembly should meet strict requirements concerning the appropriate dynamic stability, which generally has a determining influence on the overall stability of machine tools. Many research efforts have made significant contributions to modeling dynamic behavior of a spindle – holder – tool assembly [1-6].
The problem of considering dynamic properties of the machine tool – spindle – holder – tool system can be

Symbols/Oznake

$\mathbf{A}$	- receptance matrix of the tool - matrica odgovora alata
c	- damping coefficient, kgs^{-1} - koeficijent prigušenja
$\mathbf{D}$	- receptance matrix of the holder - matrica odgovora držača alata
E	- Youngs modulus, Nm^{-2} - modul elastičnosti
$f(\boldsymbol{\theta})$	- objective function - funkcija cilja
$\mathbf{G}$	- receptance matrix of the global system - matrica odgovora globalnog sustava
$\mathbf{H}$	- receptance matrix - matrica odgovora
$\breve{\mathbf{H}}$	- Hessian approximation - aproksimacijska matrica Hessian matrice
i	- imaginary number - imaginarni broj
$\mathbf{I}$	- identity matrix - jedinična matrica
$\mathbf{J}$	- Jacobian matrix - Jakobijeva matrica
k	- stiffness coefficient, Nm^{-1} - koeficijent krutosti
$\mathbf{K}$	- stiffness matrix - matrica krutosti
$\mathbf{r}$	- vector of residuals - vektor ostatka
μ	- dynamic viscosity, - dinamička viskoznost
ρ	- density, kgm^{-3} - gustoća
ω	- angular velocity, rad s^{-1} - kutna brzina

Subscripts/Indeksi

i	- internal node on substructure - vanjska koordinata podsustava
c	- coupling node on substructure - unutarnja koordinata podsustava
t	- tranverse displacement/excitation - translatorni odgovor/pobuda
r	- rotational displacement/excitation - rotacijski odgovor/pobuda

simplified so that instead of viewing it as a single one, the specified system is regarded as composed of three separate subsystems as follows: a machine tool – spindle, holder and tool. Out of these three subsystems, the tool and holder are the most suitable for modeling because they are not structurally complex. On the other hand, modeling of dynamic behavior of the machine tool – spindle system is much more complicated. Modeling of dynamic behavior of the spindle assembly is done mainly using finite element method, but it requires detailed knowledge of dimensions of a spindle, stiffness of bearing as well as damping. When it comes to commercial machine tools, these data are unknown or are only partially known to the end user. Additionally, information on damping of the spindle assembly, because of its importance, still remains in the active area of research and is usually not available. All this points to a scenario where we can consider the modeling of structural components that are not complicated: a holder and tool, and experimental identification of those components which are difficult to model: a machine tool – spindle. The most important requirements of spindle assembly exploitation are parameters of dynamic behavior, so the main aim of this paper is development of a mathematical model for modeling of dynamical behavior of a spindle - holder – tool assembly.

2. Mathematical model of a spindle – holder – tool assembly

Since the spindle assembly is one of the most important machine tool components, it is necessary to develop an appropriate mathematical model that will be the most suitable and reliable one for the given physical system and in accordance with that, this chapter describes the complete procedure for mathematical modeling of the spindle - holder - tool assembly. It is generally accepted that an analysis of complex dynamical systems can be simplified by breaking a system down to a set of interconnected subsystems. In this sense, the problem referring to dynamic properties of the spindle system - holder - tool system can be simplified so that instead of viewing it as single, the specified system is regarded as the one composed of three separate subsystems, namely: a spindle, holder and tool.

Components of the spindle – holder – tool assembly should be coupled elastically due to flexibility and damping introduced by contacts at spindle – holder and holder – tool interfaces. Furthermore, we have applied the approach [7,8], where part of the holder inside the spindle is considered as integrated to the spindle (Figure 1). Some authors [9] applied somewhat different approach, where the spindle and holder are connected with a series of parallel springs. However, the approach presented in [7,8] provides a more realistic model, because only the dynamics due to the masses of these subsystems will be included into the model or it will be required to include their stiffness effects with distributed springs.

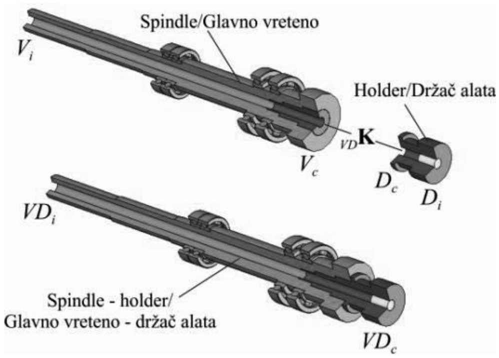

Figure 1. Elastic coupling of the spindle – holder system

Slika 1. Elastično spajanje glavnog vretena s držačem alata

Complex stiffness matrix, representing the spindle – holder interface dynamics has the following form:

$$ {}_{VD}\mathbf{K} = \begin{bmatrix} {}_{VD}k_t + i\cdot\omega\cdot {}_{VD}c_t & 0 \\ 0 & {}_{VD}k_r + i\cdot\omega\cdot {}_{VD}c_r \end{bmatrix}, \tag{1}$$

where: ${}_{VD}k_t$ – translational stiffness, ${}_{VD}c_t$ – translational damping, ${}_{VD}k_r$ – rotational stiffness and ${}_{VD}c_r$ – rotational damping at the spindle – holder interface.

Assuming that response matrices of the subsystem *V* (spindle with bearings) and subsystem *D* (holder) are known, then it is possible by using a method of receptive coupling, to obtain the global system response matrix *VD* (spindle – holder) at the holder tip:

$$\mathbf{VD}_{ii} = \mathbf{D}_{ii} - \mathbf{D}_{ic}\cdot\left(\mathbf{D}_{cc} + \mathbf{V}_{cc} + {}_{VD}\mathbf{K}^{-1}\right)^{-1}\cdot\mathbf{D}_{ci}\,. \tag{2}$$

Similarly, the part of the tool inside the holder is considered rigidly joined to the holder, so the receptance matrix of the tool can be coupled with the rest of the system, as depicted in Figure 2.

