MULTI-EQUILIBRIA REGULATION AGENT-BASED MODEL OF OPINION DYNAMICS IN SOCIAL NETWORKS

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ABSTRACT

This article investigates the Multiple Equilibria Regulation (MER) model, i.e., an agent-based simulation model, to represent opinion dynamics in social networks. It relies on a small set of micro-prerequisites (intra-individual balance and confidence bound), leading to emergence of (non)stationary macro-outcomes. These outcomes may refer to consensus, polarization or fragmentation of opinions about taxation (e.g., congestion pricing) or other policy measures, according to the way communication is structured. In contrast with other models of opinion dynamics, it allows for the impact of both the regulation of intra-personal discrepancy and the interpersonal variability of opinions on social learning and network dynamics. Several simulation experiments are presented to demonstrate, through the MER model, the role of different network structures (complete, star, cellular automata, small-world and random graphs) on opinion formation dynamics and the overall evolution of the system. The findings can help to identify specific topological characteristics, such as density, number of neighbourhoods and critical nodes-agents, that affect the stability and system dynamics. This knowledge can be used to better organize the information diffusion and learning in the community, enhance the predictability of outcomes and manage possible conflicts. It is shown that a small-world organization, which depicts more realistic aspects of real-life and virtual social systems, provides increased predictability and stability towards a less fragmented and more manageable grouping of opinions, compared to random networks. Such macro-level organizations may be enhanced with use of web-based technologies to increase the density of communication and public acceptability of policy measures.

KEY WORDS

CLASSIFICATION

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INTRODUCTION

In recent years, social sciences have embraced simulation techniques as a new powerful tool to explore the dynamics of social systems. Agent-based models (ABMs) constitute a fruitful approach to simulate and analyze complex phenomena observed in social networks. They typically rely on a set of simple rules pertaining to the behavior of agents, in order to determine the minimal conditions under which these phenomena emerge. A basic problem encountered by researchers is that of understanding emergence and, especially, the relationship between micro and macro properties of complex systems [1, 2]. Such systems can be described either in terms of the properties and behavior of their individual agents or the system as a whole. The explanation of the emergence of macroscopic societal regularities, such as norms or price equilibria, from the micro level behavior of agents requires some *generative* ('bottom-up') mechanism [3], through which decentralized local interactions of heterogeneous autonomous agents generate the given regularity.

In this context, ABMs of social networks can simulate the emergence of community-wide economic and political outcomes, based on the individual behavior and interaction dynamics of network agents. The agents can refer to consumers/voters, firms/political parties, and market, regulatory and administrative authorities. The outcomes may correspond to a diverse range of (desired or strategic) states, like the resolution of conflict situations and achievement of consensus to economic measures, political decisions or social actions concerning specific population groups. Other applications with economic perspective encompass the study of interaction dynamics among consumer agents [4] as well as among company executives within a firm and between different firms [5], to represent changes in organizational structure, price formation and competition conditions in the market. Furthermore, such models can provide insight into agents' voting behavior, the rise and fall of political parties and others.

The interaction dynamics depends on the topology of communication between agents, as the degree of connectedness and position (or centrality) of each (type of) agent in the network can decisively affect final outcomes, in terms of efficiency, equilibrium and other network properties [6-10]. Specifically, agents change/update their own opinion about a subject (e.g., an economic perception about an investment decision or a political view), in accordance with some type of learning process, which will lead to the formation of a belief on that subject and affect their final decision. At the macro level, this process, referred to as *social learning* [11], effectively aggregates information about individual opinions and beliefs, based on own-experience, communication with others, and observation of others' actions, to result in a (range of) uniform opinion(s) or social belief(s) about some economic or social situation. The ABM simulation of that process in social networks can help us to obtain a deeper understanding on how information propagates through the network and people form their beliefs and learn from each other. In particular, it allows investigating how the action of different hierarchical corporate structures, advertising, media and political and other institutional agents (opinion leaders), which give rise to alternative communication topologies, can influence opinion and belief formation (social learning process) in the network.

In the current literature of social ABMs, the final state that represents a specific economic or social situation typically emerges as a single system-aggregate and stationary equilibrium regime. On the contrary, this article builds on the concept of Multiple Equilibria Regulation (MER), which allows for the impact of both the regulation of intra-personal discrepancy and the interpersonal variability of opinions between agents on the social learning and network dynamics. The MER model constitutes an agent-based simulation model of opinion dynamics, which generates some types of macro-outcomes that have not been observed

before in the literature. These outcomes emerge from a small set of local-micro prerequisites and reflect the 'struggle' of agents to equilibrate their interactions both socially and internally. Although in a macro view, individuality (and heterogeneity) may be completely suppressed, in a micro view, individuality is always present. None of the agents used in the following simulations has the same trajectory with another. For a psychologist centered in individuality, the trajectories of all the individuals have nothing in common between them, while, for a sociologist, the formation of a 'group' closely relates to the behavior of agents and may end up in a consensus. The primary aim of the article is to investigate, through the MER model, the role of different types of network structures (topologies) on opinion formation dynamics and the overall evolution of the system.

MODELS OF OPINION DYNAMICS

This section reviews the literature and presents a concise comparison of the MER model, originally introduced in [12, 13], with three other well-known representative ABMs of opinion dynamics, i.e., those of the Axelrod's *Dissemination of Culture* (DoC) [14], Latané and Nowak's *Dynamic Social Impact Theory* (DSIT) [15] and Hegselmann and Krause's *Bounded Confidence* (BoC) [16]. The principal aim is not to investigate and compare the models in full length, but mainly to present their basic properties and characteristics (see Table 1), in order to clarify the resemblances and differences with the MER model and facilitate its analytical presentation in the next section. In this table, the properties of the MER model are primarily based on the adoption of a Cellular Automation (CA) topology (whose description is provided later in the text) to represent the position of and interactions among agents. However, it is noted that several other network structures or topologies can be well adopted (see later). DeMarzo, Vayanos, and Zwiebel [17] dealt with general network structures by assuming that agents follow a specific belief updating rule and (erroneously) treat new iterations of information as independent of previous iterations. They reported an intuitive relationship between the position of an agent in the network and the resulting impact on beliefs and opinions. The aforementioned studies constitute important steps in developing a more sound understanding of how interaction structure affects information, dissemination and belief formation.

