

REAPPRAISAL OF RATIONAL CHOICE THEORY

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ABSTRACT

The value of rational choice theory (RCT) for the social sciences has long been contested. Much time has been spent by economists and critics on the pervasive but elusive concept of rationality. The critiques mainly challenge the basis of the utility theorem. Several articles on the misuse of mathematics in economics have already appeared in the literature. As N. Bouleau stated, “On several occasions, however, one feels that the criticism is that the math is being misused and should be developed in some other direction (e.g. a statistical analysis of the financial tendencies that polarize wealth and income, or a study of the positive feedback mechanisms, etc.). This leaves certain dissatisfaction – on a philosophical level.” The aim of this paper is to present a decision theory, yields intention (logos) and valuation (existence). Here we present a new mathematical representation of RCT, which leads to a dynamic economic theory. We discuss the philosophical or meta-economical problems, which are needed for the successful applications of mathematics.

KEY WORDS

rational choice theory, dynamic economic theory

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INTRODUCTION

The value of rational choice theory (RCT) for the social sciences has long been contested. Much time has been spent by economists and critics on the pervasive but elusive concept of rationality. The critiques mainly challenge the basis of the utility theorem. Several articles on the misuse of mathematics in economics have already appeared in the literature. As Bouleau stated, “On several occasions, however, one feels that the criticism is that the math is being misused and should be developed in some other direction (e.g. a statistical analysis of the financial tendencies that polarize wealth and income, or a study of the positive feedback mechanisms, etc.). This leaves a certain dissatisfaction – on a philosophical level.” [1]. Here we present a new mathematical representation of RCT, which leads to a dynamic economic theory. We discuss the philosophical or meta-economical problems, which are needed for the successful applications of mathematics.

The basic novelty is that we start from a phenomenological description of decisions. The main question is not what to choose from the possibilities, but why we choose something and how it changes our environment. Economy is complex system. Basic interactions are the exchanges and transformations (productions, consumption, ...). Agents exchange material, energy, money and information, so they change themselves and simultaneously they change their environment [2, 3]. The economic activity is modeled as transformation and transport of commodities (materials) owned by the agents. Rate of transformations (production intensity), and the rate of transport are given by decisions. Our approach is a new formal description of economic activity which corresponds to the bookkeeping practice, instead of starting from necessity and from equilibrium. As this approach is considered as uninteresting – it was not elaborated, thus have to introduce some new concepts, which are present in the colloquial economics, but they do not appear in the textbooks.

In our previous works we accepted the false idea, that utility maximization principle is the only way to implement the rational choice theorem. We proposed our postulates to replace this. Nevertheless, a careful analysis revealed that our approach is a valid mathematical representation of the RCT. For descriptive economic investigations our results show that for the application of RCT, a time aggregation must be applied, which demands for a new model of decision makers. Depending on the level of aggregation we get two approaches. The first model is the well-known Homo Economicus (HE), which leads to the present equilibrium economics. The other approach is called Homo Sapiens Economicus (HSE), which yields dynamic economics. For HSE a force law replaces the maximization rule; the expected wealth increase behaves such as the economic driving force. The result is complex, dynamic economics. These formal dynamic equations of an economic system will be coupled non-linear equations. Forecast would need a huge quantity of empirical data. Nevertheless, as the system is more complex than physical non-linear systems, they show that generally there is no equilibrium or there are several equilibrium states. Conclusions, economic results from the pure equilibrium theory have less relevance for the real economic world, as there is no argument to assume that real economic systems tend to equilibrium in case of multiple equilibria. This questions the applicability of some aspects of the neoclassical economics. For the analysis of the markets it makes untenable the efficient market hypothesis, being the fundamental idea of mainstream. As Davidson stated “Whether they declare themselves Monetarists, Rational Expectation theorists, Neoclassical Synthesis [Old] Keynesians or New Keynesians, the backbone of their mainstream theories is the efficient market analysis where the future can be known.” [4].

In this paper first we will show a result type approach of economics, which describes the changes in stock. Introduced concepts are helping to understand the dynamic formalization of changes, but they are not answering, why an economic agent does his/her activity. The second part of the paper shows the basics of the RCT, namely the postulates and the required constraints in philosophical as well as in mathematical levels. On the basis of RCT we can interpret the wealth (Z) function, which governs our decisions in an abstract level. Function Z has several attributes. One of them is that we can observe the economic valuation of agents. In the end we get a new model for decisions. With some extra constraints on the aggregation process we get HE, which gives us price-value equilibrium. If we skip the extra constraints then we get HSE, with unique valuation, which yields a dynamic approach.

Main message of HSE is that a new economic theory is possible and needed. We hope that the present paper is a step into this direction.

