

SOME SIMPLE APPROACHES TO PLANNING THE INVENTORY OF SPARE COMPONENTS OF AN INDUSTRIAL SYSTEM

Alenka Brezavšček

University of Maribor, Faculty of Organizational Sciences
Kidričeva cesta 55a, 4000 Kranj, Slovenia
Phone: ++ 386 4 2374 255; E-mail: alenka.brezavscek@fov.uni-mb.si

Alenka Hudoklin

University of Maribor, Faculty of Organizational Sciences
Kidričeva cesta 55a, 4000 Kranj, Slovenia
Phone: ++ 386 4 2374 269; E-mail: alenka.hudoklin@fov.uni-mb.si

Abstract

Two variants of a simple stochastic model for planning the inventory of spare components supporting maintenance of an industrial system are developed. In both variants, the aim is to determine how many spare components are needed at the beginning of a planning interval to fulfil demand for corrective replacements during this interval. Under the first variant, the acceptable probability of spare shortage during the planning interval is chosen as a decision variable while in the second variant, the adequate spare inventory level is assessed taking into account the expected number of component failures within the planning interval. Calculation of the number of spare components needed depends on the form of the probability density function of component failure times. Different statistical density functions that are useful to describe this function are presented. Advantages and disadvantages of using a particular density function in our model are discussed. The applicability of the model is given through illustrative numerical examples.

Key words: *industrial system, maintenance, corrective replacement, spare components, inventory planning, stochastic modelling*

1. INTRODUCTION

During the operation, the components of an industrial system frequently fail causing the system downtime, and consequently, loss of income. In order to reduce the system downtime, various maintenance activities are performed. Maintenance costs represent a major item of the total system operating costs. Significant savings can be achieved by introducing an efficient maintenance policy. In defining the maintenance policy, a great attention should be paid to provide availability of spare components at the times of component replacements.

Many models for spare provisioning policy have been proposed in the literature. A comprehensive overview of published models is given by Kennedy et al. (2002). The optimal spare provisioning policy is obtained by minimizing the total system maintenance costs that comprise the component replacement costs (corrective

and preventive) and the inventory costs including ordering, holding and shortage costs. Such mathematical models are usually quite complex, containing a great number of parameters (see e.g. Brezavšček and Hudoklin, 2003; Diallo et al., 2008; Huang et al., 2008; de Smidt-Destombes et al., 2009; Wang et al., 2009). Therefore, such optimization models are often difficult to implement. From a practical point of view, simpler methods for defining an efficient spare provisioning policy would be desirable.

We will study a situation, existing in many industrial plants, when the cost of system downtime due to the shortage of spare components considerably exceeds all the other elements of the maintenance costs. In our opinion, in such a situation there is no need to optimize the total maintenance costs. It is enough to ensure sufficient quantity of spare components to prevent inventory shortage in a given time. Such an approach represents simplification of rather complicated optimization models, widely used in the literature.

In this paper, we propose a simple stochastic model for planning the inventory of spare components needed to support maintenance of an industrial system. The aim of the model is to determine how many spare components are needed at the beginning of a given planning interval to fulfil demand for component corrective replacements during this interval. Based on the renewal theory, two variants of the model are presented. Under the first variant, the adequate number of spare components is calculated considering the acceptable probability of spare shortage during the planning interval. In the second variant, the adequate spare inventory level is assessed taking into account the expected number of component failures during the planning interval.

Calculation of the adequate number of spare components in the inventory according to our model depends on the form of the probability density function of component failure times. Different statistical density functions that can be used to describe the probability density function of component failure times are presented. Advantages and disadvantages of using a particular density function in our model are discussed. The applicability of the model is shown through illustrative numerical examples.

2. PRELIMINARIES

The inventory of spare system components includes the spares needed for component preventive replacements, and the spares needed for corrective replacements. The number of spare components needed for preventive replacements in a given planning interval is known in advance. Therefore, in defining an efficient spare provisioning policy, the essential task is to ensure sufficient number of spare components needed for corrective replacements during a given time interval.