All four models of opinion dynamics generate group formations, that is, distinct patterns of opinions' holders. More specifically, Axelrod's model generates clustering and survival of a number of cultures, by supposing that agents who are similar to each other are likely to interact and then become even more similar. Latanè and Nowak's model generates the survival of the minority and is organized in spatial clusters, by supposing that agents are influenced by the persuasiveness of the group members, the 'social distance' from the other agents and the number of group members. The Bounded Confidence model generates either consensus or polarization or fragmentation, supposing that agents tend to adopt the opinions of other agents that are similar to their own (within a bound of confidence). Under certain conditions, the MER model generates a chaotic society that never rests in a final steady state. The resulting clusters are continually transformed and agents usually change clusters. The latter model allows producing and examining competing micro-specifications of patterns of opinions which have equivalent generative power [3], i.e., their generated macro-structures fit the macro-data equally well.

As it is shown in Table 1, the crucial difference of the MER model, in relation to the other models, lies on simulating the intra-agents' behavior, i.e., regulation of intra-personal discrepancies in the opinion-making of each agent in order to balance internally. According to the settings of parameters and locality in communication, the outcome of the MER model is unpredictable [18] and it may never end to a final (stationary) state, compared to all the other

Table 1. Comparison of the four agent-based models of opinion dynamics or social influence, along their sequential steps (continued on the next page).

Model I.	Model II.	Model III.	Model IV.
Axelrod's Model of Dissemination of Culture (DoC)	**Latané's and Nowak's Model of Dynamic Social Impact Theory (DSIT)**	**Bounded Confidence Model (BoC)**	**MER Model**
1. Problem addressed			
How many cultural regions will survive in a society	The problem of consolidation: how and when minorities will decline or disappear and when they will survive or even grow	The classical question of reaching a consensus or disagreement leading to polarization; how many clusters of opinions will survive	The dynamics of opinions in an agent--based simulated society (and the property of unpredictability)
2. Random initial state			
Sequential updating		**Synchronous updating**	
Algorithm with stochastic processes		**Deterministic algorithm**	
3. Number of agents			
4 up to 10,000[1]	1600	100	100
4. Properties of agents			
Discrete opinions: each agent has a culture composed of five features; each feature has ten traits and the value of the culture is discrete, an integer between 1 and 99999	Discrete opinions: each agent has a binary opinion: yes or no (the value of the opinion is discrete, 0 or 1)	Continuous opinions: each agent has an opinion that is a real number belonging to the interval [0, 1] (continuous value)	Continuous opinions: each agent has two opinions. Opinions are real numbers from the interval [0, 1] (continuous value) and are considered to be 'opposite' to one another (structure)
5. Inter-agents' behavior algorithm			
Each agent is influenced by (i) other agents in the proximity and (ii) the agents that have a similar culture (the degree of similarity increases the probability for having an interaction)	The 'impact' of a group of agents on an individual agent is a multiplicative function of the 'persuasiveness' of the group members, their 'social distance' from the individual and the number of members	Each agent is influenced by other agents that (i) have opinion inside a bound of confidence and (ii) are located in the proximity (and same locality)	Each agent is influenced by other agents that (i) have opinion inside a bound of confidence and (ii) are located in the proximity (and same locality)
6. Intra-agents' behavior algorithm			
None	None	None	Each agent assesses his/her own opinion and makes changes to it to balance internally
7. Results-emergent properties			
Clustering			
Local convergence can generate global polarization			
Predictable after simulation			*Unpredictability*; the model is chaotic[2]
Ending in a final steady state-Static equilibrium achieved			**Never ending in a final state[2] – Dynamical equilibrium**

Table 1. Comparison of the four agent-based models of opinion dynamics or social influence, along their sequential steps (continuation from the previous page).

Model I.	Model II.	Model III.	Model IV.
Axelrod's Model of Dissemination of Culture (DoC)	**Latané's and Nowak's Model of Dynamic Social Impact Theory (DSIT)**	**Bounded Confidence Model (BoC)**	**MER Model**
The number of stable regions (or cultures or clusters) reached at the final state increases when: a) the number of features decreases, b) the amount of traits increases, c) the neighborhood size decreases and d) the size of the territory increases. Then, it reaches a maximum and next the number of stable regions decreases again. The simulation ends when each zone has exactly one region. Cultural similarity between adjacent sites in the same cultural zone tends to increase. Boundaries within cultural zones tend to dissolve, but the boundaries between cultural zones tend to be stable.	Opinion clusters emerge and remain in equilibrium, over a wide range of assumptions and parameters. The agents are clustered spatially into cohesive subgroups and the minority survives with minority members located near each other, often near the border.	The number of clusters in the final state depends on a) the magnitude of the bound of confidence and b) the size of the neighborhood[3]. Extreme opinions are under one sided influence and move direction centre. At the extremes, opinions condense. Condensed regions attract opinions from less populated areas within their bound of confidence reach. The opinion profile splits at some points and the sub-profiles (clusters, opinion world, communities) do no longer interact.	The number of clusters formed has not yet been investigated in detail. Since there is not a final state (in some parameter settings) the agents' group membership is not stable. Each agent even if belongs to a cluster does not loose his/her atomism. The clusters move and exchange members on a macro level while, at the same time, the agents move constantly on the micro level as well.
After a certain number of interactions, the agents' society splits into separated 'cultural worlds' or 'opinion worlds' that do no longer interact.			The agents are interwoven with each other. At any iteration, a slight change in an agent's opinion affects the opinions of all other agents after a small number of iterations.

models which finalize in a steady state. The complex dynamics of the MER model is attributed to the facts that the agents' group membership is not stable, since the members are constantly moved and exchanged, and a slight change in an agent's opinion may affect the opinions of all other agents.

THE MULTI-EQUILIBRIA REGULATION MODEL

The main parameters of the MER model are the bound of confidence ε and the intra-regulation factor ψ. The magnitude of ε sets out the proximity rule, so that affects how many 'groups' or 'clusters of agents' opinions are formed. Consensus means that all agents reach

the same final opinion and it takes place for ε values around 0,3 or higher. The polarization signifies those agents' populations that end up divided into two clusters and fragmentation stands for a configuration of more than two clusters of opinions for smaller values of ε. The magnitude of ε does not change the dynamical behavior of the system in almost all cases. Namely, the system can be either (more or less) predictable or unpredictable (especially, when being purely chaotic) regardless of ε. Only if ε is extremely small, e.g. $\varepsilon = 0,01$, can prevent the agents from interacting, in which case the system will remain motionless. In the following example, the confidence bound is set equal to $\varepsilon = 0,1$.