DESCRIPTION OF ECONOMIC ACTIVITY IN REAL TIME – RESULT TYPE APPROACH

An important part of economic activity is related to the production and trade of the goods. In this chapter we focus on a formal description of the changes of the stocks and goods. In the description of human activities, there is a connection between decisions and changes on the goods. But this connection is pure causality. Describing the changes of the stock of goods, bookkeeping practice offers a method called the balance method. Rewriting them in the form of equations for the changes of the stock of goods it gives a bit different formulation to the accustomed. The main aim is to describe decisions related to the change of the quantity of goods (production, consumption, or trade). But what do we call goods? Goods – is anything, which can be characterized by its stock and can be changed by a process selected by decisions, and the person is not indifferent to have it. It can be material, or immaterial, as for instance money or knowledge. An agent can be an individual, or a group of individuals, firm, company, who is able to make decisions. This decision selects the activity, which will result in the expected changes after a certain time, so the role of time and the time dependence in economics is different to natural sciences. For the sake of formalization, Symbol X_i^a is the notation for the stock of the i -th good of agent a , where $a = 0, 1, \dots, N$, \underline{a} identifies the agent, $a = 0$ is used for nature, furthermore $i = 0, 1, 2, \dots, N_g$ identifies the type of the good and $i = 0$ is used for the money stock owned by the agent.

Quantity of a good change in activities selected by decisions: voluntary changes, and there are involuntary changes, called forced processes. They can be of natural or of social origin. Aging, wearing are always present. Tax paying or robbery are examples of social origin. They must be considered as exogenous changes. Formally the balance equation can be written, as

$$dX_i^a(t)/dt = D_i^a(t) + C_i^a(t), \quad (1)$$

where $dX_i^a(t)/dt$ is the change of stock of the i -th good of agent a at time t while $D_i^a(t)$ is the change of the stock of the i -th good of agent a at time t due to activities selected by decisions and $C_i^a(t)$ is the change of the stock of the i -th good of agent a at time t due to involuntary processes.

It is important here, that we use differential equations for the sake of consistency (see below), but continuity is not necessary, moreover the observed time is usually an interval.

An activity means a change in the stocks of goods. Symbol $A_i(t_0, t)$ is for the change of the i -th good at time t in the activity, which was selected at time t_0 . Here we assumed that in a moment there is one decision, which causes only one activity. At time t the activities that

started before may result in change, so change of the stock of the i -th good of agent a at time t is the sum of the results of activities started before, formally

$$D^a_i(t) = \int_0^t A_i(t_0, t) dt, \quad (2)$$

It is a very complicated formula, but well-known in the bookkeeping practice. It is not written in this exact form, but the problem can be formulated in this way. For example, an agent pays in advance and the product is delivered two weeks later, or vice versa.

Now the effect of activity selected at t_0 is investigated. During the activity the change of goods is not continuous,

$$dX^a_i(t)/dt = A^a_i(t_0, t) + C^a_i(t). \quad (3)$$

Imagine a baker. The selected activity is to make bread. The input materials are consumed at the beginning, the electricity consumption is continuous during the baking process, and the bread appears only at the end of the process. Time based description of economic activities is possible, but it needs such details, that are not important, and hardly available for an economic description. In economic description the activities are characterized by the results. For the sake of shorter forms the vector notation will be used, example, $X^a_i(t) = \underline{X}^a$. The result of the activity is the total change of stocks during the activity, that is

$$R(t_0) = \int_{t_0}^{t_0+T^0} \underline{A}(t_0, t) dt, \quad (4)$$

where $\underline{R}(t_0)$ is an n -dimensional vector and the components show the change of the stock of the relevant good. This feature gives two further characteristics of the activity: (i) the total time span of the activity, when all the changes aimed by the activity take place, denoted as T^0 and (ii) the time span of the action, T^0_s , which is defined as the actual time spent with the activity.

Stock changes caused by the this activity in the result based approach are

$$\underline{X}^a(t_0 + T) = \underline{X}^a(t_0) + \underline{r} \cdot \underline{R}^a(t_0) + \underline{C}^a(t, T), \text{ if } T < T^0, \quad (5)$$

where the components of vector \underline{r} are between 0 and 1, expressing the fact that they are not known, when the actual change occurs in the whole interval

$$\underline{X}^a(t_0 + T) = \underline{X}^a(t_0) + \underline{R}^a(t_0) + \underline{C}^a(t, T), \text{ if } T \geq T^0. \quad (6)$$

As the result type description one can define the stock change but after the end of the activity.

Nevertheless, not all \underline{R} are different. The activities selected in different moments probably have different time histories, but in result type approach the same result can occur several times. It is more convenient to use an index, which distinguishes the results. The effective decision $\underline{R}(t_0)$ can be written into a more transparent form. The results of activities can be classified. We say that activities are of the same type, if the results satisfy the relation $\underline{R}(t_0) = c R_i(t_1)$.