The process of successive corrective replacements of a particular component can be described by an ordinary renewal process. The renewal process is ordinary when all inter-renewal times are independent identically distributed random variables, all with the probability density function $f(t)$ (e.g. Cox, 1970, p. 25). In our

model two characteristics of an ordinary renewal process will be used: the number $N(t)$ of renewals in the interval $(0,t)$, and the renewal function $H(t)$ defined as the expected number of renewals in the interval $(0,t)$: $H(t) = E[N(t)]$.

The number $N(t)$ of component corrective renewals is a random variable with the probability distribution $p_r(t) = P[N(t) = r]$, $r = 0, 1, 2, \dots$ which can be calculated according to the equation

$$p_r(t) = F_r(t) - F_{r+1}(t) \quad r = 0, 1, 2, \dots \quad (1)$$

with $F_0(t) = 1$. The symbol $F_r(t)$ in the equation (1) denotes the r -fold convolution with itself of the cumulative distribution function $F(t) = \int_0^t f(x)dx$. When there are n independent identical components under observation, the process of their corrective replacements represents a superposition of n independent renewal processes. The probability distribution of the number of renewals of all n components in $(0,t)$ is given by the discrete convolution formula

$$p_r^{(n)}(t) = \sum_{i=0}^r p_{r-i}^{(n-1)}(t)p_i(t) \quad r = 0, 1, 2, \dots, n > 1 \quad (2)$$

with $p_r^{(1)}(t) = p_r(t)$. For an arbitrary n an analytical solution of $p_r^{(n)}(t)$ exists only when the analytical solution of (1) is available. Even then, the calculation of $p_r^{(n)}(t)$ is rather tedious. However, for large values of n the function $p_r^{(n)}(t)$ can be approximated by the normal density function with mean $nH(t)$ and variance $nV(t)$ ¹ (Haehling von Lanzenuer and Lundberg, 1974; Bergstrom, 2006).

The renewal function $H(t)$ can be calculated according to the equation

$$H(t) = \sum_{r=1}^{\infty} F_r(t) \quad (3)$$

For an arbitrary time t a simple solution of (3) is obtainable for some specific types of $f(t)$ only. For large values of time t a simple asymptotic formula for $H(t)$ can be used (Cox, 1970). When there are n independent identical components under observation, the expected number of renewals of all n components is equal to $nH(t)$.

¹The symbol $V(t)$ denotes the variance of the number of renewals in the interval $(0,t)$ defined by the expression

$$V(t) = E\left[(N(t) - H(t))^2\right].$$

It is evident that the calculation of $p_r(t)$, $p_r^{(n)}(t)$ and $H(t)$ depends on the probability density function $f(t)$ of inter-renewal times. In our case $f(t)$ is equal to the probability density function of component failure times. During the component lifetime, the function $f(t)$ exhibits different behaviour. It is known from the reliability theory that the lifetime of a component can be roughly divided into three regions: the region of early failures, the region of random failures (also called the region of the normal operation), and the region of wear-out failures (see Fig. 1). Early failures are usually eliminated by screening or burn-in tests before the components are put into the operation. The operating period of the component thus comprises only the region of random failures and the region of wear-out failures. Random failures are mainly due to the inherent slow-acting defects in the component, or to the sudden excessive loading. The main failure mechanism in the wear-out region is the deterioration of the component materials. The region of the random failures is characterized by a monotonically decreasing function $f(t)$, and by the constant failure rate. In the region of wear-out failures the function $f(t)$ follows a peak-shaped curve while the failure rate increases with time. During the region of random failures, only corrective replacements of components are reasonable, while in the wear-out region also preventive replacements make sense. In Fig. 1, general form of the function $f(t)$ during the component lifetime is presented.