The intra-regulation factor ψ constitutes the so called interior balance correction factor. The magnitude of ψ can affect the opinion clustering and dynamical behavior of the system. A value of $\psi = 0,5$ stands for a type of agent who under-correct his/her opinion. These agents *underestimate* the significance of internal balance and ascribe a minor importance to intra-individual equilibrium. A value of $\psi = 1$ signifies that agents correct their opinions in an *equal manner*. This type of agent has a decision-making structure that assigns an equal importance to both the social and intra-individual equilibrium. A value of $\psi = 1,5$ means that agents over-correct their opinions. Thus, they *overestimate* the significance of internal balance and ascribe a minor importance to social equilibrium.

Let us assume a society of 100 agents, each of them has two initial opinions #1 and #2 concerning the same social/economic/political issue. This contradictory structure of opinions or beliefs for the same issue may be interpreted by the antagonistic co-existence of the cognitive and affectual dimensions of an agent's personality, which may compete to each other; however, this structure may also give raise to various others debatable interpretations in the fields of social psychology, sociophysics, social simulation and complexity. For instance, Tessone and Toral [19] assumed that one preference in some individuals is stronger than the others and this structure changes through the best-fit responses of individual to population dynamics.

The two opinions here follow a structure, wherein opinion #1 goes the other direction than opinion #2. The example used here comes from the transport market and refers to the local public advisory referendum for the imposition of a congestion (or environmental) tax in the city of Stockholm. The citizens, who were asked to vote yes or no, approved (by about 52%) the permanent implementation of the measure of congestion pricing in September 2006, in conjunction with the general election that time, after a trial period of almost seven months. Let us suppose that opinion #1 concerns the no-toll regime (absence of congestion tolls) and opinion #2 the toll regime (congestion pricing). The simulation of personality traits of each agent in the social context is important for such cases and markets, since the affiliation with social networks is limiting choice by accountability to network norms; thus, it can be considered as an efficient decision-making strategy for agents [20].

The MER model relies on the two tendencies of agents towards social and intra-individual equilibrium, which allows the *joint assessment* of both opinions. Several factors Z_i may co-exist and influence the opinions of agent i towards the one or the other direction. On the one hand, an agent i can positively assess congestion pricing because of the expected travel time savings when moving or searching street parking in the city, favorable environmental attitude, anticipated gains due to changes in land values, positive own-experience from the pilot application in the trial period, and positive information or observation from other congestion pricing implementations worldwide [21]. On the other hand, the same agent can negatively assess congestion pricing because of the opposite position of the political party that he/she supports, equity issues, fear of markets, memory lapses, error of perception, stress of information gathering and pressure from social norms [20, 22, 23].
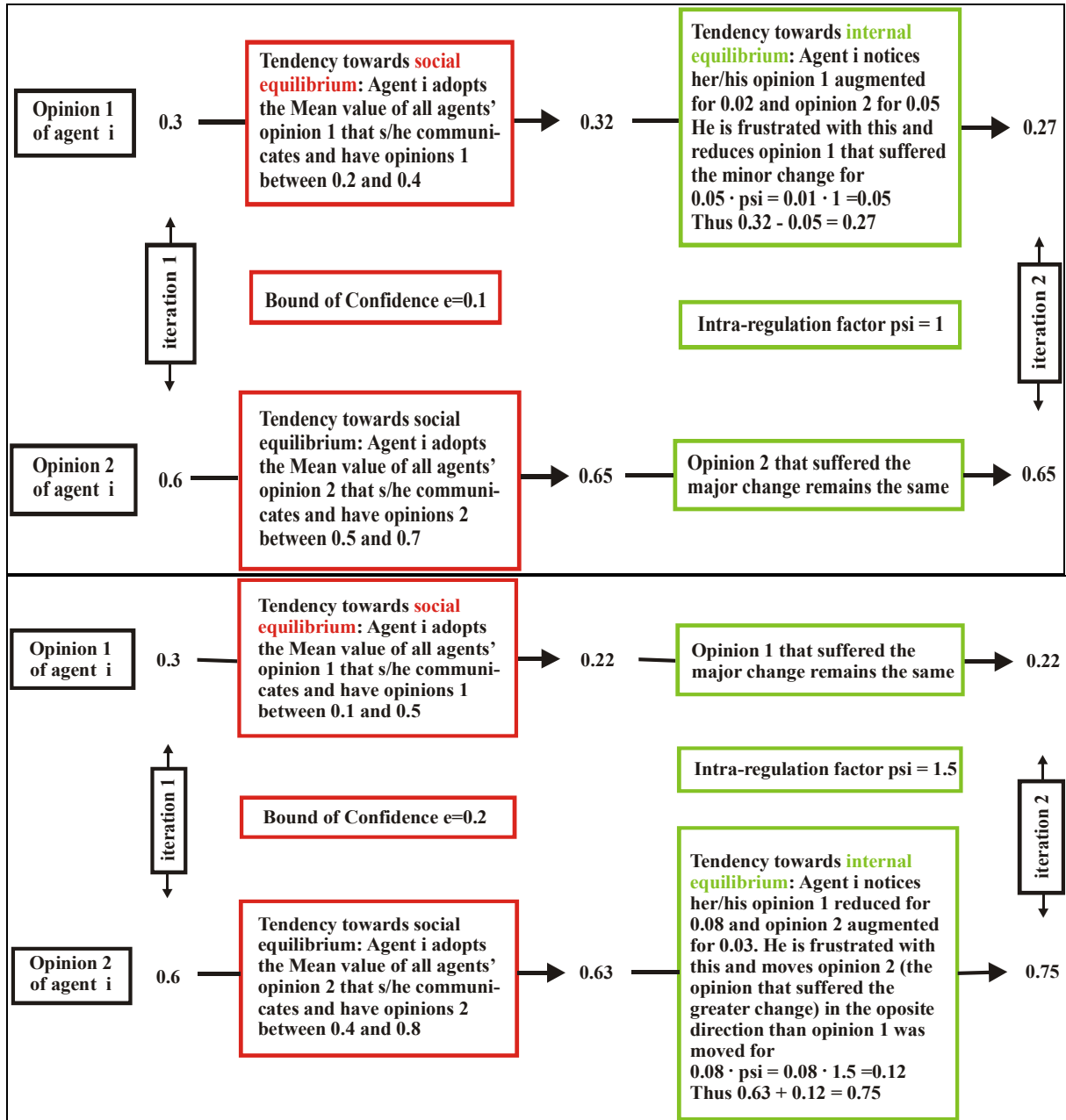
Figure 1. An example of the MER algorithm: a) $\varepsilon = 0,1$ and $\psi = 1$ for updating agent's opinions #1 and #2; opinion #2 is subject to the largest change due to the social influence imposed by other agents; b) $\varepsilon = 0,2$ and $\psi = 1,5$ for updating agent's opinions #1 and #2; opinion #1 is subject to the largest change due to the social influence imposed by other agents.