A new index, K , is introduced to characterize the type. We select one element, preferably the smallest one, as the unit activity e^{K_i} . The detailed description of the unit activity is given elsewhere [5]. The multiplying factor c can be interpreted as the intensity of the activity K . We use the symbol for it, J^K , so

$$R_i(t_0) = J^K(t_0) e^{K_i}, \quad (7)$$

or, in vector notation,

$$\underline{R}(t_0) = J^K(t_0) \underline{e}^{K_i}. \quad (8)$$

Sometimes it is worthwhile to distinguish the activities by the partners. Instead of K a double index b, k can be applied, where b identifies the partner. Then e^{ab, k_i} is the k -th unit activity with partner b , aa is for activities without partner, and $J^{ab, k}$ is the intensity of this activity.

Every agent activity is characterized by the unit activity e^{aK_i} , by the intensity of activity J^{aK} , by the total time T^{aK} and by the time spent on the activity, t^{aK_s} . If the agent a selects the activity K with intensity J^{aK} , then the result is:

$$X_i^a(t + T^K) = X_i^a(t) + J^{aK} e^{aK_i} + C_i^a(t, T). \quad (9)$$

In result type description, it is a must that the activity selected is performed with the selected stock changes. Changes of activities during the activity process are out of the scope of the result-type description. We can see that in the result type approach time must be quantized, because the overlapping of activities, their consequences and realizations are not in the same time. Nevertheless there is no information about the timing of the changes inside the quantum. In a quantum several different activities are present. For a real time dependent description a time aggregation is needed, which means that we ask the stocks at t , $t + T$ and $t + 2T$ moments, and the changes inside T intervals is not considered except the activities, namely here it is assumed that the environment does not change in the T intervals. In result type description, that is in an economic approach, one has to neglect the possibility that activity type is modified during the activity process itself. Similarly, the changes of environment, the technological change, the market changes, simply all the changes have to be neglected for the selected activity.

BALANCE EQUATION IN RESULT-TYPE APPROACH, IN THE QUANTIZED TIME

Change of the stock of the i -th good of agent a at time interval $(t, t + T)$ is

$$D_i^a(t, T) = \int_t^{t+T} \int_0^{t'} A_i(t_0, t') dt_0 dt', \quad (10)$$

and in result-type description

$$D_i^a(t, T) = \int_t^{t+T} R_i(t', t') dt' = \sum_{kb} \left[J^{abk}(t, T) e^{abk_i} + d^{abk_i} \right], \quad (11)$$

where d is for the uncertainty, which originates in the activities that started before but finished in the interval, or that were started but were not finished. In the practical life d is well-known. In bookkeeping practice, in time of balance closing they introduced some operations to diminish this effect. The larger the T , the smaller the d , but on the other hand the time resolution decreases, so for a real description an optimal choice has to be defined.

Stock of the i -th good at time $t + T$ can be written as

$$X_i^a(t + T^K) = X_i^a(t) + \sum_{kb} \left[J^{abk}(t, T) e^{abk_i} + d^{abk_i} \right] + C_i^a(t, T). \quad (12)$$

where $X_i(t)$ is the stock of the i -th good at time t , $J^{aK}(t, T)$ is the intensity of activities started and finished in the $(t, t + T)$ time interval, C is for the changes not selected by decisions, and d is uncertainty.

In the following we assume, that T is selected such a way, that one can neglect the effect of uncertainty. T' must not be bigger than the maximal quantum time. In quantized time the change of stocks has the form.

$$X_i^a(t + T) = X_i^a(t) + \sum_{kT} J^{aK}(t, T') e^{aK_i} + C_i^a(t, T). \quad (13)$$

In quantized time, an agent has a different role compared to real time models. In reality, activity is selected. In economic description, that is in quantized time, the decision is to select

$\sum_{kT} J^{ak}(t, T^a) e^{ak_i}$. But it is another question, how to choose the activity. In the following section the goal is to make formal description of the choice.

Before proceeding, let us emphasize that there is a difference between describing individual and quantized decision. The acting laws are different. Physics showed that there are difference laws in microscopic and macroscopic – aggregated behaviour. In physics the microscopic level is reversible, and can be described with optimization principles, while the aggregated macroscopic level is irreversible, and governed by the entropy production principle. However, this aggregation is not the same as in physics, where the aggregation is done for the large number of particles. In economics the agents are different; this type of aggregation cannot be applied, only in some special cases.

Time aggregation has more resemblance to the quantum mechanic problem, the transition from microscopic to macroscopic behavior. The analogy is not perfect, but important, not perfect because the real time description refers to the microscopic level (to the quantum world), while the aggregated description to the macroscopic one. Important, because quantum mechanic like description perhaps can give new insights to the modeling of individual decisions. Nevertheless, it is out of the scope of the present paper. We cite only the basic result of quantum mechanics. The laws of physics are different in the micro and in the macro world. The economic man has to introduce for the model of economic activity, and it not simply the aggregation of the individual behavior. First the model of individual decisions is needed.