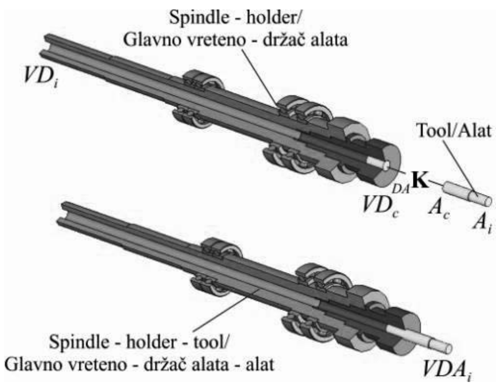

Figure 2. Elastic coupling of the spindle – holder – tool system

Slika 2. Elastično spajanje glavnog vretena – držača alata s alatom

Receptance matrix of the global system *VDA* (spindle – holder – tool) at the tool tip has the following form:

$$\mathbf{VDA}_{ii} = \mathbf{A}_{ii} - \mathbf{A}_{ic}\cdot\left(\mathbf{A}_{cc} + \mathbf{VD}_{cc} + {}_{DA}\mathbf{K}^{-1}\right)^{-1}\cdot\mathbf{A}_{ci}\,. \tag{3}$$

In the equationabove,**A** is a subsystem of a tool and ${}_{DA}\mathbf{K}$ is the complex stiffness of holder – tool interface dynamics:

$$ {}_{DA}\mathbf{K} = \begin{bmatrix} {}_{DA}k_t + i\cdot\omega\cdot {}_{DA}c_t & 0 \\ 0 & {}_{DA}k_r + i\cdot\omega\cdot {}_{DA}c_r \end{bmatrix}, \tag{4}$$

where: ${}_{DA}k_t$ – translational stiffness, ${}_{DA}c_t$ – translational damping, ${}_{DA}k_r$ – rotational stiffness and ${}_{DA}c_r$ – rotational damping at the holder – tool interface.

In order to be able to use equation (3) to predict the frequency response function of the tool tip, it is necessary to know translational and rotational dynamic response for each of the components of the spindle - holder – tool assembly. Response matrix of the tool and holder can be obtained by an analytical method, using some of the beam theories or through the FEM analysis. Defining spindle response poses a problem because data regarding dimensions, material, the manner of bearing, the number, and type of bearings are unknown so their modeling is critical. On the other hand, it is possible only experimentally to measure translational dynamic response of the spindle, whereas to complete the receptance matrices it is necessary to know rotational response. The following section presents methodology for identification of rotational dynamic response of the spindle – holder – tool assembly.

2.1. Calculation of rotational degrees of freedom

In many areas of structural dynamics, rotational degrees of freedom – RDOF play an important role in receptance coupling of subsystems, and therefore they have to be considered as independent coordinates. As the possibility to measure RDOF is very limited, only translational degrees of freedom – TDOF are mostly considered. However, in receptance coupling of the spindle – holder – tool assembly, information on RDOF plays an important role, and their neglect may result in an unreliable final model.

Silva [10] presented a method to determine the rotational response of an arbitrary system without their direct measurements. It is assumed that a spindle assembly (Figure 3) consists of subsystems *A* and *B*. The objective is receptance coupling of these two subsystems with inclusion of RDOF in the synthesis.

In deriving equations for calculating RDOF, for generalization purposes, a label *B* is used instead of the spindle – holder system (*VD*), while for an additional part of the holder that is rigidly connected with it, a label *A* is used. Two subsystems under consideration are shown in Figure 3, where ${}_AF_i$, ${}_AF_c$, ${}_BF_c$ and ${}_Ax_i$, ${}_Ax_c$, ${}_Bx_c$ mark excitation force and translational displacements, respectively. With ${}_AM_i$, ${}_AM_c$, ${}_BM_c$ indicate excitation torque, while ${}_A\theta_i$, ${}_A\theta_c$, ${}_B\theta_c$ represent rotational displacements.

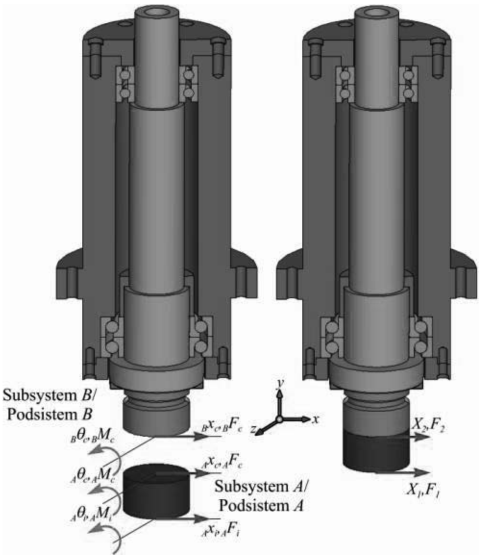

Figure 3. Substructuring of the spindle assembly

Slika 3. Podstrukturiranje sklopa glavnog vretena

As noted, RDOF must be considered for good prediction of FRF, and in accordance with these responses of the global system $\mathbf{G}_{11}$ and $\mathbf{G}_{12}$ can be considered using the following equations:

$$\mathbf{G}_{11}=\begin{bmatrix} {}_A\mathbf{H}_{tt}^{ii} & {}_A\mathbf{H}_{tr}^{ii} \\ {}_A\mathbf{H}_{rt}^{ii} & {}_A\mathbf{H}_{rr}^{ii} \end{bmatrix}-\begin{bmatrix} {}_A\mathbf{H}_{tt}^{ic} & {}_A\mathbf{H}_{tr}^{ic} \\ {}_A\mathbf{H}_{rt}^{ic} & {}_A\mathbf{H}_{rr}^{ic} \end{bmatrix} \left(\begin{bmatrix} {}_A\mathbf{H}_{tt}^{cc} & {}_A\mathbf{H}_{tr}^{cc} \\ {}_A\mathbf{H}_{rt}^{cc} & {}_A\mathbf{H}_{rr}^{cc} \end{bmatrix}+\begin{bmatrix} {}_B\mathbf{H}_{tt}^{cc} & {}_B\mathbf{H}_{tr}^{cc} \\ {}_B\mathbf{H}_{rt}^{cc} & {}_B\mathbf{H}_{rr}^{cc} \end{bmatrix}\right)^{-1}\begin{bmatrix} {}_A\mathbf{H}_{tt}^{ci} & {}_A\mathbf{H}_{tr}^{ci} \\ {}_A\mathbf{H}_{rt}^{ci} & {}_A\mathbf{H}_{rr}^{ci} \end{bmatrix}, \quad (5)$$

$$\mathbf{G}_{12}=\begin{bmatrix} {}_A\mathbf{H}_{tt}^{ic} & {}_A\mathbf{H}_{tr}^{ic} \\ {}_A\mathbf{H}_{rt}^{ic} & {}_A\mathbf{H}_{rr}^{ic} \end{bmatrix}-\begin{bmatrix} {}_A\mathbf{H}_{tt}^{ic} & {}_A\mathbf{H}_{tr}^{ic} \\ {}_A\mathbf{H}_{rt}^{ic} & {}_A\mathbf{H}_{rr}^{ic} \end{bmatrix} \left(\begin{bmatrix} {}_A\mathbf{H}_{tt}^{cc} & {}_A\mathbf{H}_{tr}^{cc} \\ {}_A\mathbf{H}_{rt}^{cc} & {}_A\mathbf{H}_{rr}^{cc} \end{bmatrix}+\begin{bmatrix} {}_B\mathbf{H}_{tt}^{cc} & {}_B\mathbf{H}_{tr}^{cc} \\ {}_B\mathbf{H}_{rt}^{cc} & {}_B\mathbf{H}_{rr}^{cc} \end{bmatrix}\right)^{-1}\begin{bmatrix} {}_A\mathbf{H}_{tt}^{cc} & {}_A\mathbf{H}_{tr}^{cc} \\ {}_A\mathbf{H}_{rt}^{cc} & {}_A\mathbf{H}_{rr}^{cc} \end{bmatrix}. \quad (6)$$

The objective is receptance coupling of these two subsystems with inclusion of RDOF in synthesis. The assumption is that a subsystem *A* can be modeled using the finite element software, and thus determine the complete FRF response matrix with translational and rotational dynamic responses, while the subsystem *B* cannot be modeled, but only experimentally measured. Thus, with FEM simulation of the subsystem *A*, dynamic responses ${}_A\mathbf{H}_{tt}$, ${}_A\mathbf{H}_{tr}$, ${}_A\mathbf{H}_{rt}$, ${}_A\mathbf{H}_{rr}$can be obtained as to complete the following FRF matrices: ${}_A\mathbf{H}_{ii}$, ${}_A\mathbf{H}_{ic}$, ${}_A\mathbf{H}_{ci}$ and ${}_A\mathbf{H}_{cc}$. As regards the subsystem *B*, only translational response ${}_B\mathbf{H}_{tt}$ can be experimentally measured in a reliable way, because responses ${}_B\mathbf{H}_{tr}$ and ${}_B\mathbf{H}_{rr}$ are related to RDOF and, practically, it is impossible to measure them. The methodology presented in [10] defines the rotational responses of only one FRF, whereas in this case there are two FRF to be determined as follows: ${}_B\mathbf{H}_{tr}={}_B\mathbf{H}_{rt}$ and ${}_B\mathbf{H}_{rr}$. In this sense, the equation presented in this paper is extended by the system of two equations with two unknowns.

After appropriate mathematical transformations, with rotational responses of the subsystem *A* using a finite element method, it is possible to derive expressions for the rotational dynamic responses of the subsystem *B* using the MATLAB program system and its symbolic nonlinear analytical toolbox. According to [11] derived expressions for the rotational dynamic responses of the subsystem B are:

$${}_B\mathbf{H}_{rt}^{cc}=\mathbf{B}_{rt}-{}_A\mathbf{H}_{rt}^{cc}, \quad (7)$$

$${}_B\mathbf{H}_{rr}^{cc}=\mathbf{B}_{rr}-{}_A\mathbf{H}_{rr}^{cc}, \quad (8)$$

where:

$$\mathbf{B}_{rt}=\frac{kfv-kug+kag-kfb+fdb-cbg}{ad-ud-cb+cv}, \quad (9)$$

$$\mathbf{B}_{rr}=\frac{1}{ad-ud-cb+cv^{\;2}}\Big[kf^2v^2+\;2kagf+bf^2d- \\ 2kugf-ec^2g+decf-2bkf^2-bfcg\;\;v-d^2efu+ \\ d^2efa+g^2ka^2-decga+decgu+g^2ku^2+bdgfa- \\ 2g^2kua+bec^2g-bdgfu-bdefc+2bkugf+b^2kf^2- \\ 2bkagf+bg^2cu+b^2fcg-b^2f^2d-bg^2ca\Big], \quad (10)$$

where: $a={}_A\mathbf{H}_{tt}^{ii}$, $b={}_A\mathbf{H}_{tt}^{ic}$, $c={}_A\mathbf{H}_{tt}^{ci}$, $d={}_A\mathbf{H}_{tt}^{cc}$, $e={}_A\mathbf{H}_{rt}^{ic}$, $f={}_A\mathbf{H}_{rt}^{ci}$, $g={}_A\mathbf{H}_{rt}^{cc}$, $k={}_A\mathbf{H}_{tt}^{cc}+{}_B\mathbf{H}_{tt}^{cc}$, $u=\mathbf{G}_{tt}^{11}$, $v=\mathbf{G}_{tt}^{12}$.