The opinions are normalized between 0 and 1 and may receive all possible values in this interval. The initial state, as defined by the set of initial values of opinions #1 and #2, can be empirically estimated through a random utility maximization framework, e.g., using a logit-type econometric model, on the basis of a specified utility function $U_i = f(Z_i)$ [23, 24][4].

Due to lack of empirical data, the initial state is produced here by randomly assigning to all agents with two numbers belonging to the interval [0, 1]. These $2 \times 100 = 200$ numbers are produced by a random number generator, namely, a random initial profile is adopted. If an agent's opinion #1 equals 0, then he/she is totally not in favor of the no-toll regime; the opposite holds if his/her opinion #1 equals 1, which means that he/she is a fervent supporter of the no-toll regime.

Let us assume that agent i has opinion #1 equal to 0,3 and opinion #2 equal to 0,6. That means he/she is in loosely favor of the toll regime, but he/she does not reject completely the no-toll regime. Agent i is influenced by all other agents whose opinions is aware of and belong to his/her own proximity and geographic locality (depending on the social and spatial topology of the network, respectively, as will be analyzed later). The proximity/closeness of agents' opinions is regulated by the bound of confidence ε, as suggested in the model of Hegselmann and Krause [16]. It is noted that such continuous opinion dynamics models as the BoC, which are related to negotiation problems or fuzzy attitudes that do not actually match with a yes or no decision, have also been suggested in different versions in the existing literature [25, 26]. In the latter case, the concept of repeated averaging under bounded confidence can involve multidimensional opinions and heterogeneous bounds which may drift the average opinion to extremal opinions.

In the current example, one agent is influenced by those agents with opinion #1 between 0,2 and 0,4 (if $\varepsilon = 0,1$) and with opinion #2 between 0,5 and 0,7. Therefore, the confidence interval ε for opinion #1 is [0,2; 0,4] and for opinion #2 is [0,5, 0,7]. Due to the social influence, the agent i temporarily changes/updates his/her opinion #1 to 0,32 and opinion #2 to 0,65, by calculating the mean values of the same and local others for opinions #1 and #2, respectively. After that, the agent feels frustrated, since he/she believes that both the no-toll regime and the toll regime are better policy options than they were before. The frustration is attributed to the structure (yes or no) of opinions, i.e. opinion #1 goes the other direction than opinion #2. In order to address this frustration, the agent chooses to keep opinion #2, which experienced the largest change (by 0,05), and updates opinion #1 at the opposite direction, by a magnitude equal to the product between the change of opinion #2 and the intra-regulation factor (here, $\psi = 1$), i.e., $0,05 \cdot \psi = 0,05 \cdot 1 = 0,05$; thus, opinion #1 becomes $0,32 - 0,05 = 0,27$. In other words, this opinion-making process gradually makes agent i to weaken the support for the no-toll regime and strengthen the support for the toll regime.

The whole algorithm is described in Figure 1a, while Figure 1b shows a corresponding example where opinion #1 experiences the largest change. In the latter case, where $\psi = 1,5$, the dissonant opinion (i.e., opinion #2) is adjusted by multiplying the maximal difference (of opinion #1), i.e. 0,08, with 1,5, and adding this product to its value, i.e., $0,63 + 0,08 \cdot 1,5 = 0,75$. The addition is due to the move of opinion #1 to the opposite direction. As a result, in the latter example (Figure 1b), the opinion-making process makes agent i to even more weaken the support for the no-toll regime and even more strengthen the support for the toll regime, compared to the former example (Figure 1a). The parameter ψ can take values between zero (where the tendency to intra-individual equilibrium is absent) and infinity. Nevertheless, ψ is considered to be limited theoretically, since values above 2 would be rather 'unrealistic'. This is because by adding or subtracting the double of the maximal difference found in one opinion to the other can be characterized as 'over-reaction'. In order to prevent opinions escaping from the predefined interval [0, 1] and, at the same time, retain the dynamical behavior of the system, a procedure called *rescaling* is applied (for details, see [12]).

DESCRIPTION OF ALTERNATIVE TOPOLOGIES

The MER model is implemented through the use of five typical network structures, which depict alternative topologies of communication between agents. In each case, the agents lie on the nodes of the graph and the edges (links) denote communication. These topologies, which are illustrated in Figure 2, are as follows:

(i) The complete graph topology (CGT), where every agent communicates with and is aware of the opinions of all the others, Fig. 2a). However, the agent is influenced only by those that have opinion included in his/her own proximity, based on confidence interval ε.

(ii) The star (or one-to-all) graph topology, where the central agent has a 'global' view of the system (knowledge of the opinions of all other agents). He/she affects and is influenced by all of them, conditional upon their proximity (Fig. 2b)), while the other agents are (explicitly) influenced only by him/her.

(iii) The Cellular Automata (CA) topology [27, 28], where each agent is posed on a different cell and communicates only with those agents located within a 3 × 3 locality pattern (also known as Moore neighbourhood). This CA topology is shown in Figure 2c), where the larger size indicates nodes with more connections.

(iv) The small-world network topology [29], where most agents are not neighbors, but they can be reached from every other through a small number of hops or steps (denoted as L). Figs. 2d) and 2e) depict two small-world networks with $L = 3$ and $L = 6$, respectively[5].

(v) The random graph topology results from randomly assigning links to various nodes (agents). Figures 2f) and 2g) illustrate two random graphs which have been generated by assuming that every possible link occurs independently with (uniform) wiring probability $wp = 0,10$ and $wp = 0,50$, respectively[6].

The CGT, star and CA networks can be generally regarded as theoretically extreme cases of real-life social networks. In practice, two (or more) individuals may never communicate just because they will never meet each other. Even with the advent of high-technology communication devices and internet/software, such as the web 2.0, the ubiquitous interaction of all agents in a society (as reflects in CGT) can be considered as practically impossible. Besides, agents are not typically isolated and forced to communicate with just a 'leader' agent. Such an extreme case (as reflects in star topology) would possibly happen in the presence of a powerful central leader (e.g., a 'dictator') who prohibits any physical (face-to-face) contact and cuts every possible distant communication among individuals. Lastly, geographic locality cannot completely constrain the interactions among agents within a community (as implied in CA), since the information and communication technologies have reduced the role of spatial friction on social networking. In contrast with these three types of networks (which are undirected graphs), the small-world and random topologies (which are directed graphs) constitute closer representations of social networks in real-life communities. This is because they consider both geographically close as well as distant interactions between agents with varying degrees of connectivity. Especially the small-world network, through parameter L, can properly take into account the relative influence of geographic proximity (neighborhood) on the formation of network-level interactions among agents. For demonstration purposes, a set of 100 agents is assumed in each network setting. A relatively moderate value of $\varepsilon = 0,2$ is adopted for the confidence bound, and a value of $\psi = 1$ is set for the intra-regulation factor.

