A simple method to estimate the maximum liquid production rate using plunger lift system in wells

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REVIEW

Plunger Lift is a cyclic method of artificial lift that uses a plunger to establish an interface between the liquid accumulated in the production tubing and the reservoir or annulus gas pressure that will be used to lift the fluid in wells. In this work, the maximum possible liquid production rate that plunger lift will tolerate for a given depth and tubing size has been placed on a quantitative basis by means of simple equations obtained from correlations of field data. The predictive tool developed in this study can be of immense practical value for petroleum engineers to have a quick check on the maximum possible liquid production rate that plunger lift will tolerate for a given well depth and tubing size at various wells without opting for any expensive field trials. In particular, petroleum and production engineers would find the proposed method to be user-friendly with transparent calculations involving no complex expressions.

Key words: plunger lift, artificial lift, tubing size, oil production, Vandermonde matrix, predictive tool

1. Introduction

Plunger lift is an intermittent artificial lift method that usually uses only the energy of the reservoir to produce the liquids.[12] A plunger is a free-travelling piston that fits within the production tubing and depends on well pressure to rise and solely on gravity to return to the bottom of the well. Plunger lift operates in a cyclic process with the well alternately flowing and shut-in.[6] Many low-volume gas wells produce at suboptimum rates because of liquid loading caused by an accumulation of liquids in the wellbore that creates additional backpressure on the reservoir and reduces production, therefore plunger lift can use reservoir energy to remove these accumulated liquids from the wellbore and improve production. Lacking a thorough understanding of plunger lift systems leads to disappointing results in many applications.[9,8]

One type of a typical installation of plunger lift is shown in figure 1. Plunger-lift operations are difficult to optimize owing to a lack of knowledge concerning tubing, casing, and bottom hole pressures; liquid accumulation in the tubing; and the location of the plunger.[14,15]

Because expense is involved in trying out some method of lift in a well, it is desirable to be able to predict in advance if plunger lift will work or not in a well. Even though plunger lift is not too expensive, additional equipment options can increase the initial costs.[8,9] Also, downtime for installation, adjustments to see if the plunger installation will perform, and adjustments to optimize production well all add to the costs[5], therefore it worthwhile to be able to predict in advance the maximum possible production rate using a plunger lift system.

In view of the above mentioned issues and the importance of plunger lift in petroleum engineering, it is necessary to develop an accurate and simple correlation to permit mathematical solution of the problem of plunger lift performance for a given well. In this work, the maximum possible liquid production rate using plunger lift has been developed as a function of well depth and tubing size.

This paper discusses the formulation of such a predictive tool in a systematic manner along with an example to show the simplicity of the model and usefulness of such a tool. The proposed method is exponential function which leads to well-behaved (i.e. smooth and non-oscillatory) equations enabling more accurate and non-oscillatory predictions and this is the distinct advantage of the proposed method.

2. Methodology for the development of novel correlation

The primary purpose of the present study is to accurately correlate the maximum oil production rate using plunger lift, as a function of tubing size and the well depth.

Vandermonde matrix is a matrix with the terms of a geometric progression in each row, i.e., an $m \times n$ matrix.[4]

$$V = \begin{bmatrix} 1 & \alpha_1 & \alpha_1^2 & \dots & \alpha_1^{n-1} \\ 1 & \alpha_2 & \alpha_2^2 & \dots & \alpha_2^{n-1} \\ 1 & \alpha_3 & \alpha_3^2 & \dots & \alpha_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_m & \alpha_m^2 & \dots & \alpha_m^{n-1} \end{bmatrix} \quad (1)$$

or

$$V_{i,j} = \alpha_i^{j-1} \quad (2)$$

For all indices i and j the determinant of a square Vandermonde matrix (where $m=n$) can be expressed as[4]:

$$\det(V) = \prod_{1 \le i < j \le n} \left(\alpha_j - \alpha_i\right) \quad (3)$$