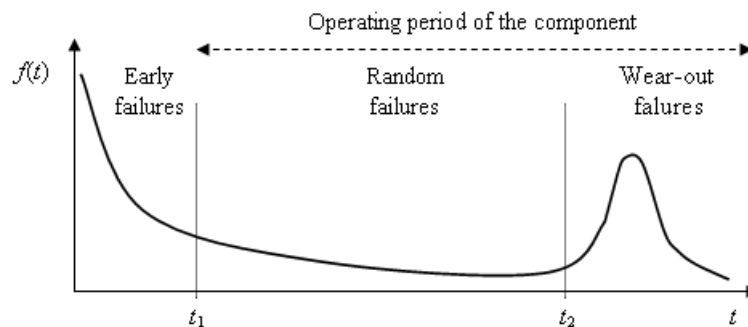


Figure 1: General form of the probability density function of failure times during the component lifetime

3. MODEL DEVELOPMENT

A simple stochastic model for planning the inventory of spare components needed for corrective replacements of system components is developed. The model addresses the situation when the cost of system downtime due to the shortage of spare components considerably exceeds all the other elements of the maintenance costs. The model is based on the renewal theory.

In developing the model the following assumptions are considered:

- The system includes n independent identical components operating in similar conditions.
- An inventory of spare components is replenished periodically every T units of time. At the beginning of each planning interval T , Q spare components should be available to fulfil demand for corrective replacements during T .

- A failed system component is replaced immediately by a new one if a spare component is available. The replacement time is negligible, there is no system downtime.
- If the replacement of the failed component cannot be performed (due to the shortage of spare components) the system downtime occurs.
- At the beginning of the interval T , all system components under consideration are new.

The aim of the model is to determine the minimal number Q of spare components in the inventory at the beginning of the planning interval T to fulfil demand for component corrective replacements during T . Two variants of the model are presented. Both variants are applicable to the components operating in the region of wear-out failures when the preventive replacements are performed according to the block replacement policy. Besides, the first variant is useful also when the components operate in the region of random failures. In a particular variant of the model, different decision variable for determining the number Q is chosen. In the first variant, the number Q is determined considering the acceptable probability of spare shortage during T , while in the second variant the number Q is assessed taking into account the expected number of component failures within T .

Variant 1:

The inventory of spare components needed for corrective replacements is planned for an arbitrary interval T . Let the acceptable probability of spare shortage during T be $P_s(T)$. The value $P_s(T)$ is predetermined considering the specific requirements of the system operation. The idea to determine the minimal number of spare components Q at the beginning of T which ensures that the probability of spare shortage during T does not exceed the value $P_s(T)$. The number Q is the minimal integer that satisfies the relation

$$\sum_{r=Q+1}^{\infty} p_r^{(n)}(T) \leq P_s(T) \quad (4)$$

where the symbol $p_r^{(n)}(T)$ denotes the probability distribution of the number of corrective replacements of n components, given by the equation (2).

Variant 2:

The components under consideration operate in the region of wear-out failures. Preventive replacements of the components are performed according to the block replacement policy every τ units of time. The inventory of spare components needed for corrective replacements is planned for the interval $T = k\tau$, where $k \gg 1$. The idea to determine the number of spare components Q at the beginning of T in such a way that Q will be at least equal to the expected number of component failures within T . Considering the maintenance policy proposed, the expected number of failures of n components in T is equal to $knH(\tau)$. The number Q is than the minimal integer satisfying the relation

$$Q \geq knH(\tau) \quad (5)$$

If the condition $k \gg 1$ is fulfilled, and there are Q spare components available for corrective replacements at the beginning of T , the expected system downtime due to spare shortage during T approaches to zero.

Calculation of the number of spare components Q needed depends on the form of the probability density function $f(t)$ of component failure times. There are many of well known probability density functions which have been found in practice to describe the function $f(t)$. Exponential, normal, Weibull, and Gamma density functions are frequently used (see e.g. Jardine and Tsang, 2006; Kececioglu, 1995).

Advantages and disadvantages of using particular density function in our model are discussed.

Exponential density function

The exponential density function is given by the formula

$$f(t) = \lambda e^{-\lambda t} \quad t \geq 0, \lambda > 0$$

where the parameter λ denotes the constant failure rate. The form of the function $f(t)$ for different values of λ is shown in Fig. 2.