Equations (7) and (8) define calculated RDOF responses of the subsystem *B*, or in this case the spindle – holder system (*VD*). To get response at the tip of the spindle, it is necessary to use inverse receptance coupling for substracting a part of the holder from the spindle – holder system. Returning to the notation, in which tags *VD*, *V* and *D* denote spindle – holder subsystems, the spindle and the holder, respectively, desired response is obtained at the top of the spindle:

$$\mathbf{V}_{cc}=\mathbf{D}_{ci}\cdot\;\mathbf{D}_{ii}-\mathbf{VD}_{ii}\;^{-1}\cdot\mathbf{D}_{ic}-\;\mathbf{D}_{cc}+{}_{VD}\mathbf{K}^{-1}\;. \quad (11)$$

3. Identification of connection parameters for the spindle – holder – tool assembly

Identification of the parameters has an increasing application in many areas of engineering, where mathematical models are used to describe natural phenomena and experiments that are performed to verify these models. The advantages of mathematical models include optimization of design and production, as well as the possibility to analyze and understand system behavior subject to conditions that cannot be easily obtained in the course of an experiment. Mathematical models very often contain a number of parameters that cannot be measured directly or calculated using the established laws of nature, and therefore must be identified from experimental data.

The basic concept is to determine these parameters in a way that the differences between the experimental data and the values predicted by the model are minimal.

In most cases, synthesis of dynamic systems, as well as the spindle assembly, is considered with the rigid connection between subsystems, which is a simplification of the problem, because most of the relationship is characterized by elasticity and damping effects. In the synthesis of dynamical systems modeling of contact parameters plays a critical role, because of significant impact on response of the global system. Accordingly, neglecting the effects contact parameters between subsystems of the spindle assembly can make prediction of the entire system unreliable and inaccurate. Therefore, the accuracy of prediction of the dynamic response of the global system is largely conditioned by a lack of a reliable description of interactions between subsystems, i.e. types of connections and their behavior. For these reasons, it is important that the mathematical model of the spindle assembly incorporating the effects of connection between the subsystems.

3.1. The mathematical formulation

The assumption is that the mathematical model under consideration can be described by a system of differential equations:

$$\mathbf{D}\dot{\mathbf{y}} = \mathbf{f}(t,\mathbf{y},\boldsymbol{\theta}), \quad y(t_0,\boldsymbol{\theta}) = \mathbf{y}_0(\boldsymbol{\theta}), \tag{12}$$

where: $\boldsymbol{\theta}$ – vector of unknownparameters, $\mathbf{y}$– statevectordepending on $t$and $\boldsymbol{\theta}$, $\mathbf{f}$ –generally, nonlinearfunctions, $\mathbf{D}$ –nx$n$constantdiagonalmatrix. Applying the notation [12] eachmeasurementcan be characterizedbythreeparameters:

$$(c_i, t_i, \tilde{y}_i), \qquad i = 1,2,...,m, \tag{13}$$

where: c_i – component of the state vectory that has been measured, t_i – the time of measurement, $\tilde{y}_i$ – measured value, m – total number of measurements. The solution of the model equations (12), for the c_i-th component at time t_i, which corresponds to the i-th measurement is denoted by $y_{c_i}(t_i,\boldsymbol{\theta})$.

The general approach to the problem of identifying parameters is to minimize the differences between the results obtained by measuring and by the mathematical model, i.e.:

$$r_i(\boldsymbol{\theta}) = y_{c_i}(t_i,\boldsymbol{\theta}) - \tilde{y}_i. \tag{14}$$

Appropriate method of identification depends on the assumptions and knowledge about the errors of measurement. One of the most widely used method of identification is the method of least squares. In its simplest form, the parameters are identified such that the sum of squared residuals is minimal, i.e. the objective function is given as a sum of squared differences:

$$f(\boldsymbol{\theta}) = \frac{1}{2}\sum_{i=1}^{m} r_i^2(\boldsymbol{\theta}) \tag{15}$$

Differences between the results obtained in experimental tests and using the mathematical model can be represented as a vector **r** defined by:

$$\mathbf{r}(\boldsymbol{\theta}) = \left[r_1(\boldsymbol{\theta}) \quad r_2(\boldsymbol{\theta}) \quad \cdots \quad r_m(\boldsymbol{\theta}) \right]^T, \tag{16}$$

which is a basis to obtain an expression for the objective function of the form:

$$f(\boldsymbol{\theta}) = \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\theta})\right\|^2 = \frac{1}{2}\mathbf{r}(\boldsymbol{\theta})^T\mathbf{r}(\boldsymbol{\theta}). \tag{17}$$

3.2. Optimization procedure

Identification of the parameters can be formulated as follows:

$$\boldsymbol{\theta}^* = \arg\min_{\theta} f(\boldsymbol{\theta}), \tag{18}$$

where: $\boldsymbol{\theta}$ – vector of parameters, f – the objective function, $\boldsymbol{\theta}^*$ – vector that minimizes the objective function.