The Vandermonde matrix evaluates a polynomial $a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1}$ at a set of points; formally it transforms coefficients of a polynomial to the values the polynomial takes at the point's α_i. The non-vanishing of the Vandermonde determinant for distinct points α_i shows that for distinct points the map from coefficients to values at those points is a one-to-one correspondence and thus that the polynomial interpolation problem is solvable with unique solution; this result is called the unisolvence theorem.[7] They are thus useful in polynomial interpolation since solving the system of linear equations $Vu = y$ for u with V and $m \times n$ Vandermonde matrix is equivalent to finding the coefficients u_j of the polynomial (s).[4]

$$P(x) = \sum_{j=0}^{n-1} u_j x^j \qquad (4)$$

For degree $\leq n$-1 which has (have) the property:

$$P(\alpha_i) = y_i \text{ for } i=1\ldots m \qquad (5)$$

the Vandermonde matrix can easily be inverted in terms of Lagrange basis polynomials: each column is the coefficients of the Lagrange basis polynomial, with terms in increasing order going down. The resulting solution to the interpolation problem is called the Lagrange polynomial suppose that the interpolation polynomial is in the form[7,4]:

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots a_2 x^2 + a_1 x + a_0 \qquad (6)$$

The statement that p interpolates the data points means that

$$P(x_i) = y_i \text{ for all } i \in \{0,1,\ldots n\}. \qquad (7)$$

If we substitute equation (6) in here, we get a system of linear equations in the coefficients a_k. The system in matrix-vector form reads (Bair et al, 2006)

$$\begin{bmatrix} x_0^n & x_0^{n-1} & x_0^{n-2} & \ldots & x_0 & 1 \\ x_1^n & x_1^{n-1} & x_1^{n-2} & \ldots & x_1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ x_n^n & x_n^{n-1} & x_n^{n-2} & \ldots & x_n & 1 \end{bmatrix} \begin{bmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix} \qquad (8)$$

We have to solve this system for ak to construct the interpolant $p(x)$. The matrix on the left is commonly referred to as a Vandermonde matrix.[1]

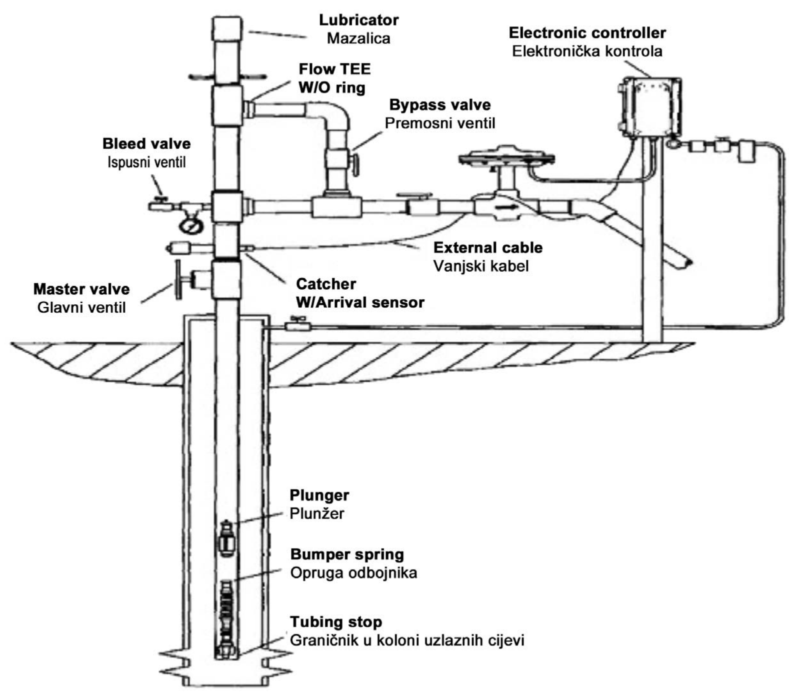

Fig. 1. Installation of Plunger lift in a well
Sl. 1. Instalacija plunžernog plinskog lifta u bušotini

2.1. Development of correlation

The required data to develop this correlation includes maximum oil production rate, as a function of tubing size and well depth. The following methodology has been applied to develop this correlation. Firstly the maximum oil production rates using plunger lift, are correlated as a function of well depth for several tubing size data, then, the calculated coefficients for these equations are correlated as a function of tubing size. The derived equations are applied to calculate new coefficients for equation (9) to predict maximum oil production rate using plunger lift. Table 1 shows the tuned coefficients for equations (10) to (13) for predicting the maximum oil production rate to support the applicability of plunger lift. In brief, the following steps are repeated to tune the correlation's coefficients using Matlab (2008).[13]

1. Correlate maximum oil production rate using plunger lift, as a function of well depth for a given tubing size.
2. Repeat step 1 for other tubing size data.
3. Correlate corresponding polynomial coefficients, which were obtained for different tubing size data versus tubing size. a= f (dt), b = f (dt), c = f (dt), d = f (dt) [see equations (10)-(13)].