RATIONAL CHOICE THEORY

Rational choice theory in this aspect is about what activity to choose. For the model of individual decisions the starting point is that an activity is selected by the agent from the set of recognized possibilities, which are characterized by the set $E^{Ra} = \{e^{aK}\}$. The agent selects the type and intensity of activity. The criterion for the selection is well-known. The best is selected [6]. There are no arguments why to select the worse. The problem is that this statement does not tell anything about the selection method, and there is no interpretation of the best. It tells only to the observer, which was the best activity for the agent in the moment t_0 , the selected one.

Rational choice theory gives a reasonable description of the selection process. It states that the best is selected by the expected result of the activity. Albeit, it is not a general truth. In reality the form of action and its details can also be important. Compulsive shopping behavior is an example, when the activity is more important than the result [7]. Nevertheless, for the economic decisions or rational choices it seems to be a reasonable assumption that the expected result is the basis of our decisions. Rational choice theory is the idea that individuals make choices to maximize benefit while minimizing cost. It postulates that when making a decision, individuals first weigh the expected positive benefits against the expected negative consequences, and then base their choice on what they think will ultimately benefit them the most. The basic law of human actions is formulated as a separate postulate.

POSTULATE I.

Postulate I.: an individual acts as if balancing costs against benefits to arrive at action that maximizes personal advantage.

The rationality postulate is generally considered the paradigmatic core of economics, but there is no unique understanding. As Vanberg summarized it: “The status of the postulate is very different in the different branches. It is considered either as an empirically testable explanatory theory, or as a non-refutable axiomatic doctrine or as a ‘metaphysical statement’,

while still others regard it as a normative principle that ‘tells us how, as rational agents, we ought to choose’ [8]. These approaches are different by the representation of the cost benefit analysis. The cost and benefit are defined by the agent and also the meaning of the personal advantage. In normative theories there are attempts to give objective evaluations, but in economic description the subjective evaluation is a must. We follow L. von Mises’ approach to economics. The rationality principle is a mere definition. To say that human action is ‘rational’ according to Mises is the same as saying that it is subjectively meaningful, purposeful or goal directed. He therefore concludes: “Human action is necessarily always rational. The term ‘rational action’ is therefore pleonastic and must be rejected as such” [9; p.18]. There can be no irrational purposeful economic actions. “Behavioral responses that do not qualify as purposeful actions, such as accidental body movements or purely mechanical reflexes, simply fall outside the explanatory domain of rational choice theory” [9; p.20].

Mises comment states that for every observed decision there is a hidden cost-benefit analysis. The success of a model is defined by the validity of the cost-benefit model. Economic decisions give a model for that.

ECONOMIC DECISIONS

The better or worse distinction is a subjective evaluation of the agent. These decisions can be divided into two groups, economic and non-economic decisions. In case of economic decisions the cost and benefit arise from the relevant changes of stocks. As for instance, when the baker makes the bread the input materials and the services (electricity, ...) represent the cost and the benefit is the bread, or when shopping the money is the cost and the product purchased is the benefit. In non-economic activities the cost or the benefit is not represented by them. A trivial example is to give money to a beggar. Nevertheless, it is well discussed, that this action is not irrational, if one considers the psychological welfare of the agent. If it is incorporated into the symbolic goods, then the result of the action, to give money to the beggar, leads to a better state for the agent. Whether a decision is economic or does not depend on the modeler, which goods are included into the description. If the symbolic goods are not taken into account, then such type of actions are not in the decision part, but they belong to the constrained processes. Similarly, the consumption can be a decision, if the symbolic goods contain the physiological state, or a constrained process, if it is not present. There are mixed cases also, when the decision is partially economic, but not fully. Scientists, especially the economists, give an example. On the basis of economic price evaluation it is much better to do business than science. The question is, what kind of goods are included by the modeler.

For that part of economic decisions, which concern the activities of changes of the stock of goods a rationality criteria is that the agent selects the activity if the result leads him/her to a better state.

POSTULATE II.

Postulate II: An economic activity is selected by the agent in the hope that leads to a better state.

This postulate is universally valid, if it is violated, then there is an error in the model of the agent, something is missing from the description. Economic decisions are based on RCT, so the agent is able to evaluate the effect of an activity, that is she/he can answer for the question, whether \underline{X} or $\underline{X} + \underline{R}$ is better, it means that there is a preference order in the set of the stocks of goods.