THE EFFECT OF TOPOLOGIES ON OPINION DYNAMICS

EXPERIMENTAL SETUP

This section investigates the opinion grouping, dynamics and macro-behavior resulting from running the MER model with the alternative communication topologies (as described in the previous section). Figure 3 shows a three-dimensional representation of the dynamics of opinion 1 and opinion 2 with respect to the number of iterations. In the current context, each iteration can be viewed as a time interval lasting several hours (e.g., day period). In addition to the three undirected graph topologies, i.e. the CGT (Fig. 3a)), the star topology (Fig. 3b))
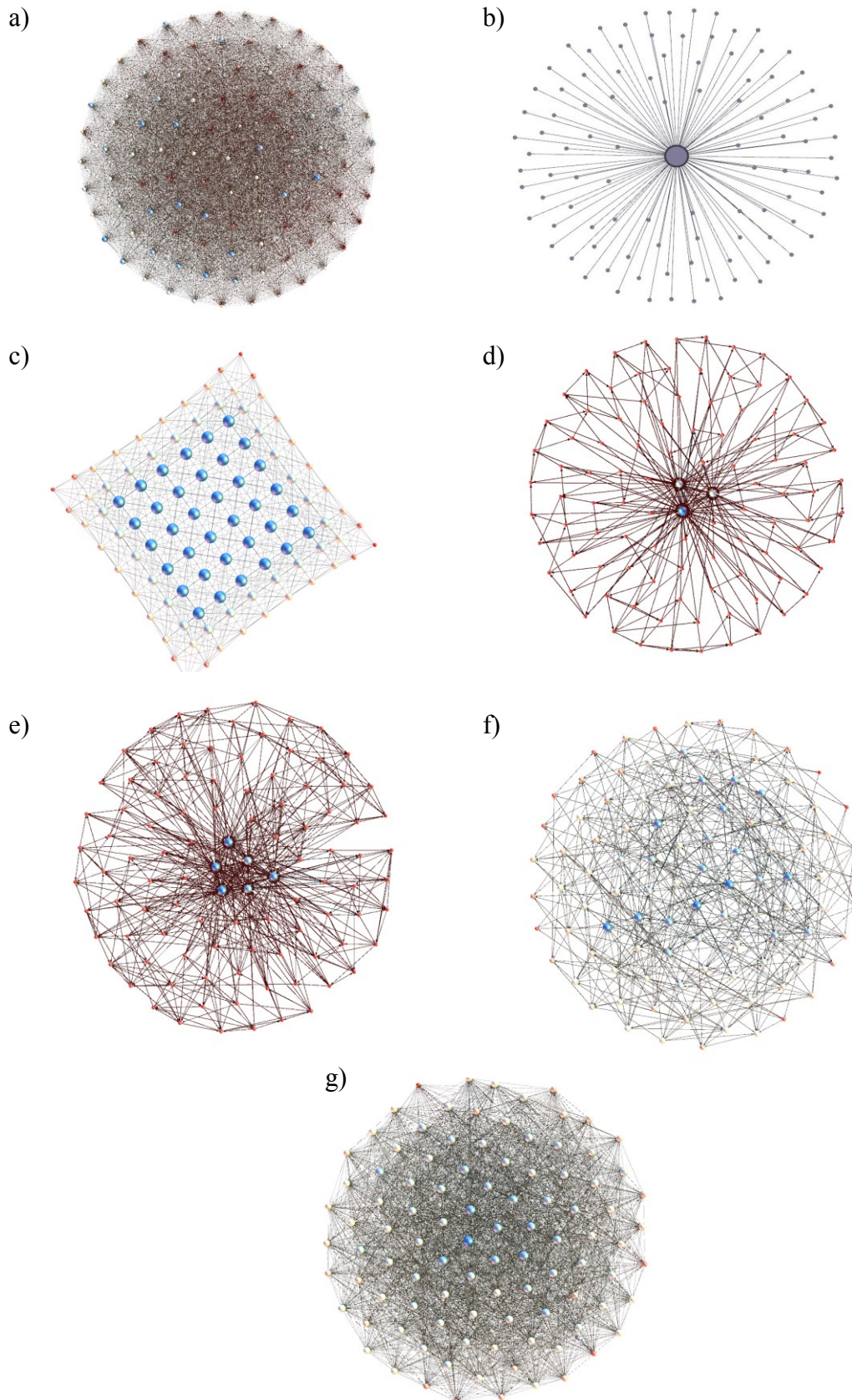
Figure 2. Alternative topologies of communication between agents: a) Complete Graph Topology (CGT); b) Star (one-to-all) topology; c) Cellular Automation (CA) topology; d) Small-world topology ($L = 3$); e) Small-world topology ($L = 6$); f) Random graph topology (wp = 0,10) and g) Random topology (wp = 0,50).
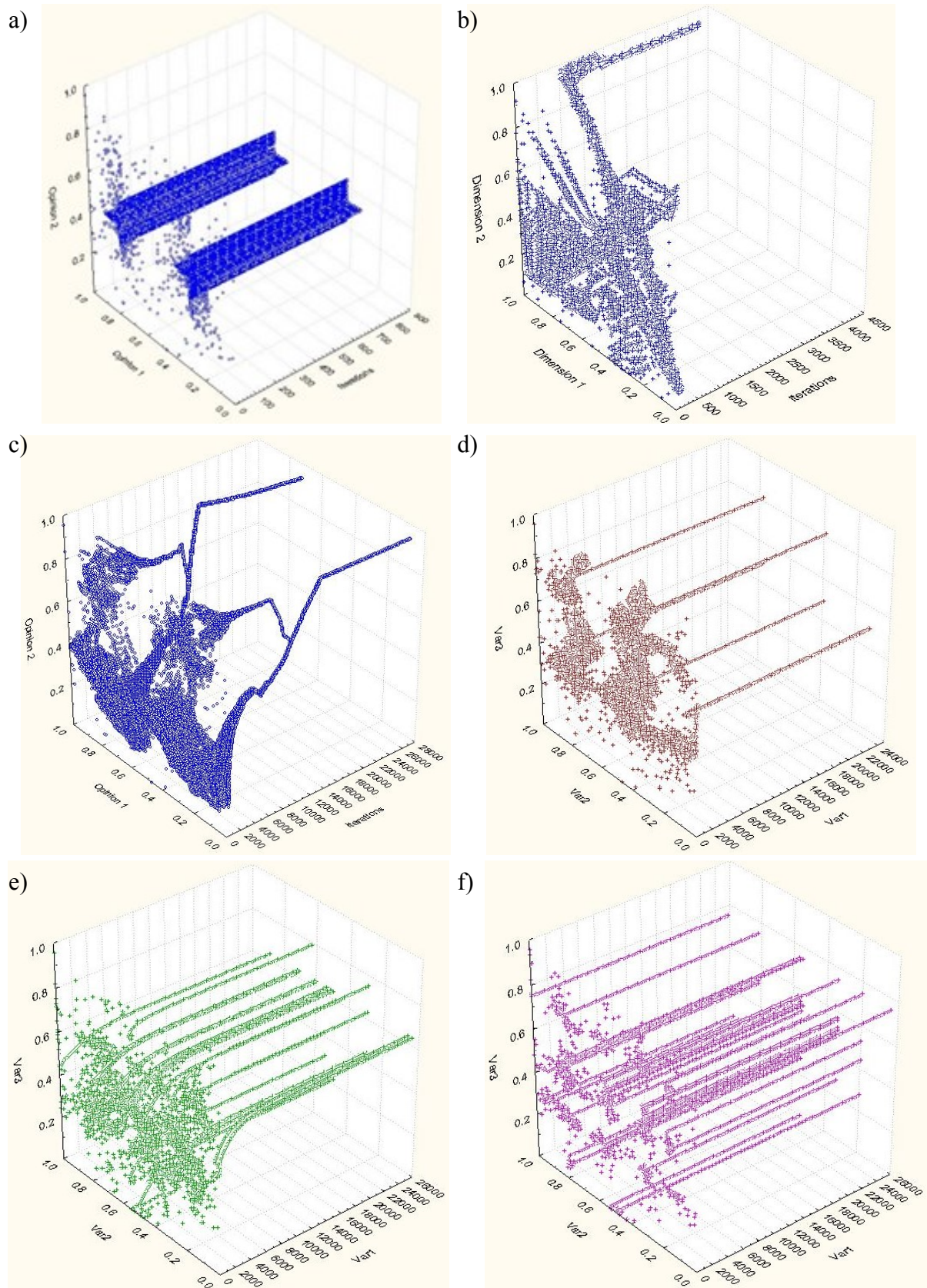
Figure 3. Full dynamics and macro-behavior of the MER model for: a) Complete Graph Topology (CGT); b) Star (one-to-all) topology; c) Cellular Automation (CA) topology; d) Small-world topology (*L*=3); e) Small-world topology (*L* = 6); f) Random graph topology (*wp* = 0,10); g) Random topology (*wp* = 0,50).