If f is twice continuously differentiable, then the following Taylor expansion for f applies:

$$f(\boldsymbol{\theta}+\mathbf{h}) = f(\boldsymbol{\theta}) + \nabla f^T(\boldsymbol{\theta})\mathbf{h} + \frac{1}{2}\mathbf{h}^T\nabla^2 f(\boldsymbol{\theta})\mathbf{h} + O\left(\left\|\mathbf{h}^3\right\|\right), \tag{19}$$

where the gradient **g** and the Hessian matrix **H** are defined as:

$$\mathbf{g} = \nabla f(\boldsymbol{\theta}) = \left[\frac{\partial f}{\partial \theta_1} \quad \frac{\partial f}{\partial \theta_2} \quad \cdots \quad \frac{\partial f}{\partial \theta_n} \right]^T, \tag{20}$$

$$\mathbf{H} = \nabla^2 f(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial^2 f(\boldsymbol{\theta})}{\partial^2 \theta_1} & \frac{\partial^2 f(\boldsymbol{\theta})}{\partial \theta_1 \partial \theta_2} & \cdots & \frac{\partial^2 f(\boldsymbol{\theta})}{\partial \theta_1 \partial \theta_n} \\ \frac{\partial^2 f(\boldsymbol{\theta})}{\partial \theta_2 \partial \theta_1} & \frac{\partial^2 f(\boldsymbol{\theta})}{\partial^2 \theta_2} & \cdots & \frac{\partial^2 f(\boldsymbol{\theta})}{\partial \theta_2 \partial \theta_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f(\boldsymbol{\theta})}{\partial \theta_n \partial \theta_1} & \frac{\partial^2 f(\boldsymbol{\theta})}{\partial \theta_n \partial \theta_2} & \cdots & \frac{\partial^2 f(\boldsymbol{\theta})}{\partial^2 \theta_n} \end{bmatrix}. \tag{21}$$

For the objective function the gradient and Hessian matrix are:

$$\mathbf{g} = \nabla f(\boldsymbol{\theta}) = \sum_{i=1}^{m} r_i(\boldsymbol{\theta})\nabla r_i(\boldsymbol{\theta}) = \mathbf{J}^T(\boldsymbol{\theta})\mathbf{r}(\boldsymbol{\theta}), \tag{22}$$

$$\mathbf{H} = \mathbf{J}^T(\boldsymbol{\theta})\mathbf{J}(\boldsymbol{\theta}) + \sum_{i=1}^{m} r_i(\boldsymbol{\theta})\nabla^2 r_i(\boldsymbol{\theta}), \tag{23}$$

where $\mathbf{J}(\boldsymbol{\theta})$ denotes the Jacobian matrix:

$$\mathbf{J}(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial r_1(\boldsymbol{\theta})}{\partial r_1} & \frac{\partial r_1(\boldsymbol{\theta})}{\partial r_2} & \cdots & \frac{\partial r_1(\boldsymbol{\theta})}{\partial r_n} \\ \frac{\partial r_2(\boldsymbol{\theta})}{\partial r_1} & \frac{\partial r_2(\boldsymbol{\theta})}{\partial r_2} & \cdots & \frac{\partial r_2(\boldsymbol{\theta})}{\partial r_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial r_m(\boldsymbol{\theta})}{\partial r_1} & \frac{\partial r_m(\boldsymbol{\theta})}{\partial r_2} & \cdots & \frac{\partial r_m(\boldsymbol{\theta})}{\partial r_n} \end{bmatrix}. \tag{24}$$

It is evident that the first part of the Hessian matrix consists of first order partial derivatives. This observation leads to an approximation forming the basis for the Gauss-Newton and Levenberg-Marquardt algorithms. Calculation of the first and second order derivatives of the objective function usually constitutes the most difficult part of the work required during the optimization. This is especially pronounced in the case of dynamical systems, where each gradient evaluation is a complex procedure requiring the solution of a set of differential equations. Therefore, in the context of parameter identification of dynamical systems, the incentive use of alternative methods that exploits the special structure in the least squares problem, is very important.

Levenberg-Marquardt algorithm is based on the assumption that the error $\mathbf{r}(\boldsymbol{\theta})$ around the point $\boldsymbol{\theta}^{(k)}$ may, in a satisfactory manner, approximate well by the first two members of Taylor's order:

$$\mathbf{r}^*(\boldsymbol{\theta}) \cong \tilde{\mathbf{r}}^*(\boldsymbol{\theta}) = \mathbf{r}^*(\boldsymbol{\theta}^{(k)}) + \nabla\mathbf{r}^*(\boldsymbol{\theta}^{(k)}) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}^{(k)}) \; . \tag{25}$$

Then, instead of minimizing the objective function, its approximation is minimized:

$$\tilde{f}(\boldsymbol{\theta}) = \frac{1}{2}\tilde{\mathbf{r}}^{*T}(\boldsymbol{\theta}) \cdot \tilde{\mathbf{r}}^*(\boldsymbol{\theta}) \; . \tag{26}$$

Equating previous equation to zero then, the following expression which minimizes the function (26) is obtained:

$$\mathbf{J}^T(\boldsymbol{\theta}^{(k)}) \cdot \mathbf{J}(\boldsymbol{\theta}^{(k)}) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}^{(k)}) + \mathbf{J}^T(\boldsymbol{\theta}^{(k)}) \cdot \mathbf{r}^*(\boldsymbol{\theta}^{(k)}) = \mathbf{0} \, . \tag{27}$$

Substituting (22) in (27) and adding learning coefficient $\alpha^{(k)}$, with $\boldsymbol{\theta} = \boldsymbol{\theta}^{(k+1)}$, the following equations is obtained:

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - \alpha^{(k)} \left[\mathbf{J}^T(\boldsymbol{\theta}^{(k)}) \cdot \mathbf{J}(\boldsymbol{\theta}^{(k)})\right]^{-1} \mathbf{J}^T(\boldsymbol{\theta}^{(k)}) \cdot \mathbf{r}^*(\boldsymbol{\theta}^{(k)}) \; . \tag{28}$$

In the literature, the expression (28) is calledGauss-Newtonalgorithm for $\alpha^{(k)} = 1$, that is, Gauss-Newtondampedalgorithm for variable$\alpha^{(k)} < 1$,where the Hessianmatrixis replacedby a matrix:

$$\tilde{\mathbf{H}}(\boldsymbol{\theta}^{(k)}) = \mathbf{J}^T(\boldsymbol{\theta}^{(k)})\mathbf{J}(\boldsymbol{\theta}^{(k)}) \; , \tag{29}$$