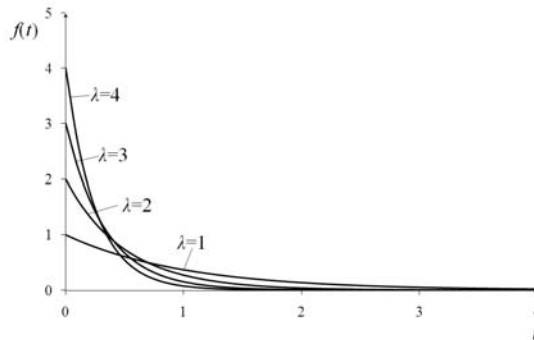


Figure 2: Form of the exponential density function for different values of λ

Advantages of using the exponential density function

- The renewal process characteristics, needed in the model, can be simply obtained in an analytical form. According to (1) we calculate the probability distribution $p_r(t)$ of the number of renewals in $(0, t)$:

$$p_r(t) = e^{-\lambda t} \frac{(\lambda t)^r}{r!} \quad r = 0, 1, 2, \dots \quad (6)$$

The function (6) is well known Poisson distribution with the parameter λt . The renewal function $H(t)$ can be simply calculated according to the equation (3) and is equal to λt . When there are n , $n > 1$, independent identical components under observation, the probability distribution of the number of renewals of all n components in $(0, t)$, $p_r^{(n)}(t)$, is also a Poisson distribution with the parameter $n\lambda t$, and the renewal function is equal to $n\lambda t$.

- The exponential probability plotting paper is available on the web site <http://www.weibull.com>. The assessment of the value of the parameter λ is therefore very easy.

Disadvantages of using the exponential density function

- The exponential density function is an appropriate mathematical model in the region of component random failures only (compare Fig. 1 and Fig. 2; see http://www.weibull.com/LifeDataWeb/the_exponential_distribution.htm).

Normal (Gaussian) density function

The normal density function is given by the expression

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \quad -\infty < t < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

with the parameters mean μ and standard deviation σ . The form of the normal density function for different values of the parameters μ and σ is shown in Fig. 3.

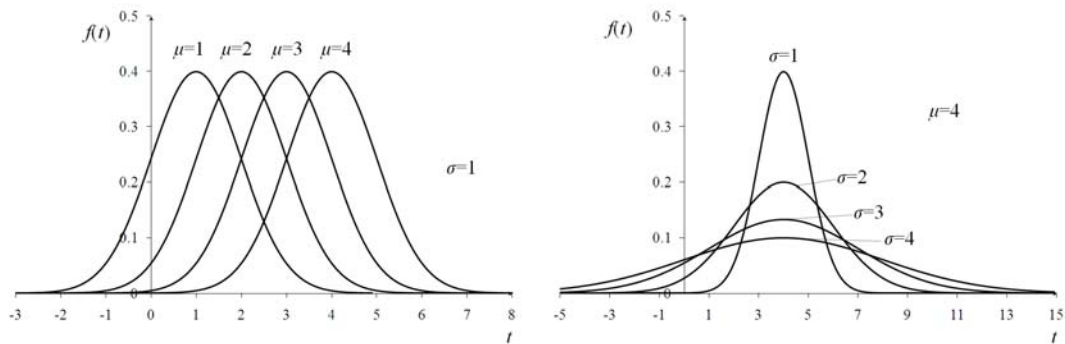


Figure 3: Form of the normal density function for different values of μ and σ

Advantages of using the normal density function

- Although the analytical solutions for $p_r(t)$, $p_r^{(n)}(t)$ and $H(t)$ do not exist, the numerical calculation of all three quantities is rather simple because the r -fold convolution of the normal distribution function $F(t)$ with the parameters μ and σ is also a normal distribution function with the parameters $r\mu$ and $\sigma\sqrt{r}$.
- The normal probability plotting paper is available on the web site <http://www.weibull.com>. The assessment of the values of the parameters μ and σ is easy.

Disadvantages of using the normal density function

- Since time to component failure is a positive random variable the area under the normal curve for negative values of time should be negligible. This is true when the ratio between μ and σ is significantly higher than 1. Otherwise a truncated normal distribution should be used.
- The normal density function is an appropriate mathematical model in the region of wear-out failures only (compare Fig. 1 and Fig. 3).