We make two further mathematical assumptions, which are needed for the formalism, namely:

- **Partial Completeness** is the known part of the set of good stock states can be ranked in an order of preference (indifference between two or more is possible),

- **Transitivity**, if state \underline{X}_1 is preferred to \underline{X}_2 , and state \underline{X}_2 is preferred to \underline{X}_3 , then \underline{X}_1 is preferred to \underline{X}_3 .

As these preference ordering is the result of a learning process the transitivity is not general truth, but the effect of a recognized in-transitivity can causes a cognitive dissonance, and everybody tends to eliminate it, so it is an acceptable assumption.

The agent is characterized by the wealth function, Z with the property

$$Z(X_1) > Z(X_2). \quad (14)$$

if state \underline{X}_1 is preferred to \underline{X}_2 . Z may seem to be very similar to the utility function. The postulates are the same, except the continuity one, which is not needed. This is a basic difference in the postulates. Also in case of utility the postulate demands for the selection of the best. In the wealth function (Z) the evaluation of the result is demanded. Z is not assigned to the result of decisions, as the utility, but to the ownership of stocks. In a special case the change of Z will be the utility function, as it will be described later.

Overall, Z is quantity, which is expected to be increased in the production and trade processes, and it reflects the valuation of the total stock of goods. It coincides with the colloquial notion of the wealth, except that it is a subjective measure, and it is not measured in monetary terms. It is the measure of those parts of happiness which originates in the ownership of the goods. It is not the hedonistic concept, but the Aristotelian, eudaimonic concept of happiness; wealth is a part of happiness, which is related to the ownership of the goods [10]. Ljubormirsky [11] summarized an empirical results that for individuals psycho-genetic factors gives 50 % of the happiness, 40 % is from the voluntary actions and 10 % from the ownership of goods (material and symbolic). To distinguish it the everyday use of the word, which is bounded to the money, we call it z-wealth, or wealth function.

MEASURE OF WEALTH

Wealth function Z , is a subjective measure: that is, it is only in the human mind, it does not exist outside of the human mind. This subjectivity does not mean arbitrariness. The wealth governs the decisions, so the circumstances define which is the “fit” form, if the form does not fit to the natural and social environment then the agent (individual or firm) will lose. The agents learn the valuation, expressed by the wealth function, and they modify it reflecting the experiences. Function Z cannot be directly measured. Nevertheless it governs our decisions, and from the observations of decisions some of its properties can be revealed. Z expresses the valuation of the resources.

Properties of Z :

1. since z-wealth is a positive attribute, a function that measures z-wealth must be non-negative,
2. all resources, material, symbolic goods and money that are owned by the economic agent are included within the function. On the other hand, in economic models, one can prepare simplified models. As it was discussed, the distinction between decision governed and constrained processes depends on the model,
3. an increase in the agent's ownership of stocks of beneficial goods or money results, *ceteris paribus*, is an increase in the agent's z-wealth,
4. the z-wealth change caused by the change of the stock of goods expresses the increase of happiness, which coincides our definition of value. Starting from the definition of value given by Menger: “To have value, a good must assure the satisfaction of needs that would not be provided for if we did not have it at our command. But whether it does so in a direct or in an indirect manner is quite irrelevant when the existence of value in the general sense of the term is in question.” [12].

The z-wealth change in case of dX_i change of the i-th good in first order is

$$dZ = Z(X_1, X_2, \dots, X_i + dX_i, \dots, X_n) - Z(X_1, X_2, \dots, X_i, \dots, X_n) = w_i dX_i. \quad (15)$$

where the $w_i = dZ/dX_i$ symbol means the subjective z-wealth value of the i-th resource, as it is the increment of z-wealth, due to the infinitesimally small quantity of the i-th resource. We call it z-value¹. For normal goods the z-value is positive, and it is a decreasing function of the stock of resource in a quantum. If the supply of any class of resource is so great that every demand is met, then the increase of the resource does not mean „better life”, so it must not lead to the increase of wealth, then value is zero or negative if it causes further problems. If, in any class of resource, the supply is not sufficient to meet the demand for satisfaction then the increase of the stock increases the wealth, and value rises. For cultural goods the typical behavior is that the value increases with the quantity. It is also important, that the valuation does not depend just the Gossen laws. It has other factors also.

A special good, money has also a z-value, if $X_n = M$ the symbol for money, then the z-wealth change with dM money change is:

$$dZ = Z(X_1, X_2, \dots, X_i, \dots, M + dM) - Z(X_1, X_2, \dots, X_i, \dots, M) = w_M dM. \quad (16)$$

With this definition of z-wealth and z-value the money has also a subjective value. It is the change of wealth due the increase of money stock. Neoclassical economics in the standard form does not deal with the value of the money. However it is a historical fact, that the subjective utility of money was introduced by Bernoulli in 1738. He proposed a solution for the St. Petersburg Paradox based on the notion of expected utility. Bernoulli proposal was that utility (of the money) is a logarithmic function of the amount of money [13, 14]. Bernoulli's utility of money coincides with the z-value of the money. The appearance of the value of money opens a new way for the optimization of economic processes [15-17].