Levenberg [13] introduced the approximate matrix of the Hessian matrix:

$$\breve{\mathbf{H}}(\boldsymbol{\theta}^{(k)}) = \mathbf{J}^T(\boldsymbol{\theta}^{(k)}) \cdot \mathbf{J}(\boldsymbol{\theta}^{(k)}) + \mu\mathbf{I} \, . \tag{30}$$

By replacing the Hessian matrix with Levenberg matrix (30), the final expression for calculation of the parameters is obtained:

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - \breve{\mathbf{H}}^{-1}(\boldsymbol{\theta}^{(k)}) \cdot \mathbf{J}^T(\boldsymbol{\theta}^{(k)}) \cdot \mathbf{r}^*(\boldsymbol{\theta}^{(k)}) \; . \tag{31}$$

Marquardt [14] is developed most commonly used method of determining the coefficient μ, so the algorithm in the literature often called the Levenberg-Marquardt algorithm. Marquardt proposes the following coefficient values: μ_0 = 0,001, μ_d = 0,1 and μ_i = 10. Based on the presented mathematical model, a program for identification of unknown parameters was written in the MATLAB software package.

4. Numerical simulation and verification of the proposed model

Geometry of the spindle – holder – tool assembly used for numerical simulation is shown in Figure 4, while dimensions of the subsystems, bearings and interface dynamics properties are given in [15]. The material is steel with Young's modulus E = 2,1e^{11} N/m^2, mass density ρ = 7800 kg/m^3 and Poisson's ratio μ = 0,3. The assembly analysis was carried out using finite element software ANSYS. Beam element BEAM188, which is based on Timoshenko beam theory, was used for modeling of the assembly components. In addition to geometric characteristics, transverse oscillations in the *xy* plane are under consideration in this case. Additional restrictions are given through the stiffness of bearings. Finite elements with spring and damping (COMBIN14) are used to represent dynamics of bearings and the spindle – holder and holder – tool interfaces.

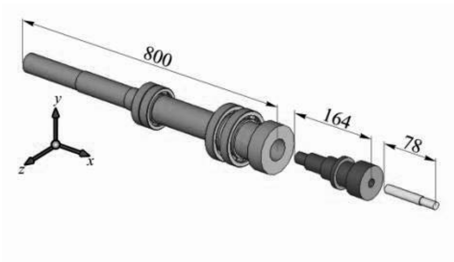

Figure 4. The spindle – holder – tool assembly for numerical simulation

Slika 4. Sustav glavno vreteno – držač alata – alat korišten u numeričkoj simulaciji

All responses ${}_A\mathbf{H}_{tt}$, ${}_A\mathbf{H}_{tr}$, ${}_A\mathbf{H}_{rt}$ and ${}_A\mathbf{H}_{rr}$of the subsystem A are obtained through FEM simulation, while the response of subsystem B ${}_B\mathbf{H}_{tt}^{cc}$ is„measured"(more accurately simulated). In a similar way, $\mathbf{G}_{tt}^{11}$ and $\mathbf{G}_{tt}^{12}$ are „measured" in the global system. When the responses above were collected, RDOF response of the spindle – holder system $\mathbf{VD}_{rt}^{ii}$ and $\mathbf{VD}_{rr}^{ii}$ can be calculated using equations (7) and (8). In order to verify accuracy of the proposed method for identifying RDOF, Figure 5 shows the calculated rotational responses of the spindle–holder system with simulated responses.

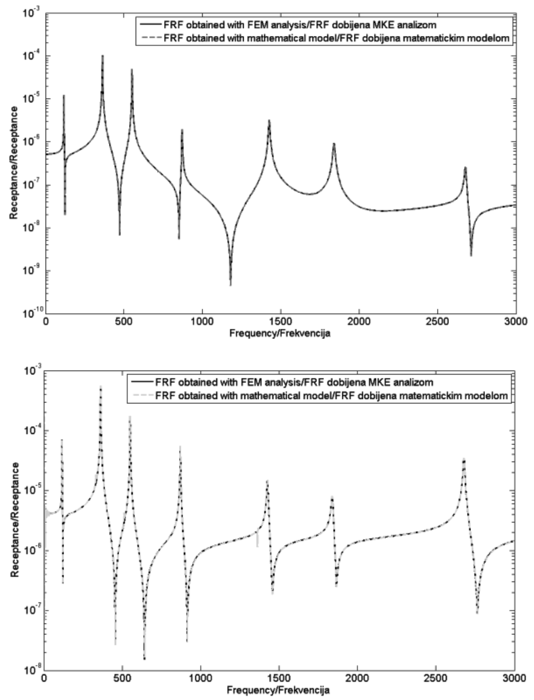

Figure 5. Comparsion between substructured and simulated responses $\mathbf{VD}_{rt}^{ii}$ (above) and $\mathbf{VD}_{rr}^{ii}$ (bellow)

Slika 5. Usporedba računskih i simuliranih odziva $\mathbf{VD}_{rt}^{ii}$ (gore) i $\mathbf{VD}_{rr}^{ii}$ (dolje)

As it can be seen in Figure 5, the simulated and sub structured FRF are identical. Error between simulated and sub structured values ranges up to a maximum of 10^{-5} (Figure 6). There are no significant differences between the predicted and obtained responses of FEM simulation, which lead us to conclude that the proposed method is accurate and can therefore be used to identify RDOF.