Weibull density function

Two-parameter² Weibull density function is given by the formula

$$f(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta} \right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^\beta} \quad t \geq 0, \beta > 0, \eta > 0$$

where β denotes the shape parameter, and η denotes the scale parameter. The form of the Weibull density function for different values of β and η is shown in Fig. 4.

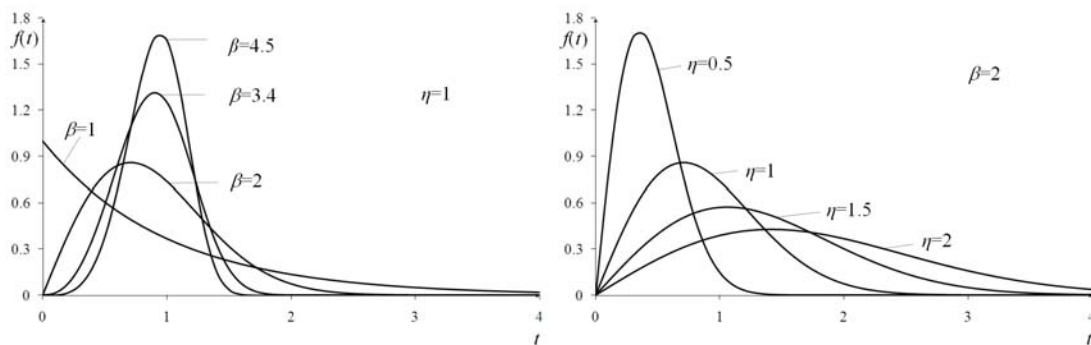


Figure 4: Form of the Weibull density function for different values of β and η

Advantages of using the Weibull density function

- The Weibull density function can describe the function $f(t)$ in both regions of the component operating time depending on the value of the shape parameter β . If β is equal to 1, the Weibull density function becomes exponential with the parameter $\lambda = 1/\eta$. If β is greater than 1, the Weibull density function exhibits a peak-shaped form. The Weibull density function with $\beta = 1$ can be therefore used for the modelling $f(t)$ in the region of random failures, while the Weibull density function with $\beta > 1$ is useful in the region of wear-out failures (compare Fig. 1 and Fig. 4).
- The Weibull probability plotting paper is available on the web site <http://www.weibull.com>. The assessment of the values of the parameters β and η is easy.

Disadvantages of using the Weibull density function

- In the case of Weibull density function with $\beta > 1$, the closed form of $F_r(t)$ is not available. Therefore, the analytical solutions for $p_r(t)$, $p_r^{(n)}(t)$ and $H(t)$ do not exist. Furthermore, the numerical calculations of these quantities are quite tedious. A comprehensive overview of different numerical calculations of Weibull renewal function is given in the book by Rinne (2009).

Gamma density function

Two-parameter³ Gamma density function is given by the formula

²In reliability theory, the three parameter Weibull density function is also used. The third parameter γ , $-\infty < \gamma < \infty$, is the location parameter. When $\gamma = 0$ the density function starts at time $t = 0$.

³In reliability theory, the three parameter Gamma density function is also used. The third parameter γ , $-\infty < \gamma < \infty$, is the location parameter. When $\gamma = 0$ the density function starts at time $t = 0$.

$$f(t) = \frac{1}{\eta\Gamma(\beta)} \left(\frac{t}{\eta}\right)^{\beta-1} e^{-\frac{t}{\eta}} \quad t \geq 0, \beta > 0, \eta > 0$$

where β denotes the shape parameter, η denotes the scale parameter, and $\Gamma(\cdot)$ denotes the Gamma function⁴. When β is integer the Gamma density function becomes Erlang density function given by the expression

$$f(t) = \frac{1}{\eta(\beta-1)!} \left(\frac{t}{\eta}\right)^{\beta-1} e^{-\frac{t}{\eta}} \quad t \geq 0, \beta > 0, \eta > 0$$

When the shape parameter is $\beta/2$ (β is any integer) and the scale parameter is equal to 2 the Gamma density function becomes Chi-square density function

$$f(t) = \frac{1}{2\Gamma(\beta/2)} \left(\frac{t}{2}\right)^{\frac{\beta}{2}-1} e^{-\frac{t}{2}} \quad t \geq 0, \beta > 0$$

The form of the Gamma density function for different values of the parameters β and η is shown in Fig. 5.