DESCRIPTION OF ACTIVITIES WITH Z

With the postulates and Z function we can describe the selection method of the activity in an abstract level. In a trade activity, when q_i quantity of the i-th good is bought for m money, the change of Z is

$$dZ = Z(X_1, X_2, \dots, X_i + q_i, \dots, M - m) - Z(X_1, X_2, \dots, X_i, \dots, M) = w_i q_i - w_M m > 0. \quad (17)$$

The inequality holds for rational choice. It can be rewritten in a more transparent form

$$dZ = w_M \left(\frac{w_i}{w_M} - \frac{m}{q_i} \right) q_i = w_M (v_i - p_i) q_i, \quad (18)$$

where p_i is the price, and v_i is the value in monetary units. If and only if the system stays in equilibrium, the values equal the prices. In equilibrium exchange the z-wealth of the agent does not change. In non-equilibrium activity $dZ > 0$ and dZ/w_M is the z-wealth production in monetary units. That is the colloquial wealth change; we call it wealth increase, or gain, or value production. The quantity

$$G^i = (v_i - p_i) q_i, \quad (19)$$

is the gain of the i-th activity (if previously we assumed that the i-th activity is this exchange). More generally,

$$G^K = \sum_i v_i J^K e^{K_i}. \quad (20)$$

which is a form valid in linear approximation, while in the general nonlinear case one has

$$G^K = Z(\underline{X} + J^K \underline{e}^K) - Z(\underline{X}). \quad (21)$$

The gain sums up the value produced and the value sacrificed, the result is the net value production. Our aim to maximize the wealth, or the gain. The choice is the selection of J^K . A simple and straightforward solution is the optimization. Quantity J^K will be selected to maximize G . The situation in the z-wealth approach is different, compared to the utility approach. For the introduction of Z it was not postulated that Z is maximized. The rational choice theorem (postulate I.) ask for the maximization of Z , but in the reality it is a conditional optimization. It would be a rather strong and artificial assumption, that G contains all the cost and benefit, that is, to assume that only stock changes are important in economics. It is well-known that the ethical norms and the cost of the violation of these norms also matter.

For the optimization of Z one has to define the boundary conditions, too:

- an agent in reality has a hierarchical decision; first he/she selects a group of activities, and selects the activity only from the group,
- ethical norms and confidence also effect the decisions. The exchange is always based on the expectations, and a well-known partner has an advantage.

Individual decisions cannot be described by Z alone. Nevertheless, observations of individual decisions give information about Z . Present formalism offers an alternative way for the description of decisions. The expected gain is the driving force for our actions. We act, if there is a hope for gain. Generally it is valid also, that the higher is the expected gain, the higher is our willingness to act, that is J^K is defined by the driving force. We define the driving force as the gain of the unit activity for a simple trade process, as

$$F^i = v_i - p_i, \quad (22)$$

which is just the value and price difference. In general form, for any activity the unit activity and the values define the force:

$$F^K = \sum_i v_i e^{K_i}. \quad (23)$$

The relation of gain and force is

$$G^K = J^K F^K. \quad (24)$$

Postulate II. states that $G \geq 0$ for actions selected. J^K is usually non-negative. In case of traders who sell and buy the same good, it can be of both signs. For traders, the postulate II. states that $J > 0$, that is the trader buys if the value is higher than the price, and $J < 0$ in the opposite case. In the other cases gain of unit activity has the property, that $J = 0$ if $F \leq 0$ and the greater the F the larger the J . The expected gain, the value production is the driving force for our activities. It suggests for the linear case

$$J^K = L^K F^K. \quad (25)$$

in which L couples the driving force to the intensity of the action. L is called motivation. The higher the motivation, the higher the action. In general case

$$J^K = J^K(F^1, F^2, F^3, \dots). \quad (26)$$

Introduction of the motivation solves the second problem, listed in the boundary condition of the choice. Confidence, ethical norms can be incorporated into the motivation or in the force law. Nevertheless, for individual decisions this approach is not easily applicable, as it does not solve the problem of the selection of K . It answers only for the determination of the intensity, if the type of activity is selected. $F > 0$ does not imply $J > 0$, as in a moment only one activity can be selected.

There is a way to overcome this difficulty: aggregation in time as we do in the result type description.

RATIONAL CHOICE IN QUANTIZED TIME

Describing the real selection of the individual decision seems impossible, because the time overlapping activities and time difference between decisions and their realization. To solve this problem we must aggregate in time.