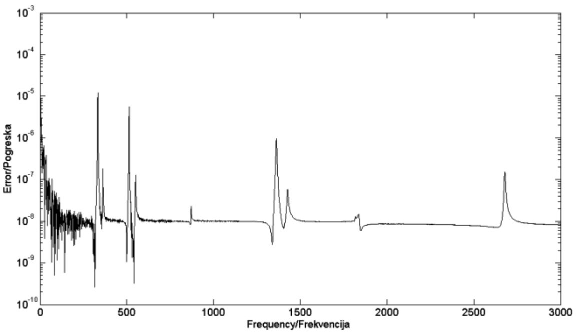

Figure 6. Error of calculated response $\mathbf{VD}_{rr}^{ii}$

Slika 6. Pogreška računskog odziva $\mathbf{VD}_{rr}^{ii}$

To create conditions that will lead to a successful experiment, it is desirable to analyze the possibility of identifying unknown parameters of the given system prior to the measuring of FRF of the spindle assembly. For this reason, described principles of identification of unknown parameters are tested on the numerical spindle – holder – tool system, as to identify unknown contact parameters within the specified system. First, the „unknown" contact parameters between the spindle and holder are identified (Table 1). In order to ensure convergence of the algorithm and reduce duration of the procedure to minimize the objective function, it is very important to determine the upper and lower bounds for the unknown variable during the process of identification. In this sense, the fact that damping does not affect the value of frequency of oscillation, but only the size of the amplitude are used. So, parameters which are first identified are translational and rotational stiffness, which are the order of 10^6 – 10^7. As initial values are set: ${}_{VD}k_t$ = 4,1·10^7 N/m, ${}_{VD}k_r$ = 2,1·10^6 Nm/rad, and bottom (d_d) and upper bound (d_g) have the following values: d_d = 5·10^5, d_g = 5·10^8. The following parameter values were the result of 36 iterations: ${}_{VD}k_t$ = 6,45644·10^7 N/m and ${}_{VD}k_r$= 3,73931·10^6 Nm/rad. After that, the identified values and the initial values of ${}_{VD}c_t$ = 100 Ns/m and ${}_{VD}c_r$ = 30 Nms/rad for the translational and rotational damping, were used for damping identification, with d_d = 1, d_g = 700. After 15 iterations, the following values were obtained: ${}_{VD}c_t$ = 43,4 Ns/m and ${}_{VD}c_r$ = 3,7 Nms/rad. Finally, identification of all parameters was performed in the end, with initial values for stiffness and damping identified in the previous steps. Values of identified parameters are shown in Table 1, together with errors of identification. Figure 7 shows the comparison of FRF at the tip of the tool holder with the identified and real values.

Table 1. Identified contact parameters of the spindle – holder system

Tablica 1. Identificirani parametri veze sustava glavno vreteno – držač alata

	Exact value / Točna vrijednost	Identified value / Identificirana vrijednost	Relative error / Relativna pogreška [%]
${}_{VD}k_t$, Nm	6,5·10^7	6,47984·10^7	0,310
${}_{VD}k_r$, Nm/rad	3,5·10^6	3,73930·10^6	6,837
${}_{VD}c_t$, Ns/m	50	44,88	10,24
${}_{VD}c_r$, Nms/rad	7	3,80	45,714

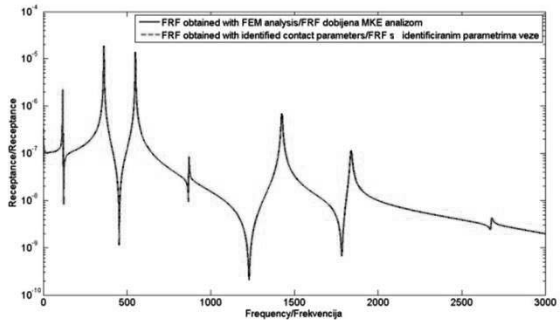

Figure 7. FRF of the spindle – holder system with identified contact parameters

Slika 7. FRF sustava glavno vreteno – držač alata s identificiranim parametrima veze

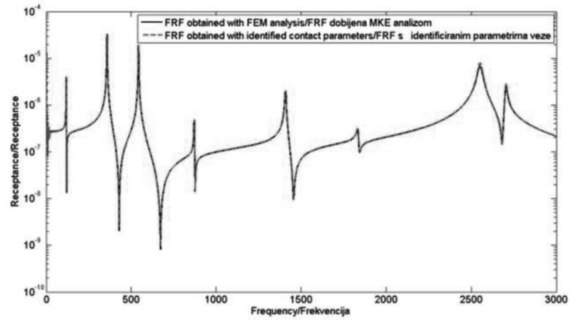

Figure 8. FRF of the spindle – holder – tool system with identified contact parameters

Slika 8. FRF sustava glavno vreteno – držač alata – alat s identificiranim parametrima veze

As shown in Figure 7, the accuracy of the identified parameters is more than satisfactory. Somewhat larger errors are encountered in the identification of the rotational stiffness, but this parameter has no significant impact in the synthesis of dynamic subsystems. The most dominant factor in the synthesis of dynamic subsystems $_{VD}k_t$ is translational stiffness, and this value is most accurately identified.
Similar procedures were performed for identification of„unknown" contact parameters between the tool and holder, i.e. spindle – holder – tool system, its respective values are shown in Table 2.

Table 2. Identified contact parameters of the spindle – holder –tool system

Tablica 2. Identificirani parametri veze sustava glavno vreteno – držač alata – alat

	Exact value / Točna vrijednost	Identified value / Identificirana vrijednost	Relative error / Relativna pogreška [%]
$_{VD}k_t$, Nm	$2{,}1\cdot10^7$	$2{,}10254\cdot10^7$	0,121
$_{VD}k_r$, Nm/rad	$1{,}4\cdot10^6$	$1{,}26983\cdot10^6$	9,298
$_{VD}c_t$, Ns/m	15	12,24	18,4
$_{VD}c_r$, Nms/rad	3	2,11	29,667

Figure 8.shows the FRF of tool tip with the identified and realvalues of the contact parameters of the spindle – holder – tool system.