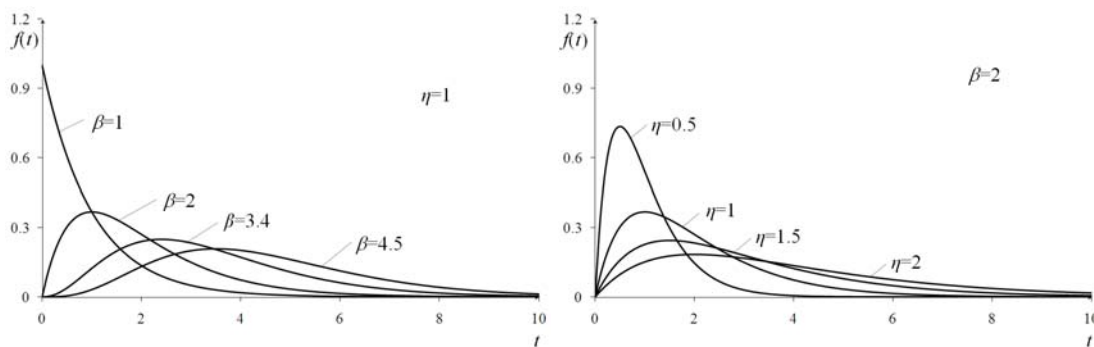


Figure 5: Form of the Gamma density function for different values of β and η

Advantages of using the Gamma density function

- Simple analytical solutions for $p_r(t)$ and $H(t)$ exist if $\beta = 2$:

$$p_r(t) = \frac{2e^{-\frac{t}{\eta}}(1+r)\left(\frac{t}{\eta}\right)^{2r} [t + \eta(1+2r)]}{\eta\Gamma(3+2r)} \quad \text{and} \quad H(t) = \frac{1}{4} \left(e^{-\frac{2t}{\eta}} + \frac{2t}{\eta} - 1 \right)$$

Using the above expression for $p_r(t)$, an analytical solution for $p_r^{(n)}(t)$ can be calculated according to (2).

- The r -fold convolution of the Gamma distribution function with the parameters β and η is also a Gamma distribution function with the parameters $r\beta$ and η . Therefore, a numerical calculation of $p_r(t)$, $p_r^{(n)}(t)$ and $H(t)$ according to (1), (2) and (3) is easy.

⁴ $\Gamma(x) = \int_0^{\infty} z^{x-1} e^{-z} dz$

- The Gamma density function can describe the probability density function of component failure times in both regions of the component operating time depending on the value of β . If β is equal to 1, the Gamma density function becomes exponential with the parameter $\lambda = 1/\eta$. If β is greater than 1, the Gamma density function exhibits a peak-shaped form. The Gamma density function with $\beta = 1$ can be therefore used for modelling $f(t)$ in the region of random failures, while the Gamma density function with $\beta > 1$ is useful in the region of wear-out failures (compare Fig. 1 and Fig. 5).

Disadvantages of using the Gamma density function

- The Gamma probability plotting paper is not commercially available. Therefore, the estimation of the parameters is not trivial. The general maximum likelihood method can be used to estimate the values of β and η (see Evans, Hastings and Peacock, 2000; Johnson, Kotz, and Balakrishnan, 1994). This is probably the reason why the Gamma density function is not widely used as a model for the probability density function of component failure times.

4. NUMERICAL EXAMPLES

We will illustrate the application of the model by some numerical examples. In all examples we assume that the probability density function of component failure times is described by the Gamma density function. In our opinion, among four statistical density functions studied, the Gamma density function is the most appropriate for calculating (although numerically) the values of $p_r(t)$, $p_r^{(n)}(t)$ and $H(t)$ in the region of random failures as well as in the region of wear-out failures.

4.1. Components operate in the region of random failures

There are $n = 40$ components of a given type under observation. The values of the shape and the scale parameter are estimated to be $\beta = 1$, and $\eta = 12500 h$. Since components operate in the region of random failures, only corrective replacements are performed.