Agents are characterized by the stock of the goods, \underline{X}^a , by the z-wealth $Z^a(\underline{X}^a)$, by the set of recognized set of possibilities for activities and by the unit activities \underline{e}^{ak} . In quantized time the change of stocks takes the form.

$$X_{i}^a(t+T) = X_{i}^a(t) + \sum_{kT} J^{abk}(t, T^a) e^{ak_i} + C_{i}^a(t, T). \quad (27)$$

In economics, that is in quantized time, an agent has to select $\sum_{bkT} J^{abk}(t, T^a) e^{ak_i}$. There are two ways of modeling that problem, namely:

- selection is done for $q = \sum_{kT} J^{abk}(t, T^a) \underline{e}^{ak}$,
- selection is done for J^{abk} .

First approach leads to the HE description, to an equilibrium world. Second approach is a new description.

THE FORM OF HOMO ECONOMICUS

In the aggregated form the summation is done for the type of activities and for the partners as well:

$$q^a = \sum_{kbT} J^{abk}(t, T^a) \underline{e}^{ak}. \quad (28)$$

The result is average quantities, gains and dehumanized activities in the quantum. Economic environment is replaced by an average, equilibrium economic system, which made the description value-blind and it is called market. This HE is bounded to equilibrium economic systems.

In this scenario selection problem for HE is formally the same as for individuals, he/she selects the best q . The RCT gives the maximization principle. As aggregation makes it impossible, to take into account the constraints, the selection principle for the aggregated decision is the maximum of Z .

With one more philosophical constraint, in neoclassical economics the money cost appears only as a budget constrain, so the selection problem is

$$\max \{Z(\underline{X} + q) - Z(\underline{X})\}, \text{ with } \Delta M = m. \quad (29)$$

As the description is bounded to equilibrium, there is no time, no stock change in time, so \underline{X} is constant, Z equals the utility function and the choice is described by the following:

$$u(q) = Z(\underline{X} + q) - Z(\underline{X}). \quad (30)$$

The price for it is the loss of dynamics. Further, which became a source of controversies, it remained hidden that the HE model is only for the aggregated decisions. Let us summarise the main properties of HE in this approach:

1. HE is a model for the individuals, for the consumers. For producers there is different decision formalism,
2. HE sees only the aggregated, averaged, "equilibrium" market, where is one unique price for every good,
3. choice possibilities are defined by the set of the consumption baskets $\{q\}$,
4. budget constraint is externally fixed,
5. perfect information, which is an euphemism, as it implies that HE knows all the prices².
6. the selection is done by the utility maximization.

Nobel Laureate Robert Solow [18] has characterized – tongue at least partially in cheek, one supposes – three central ‘structural’ pillars of economic theory as “greed, rationality and equilibrium”. By ‘greed’ Solow apparently means ‘selfishly purposeful behavior’, that is the essence of utility maximization individuals, with price based budget constraint. By ‘rationality’ Solow apparently means that agents belonging to the subspecies Homo Economicus understand their own preferences and make optimal, utility maximizing decisions based on that understanding and on whatever budgetary constraints are applicable.

THE FORM OF HOMO SAPIENS ECONOMICUS

We return to that aggregation, which is without activity aggregation. HSE selects J^{abK} , and for every activity the agents assigns the driving force, $F^{ab,k}$. The objections against force law for individual decisions disappear in the aggregated decisions. In quantized time generally it is valid, that if $F > 0$ then $J > 0$. We assume that the agent selects the activity based on the driving force.

This gives us the properties and results of HSE. Postulate II. is valid for all type of economic agent, so the HSE is model of individuals as well as firms, companies and organizations. HSE is a model for activities concerning the changes of goods, so it is a model for production and trade decisions. Its characteristics are:

1. HSE is characterized by the stock of goods \underline{X} ,
2. the valuation of the agent is done on the basis of the wealth function, Z ,
3. the activity set $\{\underline{e}^K\}$,
4. force law:

$$J^K = J^K(F^1, F^2, F^3, \dots). \quad (31)$$

When cross effects can be neglected, one obtains

$$J^{ab,k} = J^{ab,k}(F^{ab,k}). \quad (32)$$

As $J = 0$ if $F = 0$, the first approximation is the linear relation

$$J^{ab,k} = L^{ab,k} F^{ab,k}. \quad (33)$$

Quantity $L^{ab,k}$ couples the intensity and the force, we call it motivation. It reflects the ethical norms, and also the confidence. It is also a result of learning. This is an experimentally observable relation. The well-known supply-demand curves can be reinterpreted in HSE approach. Consider the case, when agent a sells the i -th good to agent b at price p . Driving force for agent a is:

$$F^a = p - v_i^a. \quad (34)$$

Driving force for agent b

$$F^b = v_i^a - p. \quad (35)$$

For small forces the linear relation can be applied, for larger forces a less steep increase is probable. These relations plotted give the usual Supply-demand curves. Difference is that instead of classical elasticity, r the motivation, L is used. The relation between L and r is

$$L = r p_0 / f(p_0). \quad (35)$$

From supply-demand curves numerical value of L , and that of the monetary values of goods can be learned.

