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ANALYSIS OF LIBRARY OPERATION USING THE QUEUING THEORY

ANALIZA KNJIŽNIČNOGA POSLOVANJA PRIMJENOM TEORIJE REDOVA ČEKANJA

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Abstract

The implementation of IT technology in operation has modernized the library methods of works and rendering services thus providing users with better and faster access to the sources of information and knowledge. However, one of the ever present problems has been the queuing. The queuing theory has been playing an important role in solving this problem over past years. In the library operation, defined as a queuing system, we applied the queuing theory in order to determine service utilization in the library operation process and waiting time to obtain services. We also derived general relations for determining the optimal number of available library materials and the prediction of its circulation.

Sažetak

Uvođenjem informacijske tehnologije u poslovanje, knjižnice su osuvremenile način rada i pružanje svojih usluga, omogućile su korisnicima kvalitetniji i brži pristup izvorima informacija i znanja. Međutim, na stalno prisutan problem, problem čekanja, proteklih je godina, imajući važnu ulogu u njegovu rješavanju, ukazala teorija redova čekanja. Budući da se knjižnično poslovanje definira kao sustav masovnog opsluživanja primijenili smo teoriju redova čekanja, kako bismo utvrdili, iskorištenost uslužnoga mjesta u procesu opsluživanja knjižnice i vrijeme čekanja za dobivanje usluge. Izveli smo i opće relacije za određivanje optimalnog broja potrebne knjižnične građe i predviđanja njegove cirkulacije.

1. INTRODUCTION

The paper addresses user serving process in a library within University of Rijeka library system, which is a university subsystem integrating library systems and, through relevant bodies, provides an integral and coordinated operation for realization of University strategic goals in librarianship. The system includes libraries operating within University of Rijeka organized to meet the needs of all university users in comparable and effective ways. Presently, the system includes:

- Nine faculty libraries meeting needs of the constituent members
- University Library Rijeka, intended for University needs regardless of the membership
- High School Library (associated member)

Operation of the Rijeka University Library system should provide quick access to information, use of library resources and modern library services suitable for users' needs and under equal condition for all users. In order to make library operation effective in any library, on the basis of the unified standards, it is necessary to provide library space, equipment suitable for users' needs in terms of size and contents offered and working hours as well space and equipment for library staff /**1**/. The objective of this paper is to determine the efficiency of library operation by applying the queuing theory. The most important indicators of effective library operation are waiting time and customer service in the system. One of the important indicators in determining the quality of library operation is the availability of library materials. In the paper we present our derivation of relations for determining the optimal number of available library materials (inventory) and prediction of its circulation.

In order to achieve the objectives pursued, our research was based on the following two hypotheses (*i*) the library has (or has not) a sufficient number of service places and (*ii*) the library has (or has not) a sufficient number of copies of the book. To test the hypotheses we carried out the research in the library at University of Rijeka library system. Indicators, such as average arrival rate and average service rate, as well as waiting time in a queuing system, the number of customers who obtained the required item and the number of customers who did not obtain the required item because it was already lent, were identified.

2. THE MODEL AND OPERATION INDICATORS FOR SERVICE IN LIBRARY

2.1 *Queuing theory*

The problem of queue occurs in practice when a certain number of units (people or objects) seeking of appropriate services must wait before being served The process consists of waiting for the arrival of units in a system that performs a particular service, waiting in line when the service center is busy, serving and leaving the system. Solving the problem of queue means to determine the optimal number of servers that will be waiting in line to be minimal, or the losses caused by waiting to be minimal. One of the first tasks is to determine the indicators of the observed serving process, and for the required level of service quality to find the minimum number of servers, in order to achieve the required quality of servicing.

Given queue has S servers ($S \geq 1$) and n users in the system, unlimited waiting time and unlimited number of customers in the queue /**2**/. Arrival and service rate correspond to Poisson distribution where interval between consecutive arrivals and service time feature the exponential distribution: $\lambda \cdot e^{-\lambda \cdot t}$ for arrivals and, $\mu \cdot e^{-\mu \cdot t}$ for services, where:

- λ is a constant value equal to the **average arrival rate in the unit of time** (also called intensity; arrival rate)
- μ is a constant value equal to the **average number of services in the unit of time**; (service rate)
- ρ is the traffic intensity (service factor) defined as $\rho = \frac{\lambda}{\mu}$.