and the CA topology (Fig. 3c)), the small-world graph topology is depicted for the cases of $L = 3$ and $L = 6$ (Figs. 3d) and 3e), respectively), and the random graph topology is represented by adopting wiring probabilities $wp = 0,10$ and $wp = 0,50$ (Figs. 3f) and 3g), respectively).Table 2 presents several calculated statistical measures which suggest underlying properties of these network topologies. These measures refer to: (i) *Average in-degree* (or *row degree*), i.e., the average of the connections leading to a node from other nodes, (ii) *average out-degree* (or *column degree*), i.e., the average of the connections leading out from a node to other nodes, which denotes how influential the node may be, (iii) *network diameter*, that is the longest graph distance between any two nodes in the network and indicates how far apart the two most distant nodes are, (iv) *network density*, which measures how close the network is to complete (the CGT has all possible edges and density equal to one), (v) *average clustering coefficient*, which provides an overall indication of the clustering in the network by measuring the probability that nodes are embedded in their neighborhood (typically used to determine whether or not a 'small-world' effect exists in the network), (vi) *average path length*, that is the average graph distance between all pairs of nodes, and (vii) *modularity*, which provides a community detection measure. A better decomposition of the network yields a higher modularity score (although it increases the computational time of processing).

Furthermore, two statistical measures, i.e., the *Lyapunov exponent* and *Information Entropy*, are calculated to determine the sensitivity to initial conditions[7] and chaotic behavior of the model. The Lyapunov exponent denotes the average exponential growth of the error at each iteration and it shows under what conditions the model is sensitive to initial conditions and thus becomes unpredictable. A positive Lyapunov exponent means that even slight perturbations in the system grow over time (nearby opinion trajectories move away), predictability diminishes and chaotic conditions arise. A negative exponent implies a fixed point (nearby opinion trajectories are attracted) or periodic cycle, and a zero exponent indicates a marginally or neutrally stable orbit [30]. The information entropy, whose calculation is based on the Shannon's entropy measure, denotes the extent of possible alternative patterns of organization of the system: as entropy increases, the system becomes less uniform and more disorganized, and vice verse [31].

RESULTS AND DISCUSSION

By and large, the DoC, DSIT and BoC models have been found to result in systems that are self-organized into opinion clusters with a rather predictable behavior. In other words, after a certain number of interactions, the agents' society splits into separated 'cultural worlds' or 'opinion worlds' that do no longer interact; this is a reason that all these models finalize in a steady final state. The resulting configurations are – although emergent – stable and unchanging. On the contrary, the MER model presents a more complex set of results, which vary from a typical steady final state to an ever-changing pure chaos, heavily depending on the social network structure.

In the CGT, the opinion trajectories are polarized in a stable final state (periodical), within the first few hundred iterations, by forming two major opinion clusters (probably due to $\varepsilon = 0,2$; see also the BoC model). The CGT creates a single community (i.e., a global common neighborhood) with the highest density and average clustering coefficient, and the shortest average path length (all equal to one), compared to the other topologies. It also yields the lowest level of organization (or the most increased disorganization), together with the random graph topology with $wp = 0,1$, as reflects the measure of information entropy (equal to 4,05 and 5,30, respectively). Besides, the CGT is the least sensitive to changes in the initial conditions (hence, the most predictable), in relation to the other two undirected graph

Table 2. Statistical measures of alternative communication topologies of the model.