The result is a new model of economic agent, Homo Sapiens Economicus. Then HE is a special case of HSE. Differences of the behaviour arise from the restrictions built in HE. Characteristics of HSE are:

1. HSE is a model for all type of economic agents; it works for individuals, as well as for firms, for companies or for organizations. It is a model for consumers and producers,

2. HSE sees the partners, so there is no need for the aggregated, averaged, “equilibrium” market, there is no one unique price for the goods. The prices are fixed by the agents. There can be bargaining, or a part of the agents are price makers, while the others are price takers,
3. choice possibilities are defined by the set of the unit activities $\{\underline{e}\}$. Technological changes, innovations and the knowledge of the agents appear in the transformation of this set. Development usually increases the set, also some elements also disappear from the set. For instance the set of the activities for my grandmother contained elements, which are missing from my set of activities. She could prepare homemade soap, which knowledge is missing for me,
4. budget constraint is present only in the form, that the money stock must be non-negative. Loans, dept has to be handled as goods, naturally with negative value,
5. reasonable information. Agents do not know all possible activities, which are available to him/her, he/she works only as the known part. Similarly he/she does not know all the partners. It is reflected in the force law, J^{ab} is zero for the unknown partners,
6. selection is done by the force law. Agents want to select outcomes that increase their wealth, and to avoid losses (on the other hand it is not assumed that more of any good is always preferable to less),
7. agent is characterized by the function $Z^a(\underline{X})$ and in linear cases with the motivation \underline{L}^a (in general case the $J = J(F)$ relation).

For the mathematical model we apply the mathematical postulate. Mathematical representation of HSE needs the knowledge of the following quantities:

1. quantities of the stock of goods, \underline{X} owned by the agent,
2. wealth function Z ,
3. activity set $\{\underline{e}^{aK}\}$,
4. motivation $\underline{L}^{a,K}$.

Unfavourable outcomes cannot always be avoided, because even if the agent has perfect knowledge, the result of decision will be the expected one, which is not always the case, circumstances and the knowledge can change between decision and outcome. The longer the delay between decision and consequence, the more likely it is that the outcome will be different from the expected outcome. The difference between expectation and reality can be unfavorable in terms of the agent’s values and priorities. In this case the agent will (or may) change the valuation, or the force law – on the basis of the reliability of the partner. This type of modifications can be done based on the knowledge change. Observing the success and failure of other agents can be sources of changes in the form of valuation expressed by Z . Knowledge changes the activity set, which may modify also the valuation and the force law. Useful knowledge (and skill) accumulates as a result of experience and inadvertent learning, but the learning process cannot be avoided.

In economic models these changes must be incorporated, that is the description is inherently in the developing path. Nevertheless it is a reliable program that in the first stage of the implementation this development is neglected, and then the equation system will describe the market mechanism.

CONCLUSIONS

Homo Sapiens Economicus offers a less restrictive mathematical representation of rational choice theorem, than Homo Economicus. Homo Sapiens Economicus allows us to observe the valuation of the agents. In this approach it is possible to build a dynamic economics, which was the intention.

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REMARK

¹It is really similar to the marginal rate of substitution (MRS), the main difference is MRS is always relative, it is comparison the goods, while Z-value is a parameter for one good.

²He is a cynic, by the definition of Oscar Wilde: “What is a cynic? A man who knows the price of everything and the value of nothing”.

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PREISPITIVANJE TEORIJE RACIONALNOG IZBORA

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SAŽETAK

Značenje teorije racionalnog izbora je u društvenim znanostima odavno osporavana. Kritičari su mnogo vremena posvetili uvjerljivom ali i nedohvatljivom konceptu racionalnosti. Kritike su najviše upućivane temeljima teorema korisnosti. Već se nekoliko radova o krivoj uporabi matematike u ekonomiji pojavilo u literaturi. Kao što je N. Bouleau izjavio, "U nekoliko slučajeva, međutim, osjeća se kritika kako je matematika krivo uporabljena i kako mora biti uporabljena u drugom pravcu (npr. u pravcu statističke analize financijskih nastojanja polarizacije bogatstva i prihoda, ili u pravcu mehanizama pozitivne povratne veze, itd.). Na filozofskoj razini to ostavlja nezadovoljstvo." Cilj ovog rada je prezentirati teoriju odlučivanja, namjere prisvajanja i vrednovanja. Predstavljamo novu matematičku reprezentaciju teorije racionalnog izbora koja vodi do dinamičke ekonomske teorije. Razmatramo filozofske i meta-ekonomske probleme koji su potrebni za uspješnu primjenu matematike.

KLJUČNE RIJEČI

teorija racionalnog izbora, dinamička ekonomska teorija