The system in which there is a single server (S=1) is called a single server queuing system with unlimited number of places in the queue ($M/M/1/\infty$). An indicator of waiting time of the $M/M/1/\infty$ is calculated using the formulas shown in Table 1. Although the library system can be described using a multi-server system ($M/M/S/\infty$) with unlimited number of places in the queue ($S>1$, $n=\infty$), in this paper we will focus only on single-system $M/M/1/\infty$. /**3**/

Table 1: Operation indicators for single server queuing system and unlimited queue

Description	Formula
Average number of customers in the system	$L = \frac{\rho}{1-\rho}$
Average number of customers in the queue	$L_Q = \frac{\rho^2}{1-\rho} = \rho \cdot L$
Average number of customers being served	$L_S = \rho = \frac{\lambda}{\mu}$
Probability that all service places are idle	$P_0 = 1 - \rho$
Average time that a customer spends in the system	$W = \frac{1}{\lambda}\frac{\rho}{1-\rho} = \frac{L}{\lambda}$
Average time that a customer spends in the queue	$W_Q = \frac{1}{\mu}\frac{\rho}{1-\rho} = \frac{L_Q}{\lambda} = \frac{\rho}{\lambda}\frac{\rho}{1-\rho} = \frac{L}{\mu}$
Average service time per customer	$W_S = \frac{L}{\lambda} - \frac{L}{\mu} = L \cdot \left(\frac{1}{\lambda} - \frac{1}{\mu}\right) = \frac{1}{\mu}$
Relative capacity for service	r_p=1
Absolute capacity of the system for service	$A_p=\lambda$

If the queue load is high, the queue is long as well as the waiting time. This results from approximately equal average arrival rate and average service rate. The closer the ρ value to one, the longer is the queue.

Queuing theory, along with its implementation in solving the problem of users visiting library and seeking certain materials, is also used to forecast the circulation of inventory requested by users.

2.2 *Morse's circulation model*

For more than 40 years, many authors have applied the queuing theory in library operation. Phillip Morse (1969) was the first who described the implementation of queuing theory to the modeling of book circulation in academic libraries /**4**/.

On the basis of queuing theory rules, Morse derived the basic relation of the circulation model as:

$$R = \lambda - \lambda \cdot \frac{R}{\mu} \Rightarrow R(\lambda) = \frac{\lambda\mu}{\lambda + \mu} \tag{1}$$

where:

- $R(\lambda)$ is expected yearly circulation rate of the book;
- λ is the arrival rate, the number of persons per year who would like to borrow the book;
- μ is expected return rate for the book (number of times the book is returned per year);
- $\frac{1}{\mu}$ is the mean circulation time, fractions of a year per loan;
- $\lambda \cdot \frac{R(\lambda)}{\mu}$ is the number of persons who search for the book but do not find it because it is on loan. /**5**/

Figure 1 shows that the value of the function $R(\lambda)$, called Morse function, never exceeds λ, regardless the value of μ. Such a conclusion results from the very definition of the function $R(\lambda)$.

If $\lambda, \mu \in N_0$ then:

$$\frac{\mu}{\lambda+\mu} \le 1 \;\Rightarrow\; \lambda\frac{\mu}{\lambda+\mu} \le \lambda \;\Rightarrow\; R(\lambda) \le \lambda .$$

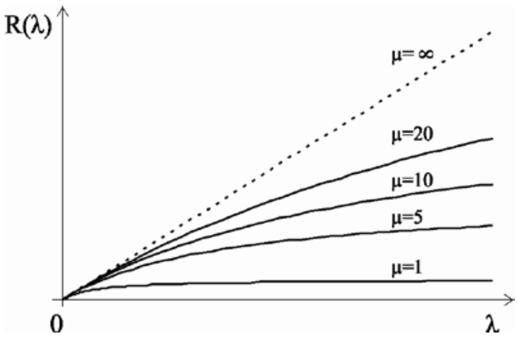

Figure 1. Value of the function $R(\lambda)$ for different values of μ

For an effective library operation, it is important that the number of satisfied customers is higher than the unsatisfied ones. The function $R_n(\lambda)$ defines the number of potential customers who will find a book in the library (number of satisfied customers), and the function $U_n(\lambda)$ gives the number of potential customers who would not find a book (number of unsatisfied customers), where n is the number of copies of the required book. Expressions for $R_n(\lambda)$ and $U_n(\lambda)$, where n = 1,2,3 were derived by Morse /**6**/, as follows:

If n=1

$$U_1(\lambda) = \lambda P_0 = \frac{\lambda^2}{s_1},\; R_1(\lambda) = \lambda - U_1(\lambda) = \frac{\lambda\mu}{s_1}$$

$$P_0 = \frac{\lambda}{s_1}, P_1 = \frac{\mu}{s_1},\; s_1 = \mu + \lambda \tag{2a}$$

If n=2

$$U_2(\lambda) = \lambda P_0 = \frac{\lambda^3}{2s_2},\ R_2(\lambda) = \lambda - U_2(\lambda) = \frac{\mu\lambda(\mu+\lambda)}{s_2}$$

$$P_0 = \frac{\lambda^2}{2s_2},\ P_1 = \frac{\lambda\mu}{s_2}\ ,\ P_2 = \frac{\mu^2}{s_2},\ s_2 = \mu^2 + \lambda\eta + \frac{1}{2}\lambda^2 \tag{2b}$$

If n=3

$$U_3(\lambda) = \lambda P_0 = \frac{\lambda^4}{6s_3},\ R_3(\lambda) = \lambda - U_3(\lambda) = \frac{\lambda\mu}{2s_3}(2\eta^2 + 2\lambda\eta + \lambda^2)$$

$$P_0 = \frac{\lambda^3}{6s_3},\ P_1 = \frac{\lambda^2\mu}{2s_3},\ P_2 = \frac{\lambda\mu^2}{s_3},\ P_3 = \frac{\mu^3}{s_3},\ s_3 = \mu^3 + \mu^2\lambda + \frac{1}{2}\lambda^2\eta + \frac{1}{6}\lambda^3 \tag{2c}$$

2.3 *The indicators of the availability of library materials*

Since quite often there are more than three copies of a book in the library, based on the Morse functions (2a, 2b, 2c), we derived a general expression for the indicators of the availability of library materials, $R_n(\lambda)$ and $U_n(\lambda)$.

For each $n \in N$ the probability that n copies are present on the shelf equals to:

$$P_n = \frac{\lambda^0\mu^n}{0!s_n}, P_{n-1} = \frac{\lambda^1\mu^{n-1}}{1!s_n}, \cdots, P_1 = \frac{\lambda^{n-1}\mu^1}{(n-1)!s_n}, P_0 = \frac{\lambda^n\mu^0}{n!s_n}, \tag{3}$$

where:

P_0 - the probability that all n copies are out;

P_1 - the probability that one copy is present on the shelf;

⋮

P_{n-1} - the probability that n-1 copies are present on the shelf;

P_n - the probability that n copies are present on the shelf;

s_n - factor in the denominator of Morse function.

Since $P_n + P_{n-1} + \ldots + P_1 + P_0 = 1$, it follows:

$$\frac{\mu^n}{s_n} + \frac{\lambda\mu^{n-1}}{s_n} + \ldots + \frac{\lambda^{n-1}\mu}{(n-1)!s_n} + \frac{\lambda^n}{n!s_n} = 1 \Rightarrow s_n = \sum_{i=0}^{n}\frac{\lambda^{n-i}\mu^i}{(n-i)!} \tag{4}$$

i.e.,

$$P_i = \frac{\lambda^{n-i}\mu^i}{(n-i)!s_n}\ ,\ i = 0,1,2,\ldots,n. \tag{5}$$

Therefore, for each $n \in N$:

- the number of potential customers who will find a book in the library (number of satisfied customers) is given as follows:

$$U_n(\lambda) = \lambda P_0 = \frac{\lambda^{n+1}}{n!s_n} \tag{6a}$$

- the number of potential customers who would not find a book in the library (number of unsatisfied customers) is given by expression:

$$R_n(\lambda) = \lambda - U_n(\lambda) = \left(1 - \frac{\lambda^n}{n!s_n}\right)\lambda \tag{6b}$$

Example 1.

Table 2 presents the functions $R_n(\lambda)$ and $U_n(\lambda)$ for given μ =20, arbitrary λ and n = 1,2,3,4,5 using equations (6a) and (6b).