Equation 9 represents the proposed governing equation in which four coefficients are used to correlate maximum oil production rates using plunger lift, as a function of well depth and tubing size where the relevant coefficients have been reported in Table 1.

$$\ln(q) = a + \frac{b}{L} + \frac{c}{L^2} + \frac{d}{L^3} \qquad (9)$$

Where:

$$a = A_1 + B_1 d_t + C_1 d_t^2 + D_1 dt^3 \qquad (10)$$

$$b = A_2 + B_2 d_t + C_2 d_t^2 + D_2 dt^3 \qquad (11)$$

$$c = A_3 + B_3 d_t + C_3 d_t^2 + D_3 dt^3 \qquad (12)$$

$$d = A_4 + B_4 d_t + C_4 d_t^2 + D_4 dt^3 \qquad (13)$$

Table 1. Tuned coefficients used in Equations (10-13)

Coefficient	Valued for 2" Plunger lift
A_1	$2.250\,690\,914\,8 \times 10^{-1}$
B_1	$8.509\,437\,393\,7 \times 10^{-1}$
C_1	$7.208\,176\,853\,9 \times 10^{-1}$
D_1	$-1.531\,736\,688\,3 \times 10^{-1}$
A_2	$-3.036\,046\,326\,9 \times 10^{4}$
B_2	$6.132\,250\,748\,3 \times 10^{4}$
C_2	$-2.7106\,472\,588 \times 10^{4}$
D_2	$3.502\,813\,224\,7 \times 10^{3}$
A_3	$2.653\,775\,026\,1 \times 10^{8}$
B_3	$-3.903\,936\,458\,1 \times 10^{8}$
C_3	$1.579\,887\,610\,4 \times 10^{8}$
D_3	$-1.958\,323\,377\,3 \times 10^{7}$
A_4	$-4.981\,559\,404\,2 \times 10^{11}$
B_4	$6.702\,434\,249\,7 \times 10^{11}$
C_4	$-2.629\,064\,897\,9 \times 10^{11}$
D_4	$3.209\,111\,827\,9 \times 10^{10}$

These optimum tuned coefficients help to cover well depth up to 20 000 ft (6 096 m) and the tubing size up to 4 in. (10.16 cm). The optimum tuned coefficients given in Table 1 can be further retuned quickly according to proposed approach if more data become available in the future.

In this work, our efforts directed at formulating a correlation which can be expected to assist engineers for rapid calculation of maximum oil production rates using plunger lift, as a function of well depth and tubing sizes. The proposed novel tool developed in the present work is simple and unique expression which is non-existent in the literature. Furthermore, the selected exponential function to develop the tool leads to well-behaved (i.e. smooth and non-oscillatory) equations enabling reliable and more accurate predictions.

3. Results

Figure 2 illustrates the Matlab-based computer program (2008) to calculate maximum oil production rates using plunger lift, as a function of well depth and tubing sizes. Figure 3 show the proposed method results for expanding cycle-controlled plungers in comparison with data.[2,3] Figures 4 shows the smooth performance of predictive tool in the prediction of maximum oil production rates using plunger lift, as a function of well depth and tubing sizes. It is expected that our efforts in formulating a simple tool will pave the way for arriving at an accurate prediction of maximum oil production rates using plunger lift, as a function of well depth and tubing sizes which can be used by petroleum and production engineers for monitoring the key parameters periodically. Typical example is given below to illustrate the simplicity associated with the use of proposed method for rapid estimation of maximum oil production rates using plunger lift to support the applicability of plunger lift in a well as a function of well depth and tubing size. The tool developed in this study can be of useful for experts and engineers to have a quick check on maximum oil production rates using plunger lift, as a function of well depth and tubing size to support the applicability of plunger lift in a well at various conditions without opting for any experimental trials. In particular, petroleum engineers would find the approach to be user-friendly with transparent calculations involving no complex expressions.

3.1 Example:

A given well is 7 000 ft (2 134 m) deep and is to be produced by plunger lift through 2 in. (5.1 cm) tubing. What is the maximum liquid that can be produced?

Solution:

a= 3.584 83 (from equation 10)

b= 1.188 11 (from equation 11)

c= -4.012 061 (from equation 12)

d= 4.743 389 (from equation 13)

q= 100 bbl/d (16 m^3/d) (from equation 9)

In this example, the well will produce a maximum production rate of approximately 100 bbl/d (16 m^3) to maintain plunger lift.