5. Experimental tests

In this chapter, an evaluation of the method described above will be done, combining experimental and FEM data. The spindle – holder – tool assembly shown in Figure 9 is suspended to obtain free-free end conditions for performing an impact test. In order to verify the presented mathematical model of the spindle – holder – tool system, FRF is measured or obtained with FEM simulation for each of these subsystems. First, FRF of the spindle (with and without holder part in its cone) is measured, and then FRF of the spindle – holder system is measured. Finally, measurement of the spindle – holder – tool assembly was performed. Modeling dynamics of the tool subsystem was performed by using finite element software ANSYS.

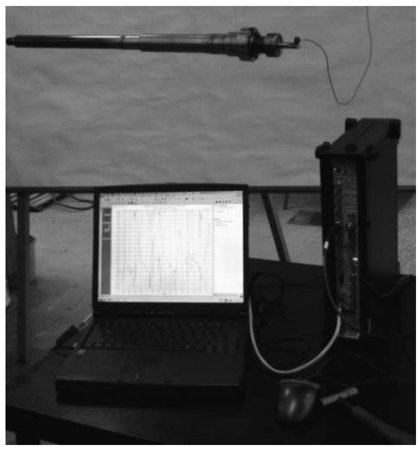

Figure 9. Measuring chain for the identification of dynamic behavior of the spindle - tool holder – tool system

Slika 9. Mjerni lanac za identifikaciju dinamičkog ponašanja sustava glavno vreteno – držač alata – alat

According to the presented mathematical model of the spindle –holder – tool assembly, accurate knowledge of complex stiffness of spindle – holder and holder – tool interface dynamics is necessary for accurate prediction of the dynamic response. First, using Levenberg-Marquardt method, for complex stiffness matrix of the spindle – holder dynamics the following parameters were identified: ${}_{VD}k_t$ = 2,971·10^8 N/m, ${}_{VD}k_r$ = 5,811·10^6 Nm/rad, ${}_{VD}c_t$ = 135 Ns/m, ${}_{VD}c_r$ = 35 Nms/rad. Figure 10 shows the result of receptance coupling of spindle and holder with identified spindle – holder interface dynamics. It can be concluded that the accuracy of identified parameters is satisfactory.

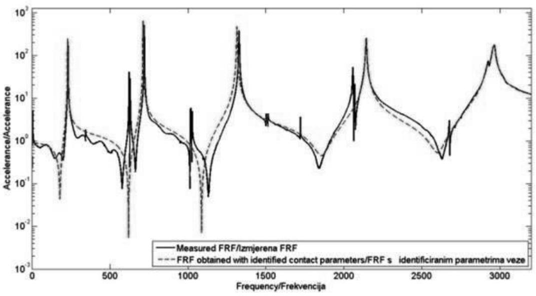

Figure 10. Comparison between measured FRF and FRF with identified spindle – holder interface dynamics

Slika 10. Usporedba izmjerene FRF i FRF s identificiranim parametrima veze između glavnog vretena i držača alata

Afterwards, the identification of holder – tool interface dynamics was carried out. For a system combination including the spindle – holder – tool with a diameter of tool D = 20 mm and tool overhang L = 40 mm, the following parameters were identified: ${}_{DA}k_t$ = 3,337·10^7 N/m, ${}_{DA}k_r$ = 1,571·10^6 Nm/rad, ${}_{DA}c_t$ = 63 Ns/m, ${}_{DA}c_r$ = 10 Nms/rad. Figure 11 shows the receptance coupling results of the spindle – holder system with tool. And in this case, it can be concluded that the accuracy of identified holder – tool interface dynamics is satisfactory.

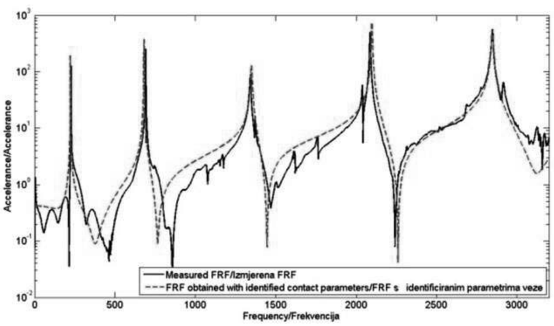

Figure 11. Comparison between measured FRF and the FRF with the identified holder – tool interface dynamics

Slika 11. Usporedba izmjerene FRF i FRF s identificiranim parametrima veze između držača alata i alata

6. CONCLUSION

The research of static and dynamic behavior of the spindle assembly poses a constant challenge for many researchers and designers of modern machine tools. One of the most important requirements in exploitation of the spindle assembly is its dynamic behavior, so the main aim of this study was to develop a mathematical model for modeling dynamic behavior of the spindle – holder – tool assembly that would take into consideration the RDOF. Based on the presented mathematical model of the spindle – holder – tool system for accurate prediction of the response system, it is necessary to know the exact stiffness matrix between the spindle and holder, and between the holder and tool. The matrix elements are stiffness and damping between these subsystems, and as the specified values cannot be experimentally measured, they need to be defined in other way. For this reason, special attention was paid to identification of the contact parameters between subsystems of the spindle assembly.

In order to verify the proposed mathematical model, numerical simulations and experimental tests of the system spindle –holder – tool were carried out. Numerical simulation confirmed that the proposed method for determining the rotational response is correct, since the difference between the results obtained by the proposed model and the results obtained by ANSYS is of the order maximum 10^{-5}. Furthermore, analysis of the identification results on unknown parameters showed that the dominant factor in the subsystems synthesis of the spindle assembly is translational stiffness, and this value was most accurately identified. It was observed that slightly larger deviations occur during identification of rotational parameters, but they do not have a significant impact on the synthesis of dynamic subsystems. The proposed mathematical model was experimentally verified on a free-free spindle – holder – tool system. Based on the results with identified rotational responses and contact parameters of the spindle – holder – tool system, satisfactory accuracy of the identified parameters was concluded.

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