Table 2: Values of the functions $R_n(\lambda)$ and $U_n(\lambda)$ for μ=20

λ	0	1	10	20	60	95	135	170
R_1	0	0.95	6.67	**10**				
U_1	0	0.05	3.33	**10**				
R_2	0	0.999	9.23	16.0	**28.24**			
U_2	0	0.001	0.77	4.0	**31.76**			
R_3	0	1	9.87	18.8	39.23	**46.37**		
U_3	0	0	0.13	1.25	20.77	**48.63**		
R_4	0	1	9.98	19.69	47.63	59.08	**65.61**	
U_4	0	0	0.016	0.31	12.37	35.92	**69.39**	
R_5	0	1	9.998	19.94	53.39	69.89	79.69	**84.51**
U_5	0	0	0.002	0.06	6.6	25.104	55.3	**85.49**

Following the data in Table 2 we can conclude: when there is only one copy of a book in the library it is necessary that λ is smaller than μ for the number of satisfied customers is greater than the sum of the unsatisfied ones. However, when there is more than one copy in the library, $R_n(\lambda)$ is greater than $U_n(\lambda)$ also when λ assumes higher values than μ. Therefore, the question is how λ and μ should relate for giving $R_n(\lambda)$ greater than $U_n(\lambda)$, (Fig. 2.).

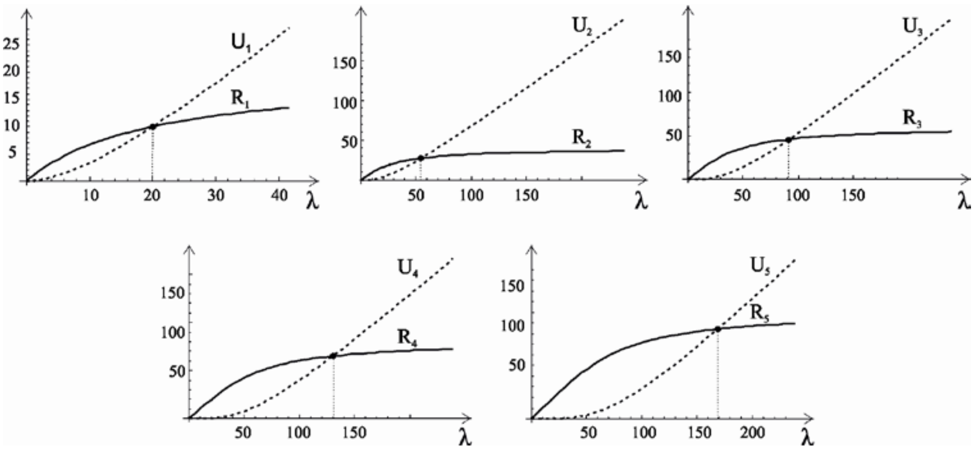

Figure 2. Value of the functions $R_n(\lambda)$ and $U_n(\lambda)$ for μ=20, and n = 1,2,3,4,5

Following Fig. 2, it is obvious that $R_n(\lambda)$ is greater than $U_n(\lambda)$ in closed interval [0, a], where a is a positive real number. The following inference proves the extent of these intervals and proves that a depends on μ.

Assuming that $R_n(\lambda) \geq U_n(\lambda)$, it follows:

$$\lambda - U_n(\lambda) \geq U_n(\lambda)$$

$$\lambda - \lambda P_0 \geq \lambda P_0 \quad \Rightarrow \quad P_0 \leq \frac{1}{2}$$

$$\frac{\lambda^n}{n!s_n} \leq \frac{1}{2} \quad \Rightarrow \quad \lambda^n \leq \frac{1}{2}n!s_n \quad \Rightarrow \quad \lambda \leq \left(\frac{1}{2}n!s_n\right)^{\frac{1}{n}} \qquad (7)$$

If relation (7) is plausible, i.e., if the probability that customer would not find a book on the shelf is smaller or equal than 0.5, then the number of satisfied customers is greater than the unsatisfied ones, respectively the library has enough copies of the book. Yet, if the number of unsatisfied customers is higher than the number of satisfied ones, i.e. $U_n(\lambda) \geq R_n(\lambda)$, it means that the library lacks sufficient number of copies of a book.

3. APPLICATION OF THE PRESENTED MODELS AND INDICATORS TO THE UNIVERSITY OF RIJEKA LIBRARY

This section explores serving process in the library in University of Rijeka Library system. Given library consists of one counter providing all types of services, such as borrowing and returning books, copying and similar. The space intended for customers has four tables, but there is no computer available to customers for searching catalog or browsing Internet. Data referring to 2010 (as obtained by library staff) were considered over 242 days, and it was established that 14523 customers required library service and arrived to library. Considering the library working hours of eight hours a day, the average number of users equaled to 60 customers per day, or 7.5 customers per hour. It was found out that the peak demand for certain literature was from Wednesday to Friday from 10.00 to 14.30. In the same period, we conducted the research with IT students that enrolled the Operations research 1 and 2, System theory and Mathematics for IT engineers 1 and 2. Obtained results provided information as to how many students succeeded to borrow mandatory literature for these subjects in 2010.