Using variant 1 of the model

The planning interval is $T = 6000 h$ of the system operating time. We want to determine the minimal value of spare components Q at the beginning of T which will ensure that the probability of spare shortage during T will not exceed 3%.

Since $\beta = 1$ the probability distribution $p_r^{(40)}(T)$ is a Poisson distribution with the parameter $40T/\eta = 19.2$. Using the equations (4) and (6) we calculate the minimal integer Q which satisfies the relation

$$\sum_{r=Q+1}^{\infty} e^{-19.2} \frac{19.2^r}{r!} \leq 0.03$$

We obtain $Q = 28$. If there are at least 28 spare components in the inventory at the beginning of the interval $T = 6000 h$ the shortage probability during T is at most 2.2%, and the requirement above is fulfilled.

4.2. Components operate in the region of wear-out failures

There are $n = 50$ components under observation. The values of the shape and the scale parameter are estimated to be $\beta = 6.5$, and $\eta = 700 h$. Preventive replacements of components are performed according to the block replacement policy every $T = 3200 h$.

Using variant 1 of the model

We want to determine the minimal value Q which will ensure that the probability of spare shortage during $T = 3200 h$ will not exceed 2%.

The probability distributions $p_r(T)$ and $p_r^{(50)}(T)$ of the number of renewals are calculated numerically according to (1) and (2). Results are shown in Tab. 1.

Table 1: Probability distribution of the number of renewals in $T = 3200 h$

r	0	1	2	3	4	5	6	7	8	9	10
$p_r(T)$	0.7621	0.2370	9.26E-04	1.82E-07
$p_r^{(50)}(T)$	1.26E-06	1.96E-05	0.0001	0.0007	0.0027	0.0078	0.0182	0.0358	0.0601	0.0877	0.1124
r	11	12	13	14	15	16	17	18	19	20	21
$p_r^{(50)}(T)$	0.1280	0.1304	0.1196	0.0993	0.0750	0.0517	0.0326	0.0189	0.0101	0.0049	0.0022
r	22	23	24	25	26	27	28	29	30	31	...
$p_r^{(50)}(T)$	0.0009	0.00037	0.0001	4.40E-05	1.36E-05	3.91E-06	1.04E-06	2.58E-07	5.93E-08	1.26E-08	...

Using the calculated values of $p_r^{(50)}(T)$, and the equation (4) we determine the minimal integer Q which satisfies the relation

$$\sum_{r=Q+1}^{\infty} p_r^{(50)}(T) \leq 0.02$$

We obtain $Q = 18$. At the beginning of each interval $T = 3200 h$ between two successive preventive replacements, 18 spare components for corrective replacements during T are required. Besides, 50 spare components are needed to perform preventive replacement of all components. The shortage probability during T is equal to 1,87%, and the requirement above is fulfilled.

Using variant 2 of the model

The inventory of spare components needed for corrective replacements is planned for the period of $T = 8 \cdot 3200 h = 25600 h$. This is approximately three years of system operation. We want to determine the minimal number of spare components Q at the beginning of T which is at least equal to the expected number of component failures within T .

The value of the renewal function $H(3200 h)$ is calculated according to (3). We obtain $H(3200 h) = 0.2389$. The number of spare components needed for corrective replacements during $T = 25600 h$ is determined according to (5) as the minimal integer satisfying the relation

$$Q \geq 8 \cdot 50H(3200 h) = 95.56$$

We obtain. $Q = 96$. If at there are 96 spares available for corrective replacements the beginning of the planning interval $T = 25600 h$, the expected system downtime due to spare shortage during T approaches to zero. Beside 96 spares for corrective replacements, 400 spare components are needed to perform eight block preventive replacement of all 50 operating components.

4. CONCLUSION

In the paper, a simple stochastic model for planning the inventory of spare components needed to support maintenance of an industrial system is proposed. The aim of the model is to determine the minimal number of spare components in the inventory at the beginning of a given planning interval to fulfil demand for component corrective replacements during this interval. Two variants of the model are presented. Both variants are applicable to the components operating in the region of wear-out failures if the preventive replacements are performed according to the block replacement policy. The first variant of the model is useful also when the components under observation operate in the region of random failures where no preventive replacements are needed. In the first variant of the model, the adequate number of spare components for corrective replacements is calculated considering the acceptable probability of spare shortage during the planning interval. In the second variant, the required number of spare components for corrective replacements is assessed taking into account the expected number of component failures in within the planning interval.