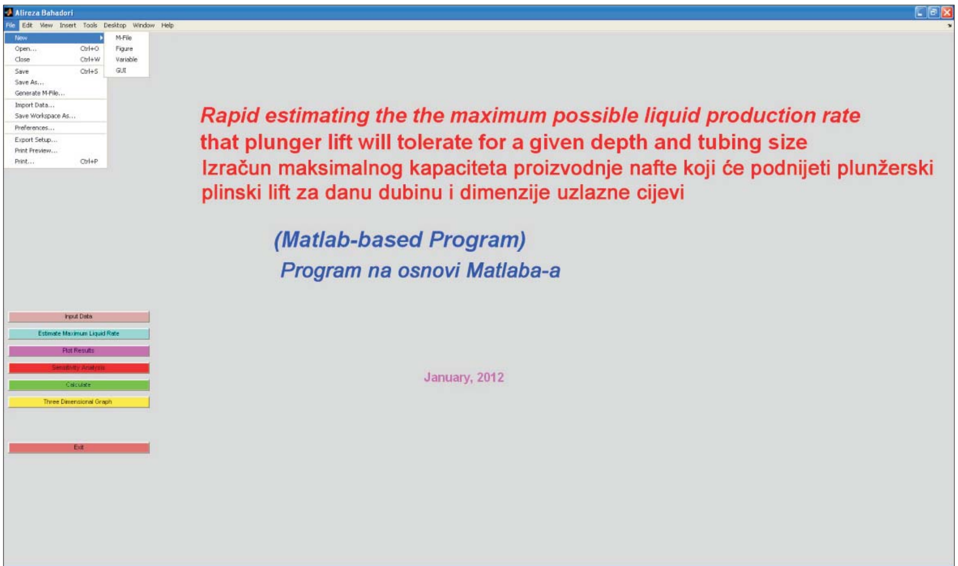

Fig. 2. Matlab-based(2008) software
Sl. 2. Softver na osnovi programa Matlab[13]

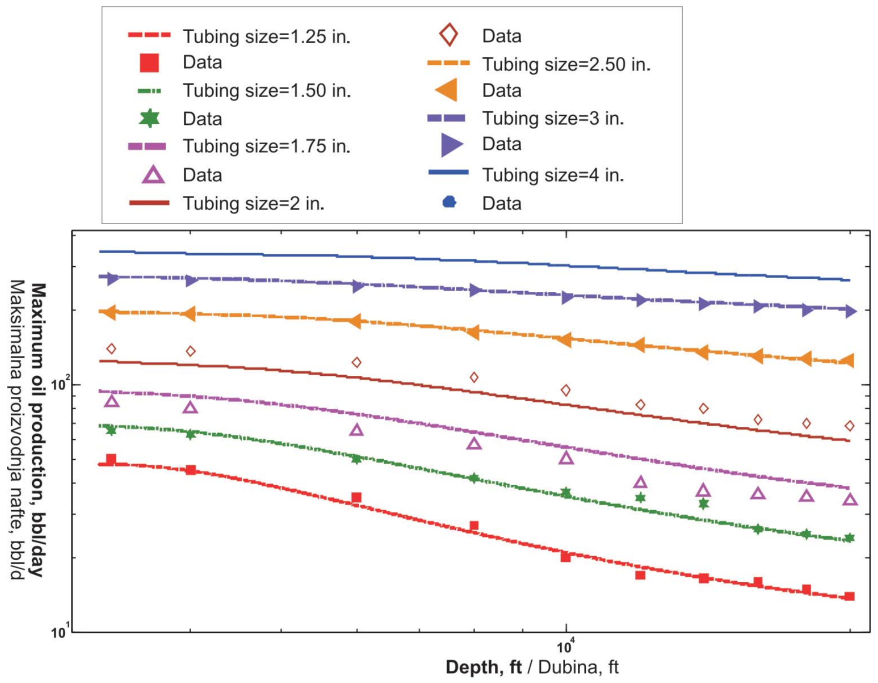

Fig. 3. The results of predictive tool for the estimation of maximum possible production rate using plunger lift in comparison with Beeson et al 1956 data

Sl. 3. Rezultati prediktivnog alata u predviđanju maksimalnog kapaciteta proizvodnje nafte korištenjem plunžernog plinskog lifta u usporedbi s podacima Beesona i suradnika (1956.)