3.1 *Results*

The number of customers requiring library service is arbitrary continuous variable as well as the duration of service. Obtained data were checked with chi-square test which revealed that distribution of number of customers and duration of service behave in line with Poisson distribution, and the library is considered as a queuing system. Observed system is of M/M/1/∞ type. Since the library is defined as a queuing system, the problem was solved by means of queuing theory.

On the basis of the formulas given in Table 1, library operation indicators were calculated and presented in Table 3. Since the service rate is higher than the arrival rate, the system is stable as the traffic intensity is less than 1. However, it is large enough to conclude that the queue is large. This is supported by other data: time spent in a queue is 18 minutes, while service time is 6 minutes. Namely, customer will be served after 24 minutes. Service place idle rate is 25%. In order to reduce the time a customer spends in a queue and the service time, another counter would be necessary for providing services.

Table 3: Library operation indicators

No.	Indicator	Unit	Service place
1.	Arrival rate (λ)	customer/hour	7.5
2.	Service rate (μ)	customer /hour	10
3.	Traffic intensity (ρ)	-	0.75
4.	Average number of customers in the system (L)	customer	3
5.	Average number of customers in the queue (L_Q)	customer	2.25
6.	Average time customer spends in the queue (W_Q)	hour	0.3h=18'
7.	Average time customer spends in the system (W)	hour	0.4h=24'
8.	Probability that all service places are idle (P_0)	%	25%

Data on number of students that succeeded to borrow the literature is presented in Table 4. We calculated values of the functions R_n and U_n using the equations (6a) and (6b).

Table 4: Data on number of students that found literature in the library

Subject	Mandatory literature	Number of copies in the library (n)	Number of students attending the subject course (λ)	R	μ	R_n	U_n
Operations research 1	D. Barković, *Operacijska istraživanja*	2	40	15	24	26.3	13.7
	D. Kalpić, *Operacijska istraživanja*	3		17	29.6	39.5	4.5
Operations research 2	D. Barković, *Operacijska istraživanja*	2	30	7	12.9	16.5	13.5
	D. Kalpić, *Operacijska istraživanja*	3		10	15	23.7	6.3
System theory	D. Radošević, *Osnove teorije sistema*	2	39	20	41.1	31.7	7.3
	V. Čerić, *Simulacijsko modeliranje*	1		13	19.5	13	26
Mathematics for IT engineers 1 & 2	M. Sošić, M. Marinović, *Repetitorij s riješenim zadacima iz matematike*	15	70	45	126	70	0

The results show that the only book borrowed by all students is *Repetitorij s riješenim zadacima iz matematike*, that is, there are no unsatisfied customers regarding this book. As to the book *Simulacijsko modeliranje*, of which only one copy is available, the number of unsatisfied customers is as much as double the number of satisfied one, meaning that there are not sufficient copies in the library. In all other cases, the number of students that succeeded to borrow the book is higher than those who did not and consequently it is fair to conclude that the library disposes with sufficient number of copies of these books.

From the relation (7) for n = 1,2,3,4,5 we define intervals where $R_n(\lambda) \geq U_n(\lambda)$, and thus the relation between λ and μ.

- For $n = 1$

$$\lambda \leq \frac{1}{2}(\lambda + \mu) \Rightarrow \lambda \leq \mu \Rightarrow \lambda \in [0, \mu]$$

- For $n = 2$

$$\lambda \leq \left(\sum_{i=0}^{2} \frac{\lambda^{2-i} \mu^{i}}{(2-i)!} \right)^{\frac{1}{2}} \Rightarrow \lambda^2 - 2\lambda\mu - 2\mu^2 \leq 0 \Rightarrow \lambda \in [0, 0.732051\mu]$$

- For $n = 3$

$$\lambda \leq \left(3 \sum_{i=0}^{3} \frac{\lambda^{3-i} \mu^{i}}{(3-i)!} \right)^{\frac{1}{3}} \Rightarrow \lambda^3 - 3\lambda^2\mu - 6\lambda\mu^2 - 6\mu^3 \leq 0 \Rightarrow \lambda \in [0, 4.59141\mu]$$