In both variants of the model, the process of successive corrective replacements of a particular component is described by the renewal process. Determination of the characteristics of the renewal process, needed in the model, depends on the form of the probability density function of component failure times. This function can be described by an appropriate statistical density function. Advantages and disadvantages of using exponential, normal, Weibull and Gamma density function in our model are discussed.

The applicability of the model is given through illustrative numerical examples considering the Gamma density function. In our opinion, among four statistical density functions studied, the Gamma density function is the most appropriate for calculating the probability distribution of the number of corrective renewals as well as the expected number of corrective renewals in the planning interval.

The model proposed represents simplification of rather complicated optimization models widely published in the literature. It is suitable for implementation in a variety of industrial systems where the cost of the system

downtime due to the shortage of spare components considerably exceed all the other parameters of the system maintenance costs.

REFERENCES

- Bergstrom, C. (2006). Lecture 5: Continuous distributions, http://protist.biology.washington.edu/biol429/Lecture%20notes/Lecture5_continuous_distributions.pdf [Accessed 03/02/11].
- Brezavšček, A. and Hudoklin, A. (2003). "Joint optimization of block-replacement and periodic-review spare-provisioning policy", *IEEE Transactions on Reliability*, Vol. 52, pp. 112-117.
- Cox, D.R. (1970), *Renewal Theory*, Methuen.
- de Smidt-Destombes, K.S., van der Heijden, M.C. and van Harten, A.(2009). "Joint optimisation of spare part inventory, maintenance frequency and repair capacity for k-out-of-N systems", *International Journal of Production Economics*, Vol. 118, pp. 260-268.
- Diallo, C., Ait-Kadi, D. and Chelbi, A. (2008). "(s,Q) Spare Parts Provisioning Strategy for Periodically Replaced Systems", *IEEE Transactions on Reliability*, Vol. 57, No. 1, pp. 134 – 139.
- Evans, M., Hastings, N. and Peacock, B. (2000), *Statistical Distributions*, 3rd. Ed., John Wiley and Sons.
- Haehling von Lanzenuer, C.. and Lundberg, W.N. (1974). "The n-Fold Convolution of a Mixed Density and Mass Function", *ASTIN Bulletin-The Journal of the International Actuarial Association*, Vol. 8, No. 1, <http://www.actuaries.org/LIBRARY/ASTIN/vol8no1/91.pdf> [Accessed 03/02/11].
- Huang, R. et al. (2008). "Modeling and Analyzing a Joint Optimization Policy of Block-Replacement and Spare Inventory With Random-Leadtime", *IEEE Transactions on Reliability*, Vol. 57, No. 1, pp. 113 – 124.
- Jardine, A.W.K. and Tsang, A.H.C. (2006), *Maintenance, Replacement, and Reliability: Theory and Applications*, Boca Raton: CRC Press, Taylor & Francis Group.
- Johnson, N.L, Kotz, S. and Balakrishnan, N. (1994), *Continuous Univariate Distributions*, Volumes I and II, 2nd. Ed., John Wiley and Sons.
- Kececioglu, D. (1995). *Maintainability, Availability and Operational Readiness Engineering Handbook*, Upper Saddle River: Prentice Hall.
- Kennedy, W. J., Pattersonb, J. W. and Fredendallb, L. D. (2002). "An overview of recent literature on spare parts inventories", *International Journal of Production Economics*, Vol. 76, No. 2, pp. 201-215.
- Rinne, H. (2009), *The Weibull Distribution: A Handbook*, CRC Press, Taylor & Francis Group.
- Wang, L., Chu, J. and Mao, W. (2009). "A condition-based replacement and spare provisioning policy for deteriorating systems with uncertain deterioration to failure", *European Journal of Operational Research*, Vol. 194, No. 1, pp. 184-205.