Character	Undirected			Directed			
Model	CGT	CA Square, Radius 2, Moore neigh.	Star	Small world, $L = 3$	Small world, $L = 6$	Random network, wp = 0,10	Random network, wp = 0,50
Implemented Social Network Characteristics[8]							
Average In-degree	49,5	9,18	1,98	3,0	6,0	4,92	24,38
Average Out-degree	49,5	9,18	1,98	3,0	6,0	4,92	24,38
Diameter	1	5	2	44	20	7	4
Density	1	0,185	0,02	0,03	0,061	0,049	0,24
Average Clustering Coefficient	1	0,638	0	0,24	0,236	0,048	0,248
Average Path Length	1	2,598	1,98	15,40	7,287	2,522	1,498
Modularity coeff. /communities	0/1	0,45/5	0/1	0,40/11	0,36/8	0,24/13	0,0/1
System Dynamics[10]							
Lyapunov Exponent[9]	$\approx 0^-$	+0,197	$\approx 0^+$	$\approx 0^+$	$\approx 0^+$	–0,11	–0,027
Information Entropy[11]	4,05	1,34	1.08	2,36	2,01	5,30	2,95
Dynamical Assessment	Stable	Pure Chaos	Transient chaos[12]			Stable	

structures (the CA and star topologies). This is because all the agents communicate with each other and have knowledge of the moves of the others, although they are unaware of the number of opinion clusters formed and how each cluster departs from their own.

The CA topology is found to yield a system that is the most unstable or sensitive to initial conditions (Lyapunov exponent is equal to 0,197), but the most organized one, together with that produced by the star topology (with entropy values equal to 1,34 and 1,08, respectively). Thus, a policy planner would possibly prefer to control the network (hence, the outcome of a referendum), through imposing a central agent[13] that communicates with and influences all the others who rest communicate only with him/her, to maximize the system's organization and make it more predictable, compared to establishing only local communication between (neighboring) agents (the case of CA). The current finding is consistent with the notion of a 'dictatorship' that ends up with a heteropolar (bipolar) equilibrium [32], as generated by a process of social influence, which was explicitly neglected in the fundamental result of social choice theory [33].

The random graph topology is found to be the least sensitive to initial conditions (Lyapunov exponent is equal to –0,11, for *wp* = 0,10, and –0,027, for *wp* = 0,50). Particularly for *wp* = 0,10, the results (Table 2) suggest that the system reaches a stable but highly disorganized final state with multiple small opinion clusters, where 13 neighborhoods are formed. Therefore, for

the given parameter settings, assuming a random communication topology would move the system far away from a socio-economic consensus of consumers/voters to fiscal measures (such as the congestion tax). On the other hand, the small-world networks (both with $L = 3$ and $L = 6$) are found to produce systems that are relatively stable (predictable, with transient chaos, having Lyapunov exponent very close to 0), and considerably more organized (less fragmented) than the random networks.

Compared to the random graph (for $wp = 0,10$), the small-world networks are composed of fewer communities (i.e., 11 for $L = 3$ and 8 for $L = 6$), but they have a considerably higher average clustering coefficient and path length. These results suggest that a small-world organization of the social network, through creating highly clustered groups of agents that are a few steps away from each other, would enhance both its predictability and stability towards a less fragmented (and hence more manageable) grouping of opinions. Such a type of organization is typically met in several real-life social and artificial networks [29, 34], particularly those of sites extracted from the web [35], since they can arguably depict more realistic aspects of them, with regard to common social relationships among individuals.

CONCLUSIONS

This article aims at offering some new insights regarding the dynamics of complex societies: stability is the word of the day in the middle of a fierce economic (and social) crisis. Several economic and social policies are designed to treat the impact of crisis and diminish their adverse effects, including opinion conflicts, to achieve the widest possible acceptance. The MER model relies on a logic of simplicity, that of formalizing two psycho-social principles in terms of a methodological individualism[14]. Simple micro-specifications, including the tendency of deterministic rational agents towards intra-individual equilibrium and their bound of confidence, as well as the topology of communication, are sufficient to generate macro-structures of interest. Equilibrium is a motive: all agents are searching to attain synchronously a state of stability, whether it is social (inter-individual) or intra-individual. However, because of this quest for two equilibria, unpredictability is generated: everything seems to be negotiated on the edge between social and individual.

On the one hand, based on the proposed methodology for simulating complex systems, different communication topologies (regarding capital flow, voting behavior or even 'simple' opinion change) can produce *radically different* dynamical social behavior patterns. The society of agents is self-organized into clusters (opinion groups in this particular case) that *emerge* at the macro level through properties and interactions from the micro level. Namely, both the agents' properties and social network structure influence the dynamics of the system, which, under certain conditions, may be chaotic, i.e., sensitive to the initial state, unpredictable and ever-changing without resulting in a steady final state.

On the other hand, given that specific topologies ('small world', 'scale free'[15]) are frequently met in real-world conditions, it can be hypothesized that the 'naturally' prevailing occurrence of these types of networks may be due to their dynamical characteristics[16]. Hence, the current findings, in conjunction with others of recent empirical studies concerning the impact of social network structures, can contribute to 'guiding' the behavior and overall stability (or instability) of such systems towards a desired state. Social networks are generally considered as being more difficult (or resistant) to be manipulated or controlled, compared to the physical and technological systems, and control attempts may lead to outcomes very different from the intended or predicted ones. Nonetheless, some topological characteristics that affect their stability and natural tendencies and (self-organizing) behavior, such as density, number of neighbourhoods and critical agents ('driver nodes') can be identified and appropriately treated [36, 37].

The MER model aspires to offer knowledge of the least prerequisites to make the system more robust and predictable. The treatment of unpredictability can be useful for horizons where a specific course of policy actions or design options may be deployed and bring about expected outcomes. It has been shown that unpredictability itself cannot be predicted for complex social systems, at least not in a traditional sense, namely, by comparing successive snapshots of a system's trajectory in the course of time [13]. This is because the esoteric interactions of a chaotic system do usually prevail upon external control or management attempts. But the present model enables the identification of the path dependency and possible occurrence of outcomes which may deviate from a single steady-state equilibrium point in the prediction horizon, in contrast with other relevant models. In the context of a congestion pricing strategy, policy planners and decision makers should organize the information diffusion and learning in the community so that enhance the predictability and stability of the desired outcome (in a final steady state), as well as the management of possible conflicts. Such a macro-level organization may involve the formation of larger-size localities-neighbourhoods and use of web-based technologies to increase the density of communication. The resulting network structure can promote acceptability (or diminish opinion fragmentation) towards the desirable pricing regime, without compromising the democratization of the voting process (e.g., through trying to impose a star communication topology).