- For $n = 4$

$$\lambda \le \left(12\sum_{i=0}^{4}\frac{\lambda^{4-i}\mu^{i}}{(4-i)!}\right)^{\frac{1}{4}} \Rightarrow \lambda^4 - 4\lambda^3\mu - 12\lambda^2\mu^2 - 24\lambda\mu^3 - 24\mu^4 \le 0$$

$$\Rightarrow \lambda \in [0, 6.50106\mu]$$

- For $n = 5$

$$\lambda \le \left(60\sum_{i=0}^{5}\frac{\lambda^{5-i}\mu^{i}}{(5-i)!}\right)^{\frac{1}{5}} \Rightarrow \lambda^5 - 5\lambda^4\mu - 20\lambda^3\mu^2 - 60\lambda^2\mu^3 - 120\lambda\mu^4 - 120\mu^5 \le 0$$

$$\Rightarrow \lambda \in [0, 8.43694\mu]$$

On basis of data from Table 4 intervals of λ where $R_n(\lambda) \ge U_n(\lambda)$ are presented (Table 5).

Table 5: Intervals of λ

No.	n	μ	λ	interval
1.	2	24	40	[0, 65.57]
2.	3	29.6		[0, 135.906]
3.	2	12.9	30	[0, 35.2435]
4.	3	15		[0, 68.8712]
5.	2	41.1	39	[0, 112.287]
6.	**1**	**19.5**		**[0, 19.5]**
7.	15	126	70	[0, 801.961]

These results confirm the previous conclusion: the library lacks sufficient copies of the book *Simulacijsko modeliranje*, while there are enough copies of other books. This is corroborated by data from Table 6 showing the probability that given *n* copies of books are available. Values Pi were calculated on the basis of relations (5).

From these results we also see that the number of copies of the book *Repetitorij s riješenim zadacima iz matematike* is sufficient (e.g. the probability that all the books are available in the library is 57 %).

Table 6: The probability that *n* copies of books are available

No.	n	μ	λ	P_i, (i=0,1,…,n)
1.	2	24	40	P_0=0.34, P_1=0.41, P_2=0.21
2.	3	29.6	40	P_0=0.11, P_1=0.25, P_2=0.37, P_3=0.27
3.	2	12.9	30	P_0=0.45, P_1=0.38, P_2=0.17
4.	3	15	30	P_0=0.21, P_1=0.32, P_2=0.32, P_3=0.15
5.	2	41.1	39	P_0=0.19, P_1=0.39, P_2=0.42
6.	1	19.5	39	P_0=0.67, P_1=0.33
7.	15	126	70	$P_0 = P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = P_7 = P_8 = P_9 = P_{10}$=0 P_{11}=0.00227732 P_{12}=0.0163967

		P_{13}=0.0885422 P_{14}=0.318752 P_{15}=0.573753

4. CONCLUSION

A queue in the library exists as at the moment customers enter the library seeking certain material until joining a queue to receive a service. The paper addresses the implementation of the queuing theory in the library operation in order to find out the waiting time for receiving service, exploitation rate of service place in serving process and circulation of library materials.

For predicting the circulation of library materials, we derived general relations for determining number of satisfied customers that found required item and unsatisfied customers who did not because the item was temporarily unavailable. The results are helpful in allocating book budgets, they are important in defining redundant books as well as those requiring additional copies.

Our future research goal is to include an additional library in the application of the models presented. This will be the new library located at the University of Rijeka Campus planned for completion by 2015. Further, we plan to extend implementation of queuing theory for determining the library operation indicators addressed in this paper, in new environment.

Notes

/1/ Tomljanović, S., Integrirani knjižnični sustav Sveučilišta u Rijeci (http://sp.uniri.hr/default.aspx)

/2/ Morse, M. P., Library Effectiveness: A Systems Approach, M.I.T. Press, Cambridge, 1969.

/3/ Zenzerović, Z., Teorija redova čekanja, Stohastički procesi (II. dio), Pomorski fakultet u Rijeci, Rijeka, 2003.

/4/ Morse, M. P., Library Effectiveness: A Systems Approach, M.I.T. Press, Cambridge, 1969.

/5/ Morse, M. P., A queuing theory, Bayesian Model for the Circulation of Books in a Library, Operations Research, Vol. 27, (1979), no. 4, pp. 693-716

/6/ Morse, M. P., Library Effectiveness: A Systems Approach, M.I.T. Press, Cambridge, 1969.