4. Conclusions:

In this work, simple-to-use equations, which are easier than existing approaches less complicated with fewer computations and suitable for engineers is presented here for the estimation of maximum oil production rates using plunger lift, as a function of well depth and tubing size. Unlike complex mathematical approaches the proposed correlation is simple-to-use and would be of immense help for engineers especially those dealing with petroleum production and operations. Additionally, the level of mathematical formulations associated with the estimation of maximum possible oil production rate to support the applicability of plunger lift can be easily handled by an engineer or practitioner without any in-depth mathematical abilities. Example shown for the benefit of engineers clearly demonstrates the simplicity and usefulness of the proposed tool. The proposed method has clear numerical background, wherein the rele-

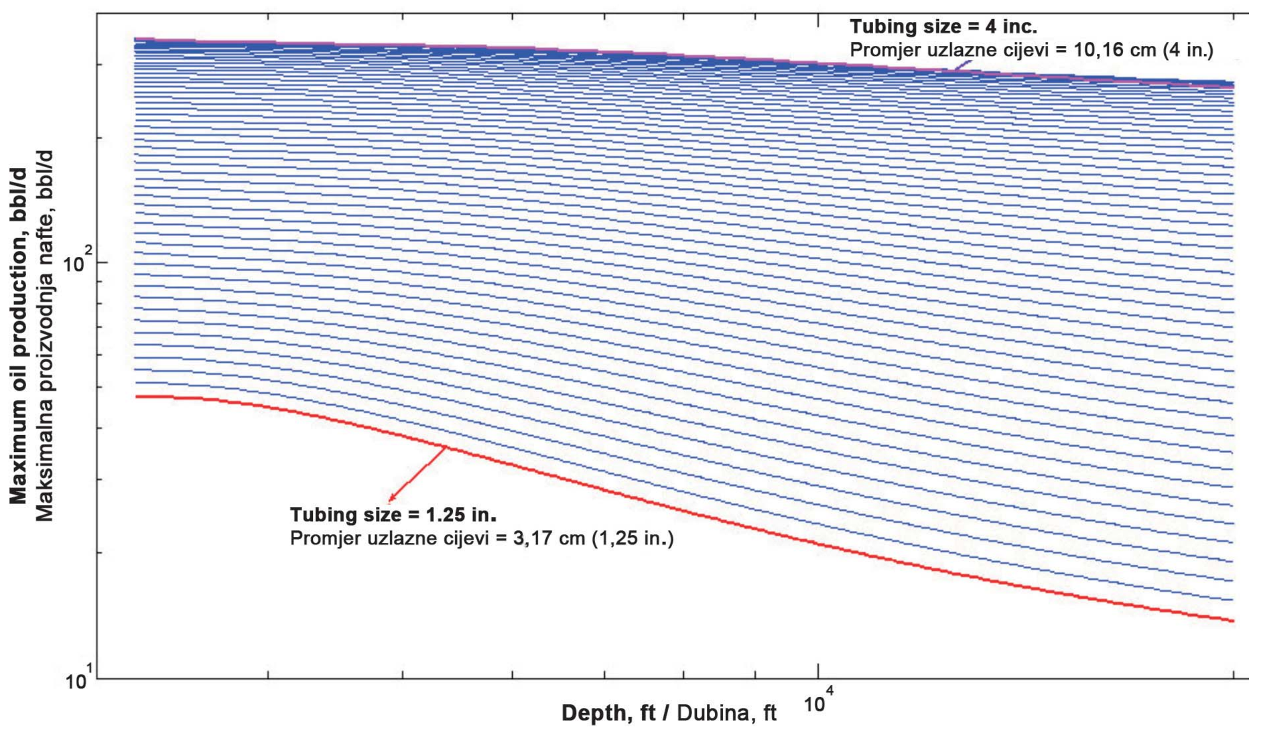

Fig. 4. The results of predictive tool for the estimation of maximum possible production rate using plunger lift in comparison with Beeson et al 1956 data

Sl. 4. Rezultati prediktivnog alata u predviđanju maksimalnog kapaciteta proizvodnje nafte korištenjem plunžernog plinskog lifta u usporedbi s podacima Beesona i suradnika (1956.)

vant coefficients can be retuned quickly if more data become available in the future.

Nomenclatures:

i	index
j	index
dt	tubing size, inch
L	well depth, ft
m	matrix row index for $m \times n$ matrix
n	matrix column index for $m \times n$ matrix
P	polynomial
q	maximum liquid production rate, bbl/day
u	coefficient of polynomial
V	Vandermonde matrix
x	data point
y	data point
α	matrix element

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