At the micro-level, the MER model can help to design targeted policy interventions, through social media campaigning, advertising and public consultation processes, to influence personality traits and relevant parameters of most critical agents in the community. In addition, such processes can affect the agents' perceptions about factors that are (positively or negatively) related to the acceptability of congestion taxation, including time savings, environmental benefits, equity concerns and political aspects. More empirical research in the field could enrich real-life knowledge on the initial opinion formation of consumers/voters, through specifying and validating a general-form utility function, and the structural parameters of the small-world network and distribution of their values.

Specifically, a top-down decision-making approach may be required to deal with practical aspects of the realistic behavior of agents, compared to the present bottom-up mechanisms. Such an approach refers to the catastrophe theory [41], which can be used to determine the set of conditions wherein the agents would finally choose one among the two (or more) competitive options (e.g., no or yes on congestion pricing). This approach can adequately explain and classify abrupt conflict phenomena when a dynamical system reaches or exceeds a bifurcation point. These phenomena are characterized by sudden and dramatic shifts in system dynamics arising from small changes in certain parameters of the agents' behavior and network structure. After the bifurcation, it can help to define multiple dynamical states in which the agents' choices are no longer superimposed and the system can reach stable equilibria or possibly enter into unstable and chaotic conditions.

Last, it is noted that there are essentially numerous potential areas of further research and practical implementation of the proposed modeling framework. In methodological terms, the model can simulate all systems composed of agents (humans, cells, neurons, facilities, institutions, etc.) who exchange information and seek both an internal and external-social equilibrium. By adopting the laws pertaining to the operation of each system, it can simulate, for instance, gene mutation and organism stability in biology, spread of diseases in epidemiology, and synchronization of neurons in memory processes. Especially useful insights can be obtained from simulating social systems operating in highly volatile environments and which relate to self-organization processes and behaviors where determinism and randomness co-exist. Such systems encompass the financial agents' transactions in national economies and stock markets, online trading and auctioning in electronic markets,

the rise and fall of political parties, urban formation dynamics guided by household and firm location choices, and the transport and inventory management in logistics networks.

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REMARKS

[1]The effect of the number of agents on the cultures formed has been investigated in the Axelrod's model; it seems that it does not play a significant role in the Latané and Nowak and the Bounded Confidence models, while this effect has not yet been examined in the MER Model, due to its heavy computational burden.

[2]For some parameters' settings and locality in communication.

[3]Although this statement has not been published, it is easy to observe when running a program with the algorithm of Bounded Confidence model for a CA topology.

[4]It is noted that existing economic models used to assess the acceptability of congestion tax or other pricing measures in transport and other network industries are typically based solely on the maximization of some measure of the utility of consumer agents, ignoring the effects of interaction topologies (social network structures) and personality characteristics at the individual and social level.

[5]Made with Gephi 0.7a© (Watts-Strogatz Small World model A).

[6]Made with Gephi 0.7a© (Random Graph).

[7]he sensitivity to initial conditions also relates to the fact that there is no error-free measurement data and it constitutes a characteristic of the system itself, not a characteristic of the measurement tool (data collection method) applied.

[8]All measurements result from Gephi 0.7a©, http://gephi.org.

[9]Zero implies a value smaller than $| 0,01 |$; the sign is shown in the parenthesis.

[10]100 agents (nodes), $\psi = 1$, bound of confidence $\varepsilon = 0,2$ and run up to 25 000 iterations.

[11]Initial value (random initial profile): 6,246.

[12]A system presents transient chaos when initially the Lyapunov exponents are positive but, after a number of iterations, they tend to zero [38]. This means that the system originally exhibits (even high) sensitivity to initial conditions but, gradually, after a tight self-organization process, it becomes stable and predictable.

[13]For instance, this may be some kind of Mass Media Communication system.

[14]The simplest way of defining methodological individualism is the thesis in which every proposition about a group is, implicitly or explicitly, formulated in terms of the behavior or interaction of the individuals constituting the group [39].

[15]In scale-free networks, some nodes-agents act as highly connected hubs (high degree), although most nodes are of low degree. Their structure and dynamics are independent of the system size. Namely, it has the same properties no matter what the number of nodes is.

[16]For instance, the prevalence of small-world networks in biological systems and the Internet may reflect an evolutionary advantage of such an architecture. One possibility is that small-world networks are more robust to perturbations (due to damage by mutation or viral infection, and random breakdowns, respectively) than other network architectures [40].

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SIMULACIJA DINAMIKE STAVOVA U DRUŠTVENIM MREŽAMA POMOĆU REGULACIJSKIH MODELA S VIŠE RAVNOTEŽNIH STANJA, TEMELJENIH NA AGENTIMA

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SAŽETAK

Rad istražuje modele regulacije s više ravnotežnih stanja, tj. simulacijske modele temeljene na agentima, za prikazivanje dinamike stavova u društvenim mrežama. Model polazi od malog broja mikro-zahtjeva (omeđena uravnoteženost i povjerenje individue) i pokazuje emergenciju (ne)stacionarnih makro-karakteristika. Ti ishodi se mogu odnositi na konsenzus, polarizaciju ili fragmentaciju stavova o oporezivanju (npr. zakrčenost cijenama) ili o drugim mjerama javnih politika već prema načinu na koji je komunikacija strukturirana. U suprotnsoti s drugim modelima dinamike stavova, model omogućuje i regulaciju diskrepancije individue kao i utjecaj varijabilnosti stavova između individua na društveno učenje i dinamiku mreže. Nekoliko je simulacijskih eksperimenata prezentirano radi pokazivanja, kroz model s više ravnotežnih stanja, uloge različitih struktura mreže (potpune mreže, mreže zvijezde, stanični automati, mreže malog svijeta i nasumične mreže) na dinamiku stvaranja stavova i cjelokupnu evoluciju sustava. Rezultati mogu pomoći identificiranju specifičnih značajki topologije (poput gustoće, broja susjeda i kritičnih čvorova-agenata) koje utječu na stabilnost i dinamiku sustava. To znanje može biti upotrijebljeno za bolju organizaciju difuzije informacija i učenja u zajednici, povećanje predvidivosti ishoda te upravljanje mogućim sukobima. Pokazano je kako organizacija malog svijeta,

koja realistično predstavlja neke vidove stvarnog života i virtualnih društvenih sustava, omogućava povećanu predvidivost i stabilnost manje fragmentiranih i više upravljivih grupiranja stavova, u usporedbi s nasumičnom mrežom. Takva organiziranja na makro-razini mogu biti pojačana uporabom mrežnih tehnologija, za povećanje gustoće komunikacije i javnog prihvaćanja mjera javne politike.

KLJUČNE RIJEČI

modeli temeljeni na agentima, društvene mreže, dinamika stavova, topologija komunikacije, nepredvidivost