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INFLUENCE OF MODELLING ON TRUSS STRESS ANALYSIS RESULTS

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Summary

In order to determine the real state of stress in a reinforced concrete truss and to determine the impact of restraint rods in the truss nodes on the stress in the beam, a theoretical and an experimental analysis of stress in reinforced concrete truss were conducted. The analytical modelling is conducted for the idealized truss system with hinges in nodes, and separately for the realistic truss system with rigid nodes. The numerical modelling is conducted using the finite-element method, and the experimental modelling is made using the concrete Warren truss, while a dimensional analysis is used for the comparison of results. In all elements of the realistic truss with rigid nodes, the stress values are greater than those of the corresponding elements of the idealized truss with hinges in nodes, especially in truss elements next to bearings. To gain a more complete insight into the real state of stress in trusses, it is recommended to view the truss structure as a frame structure with rigid nodes, which is in agreement with its real behaviour.

Key words: *truss, analytical and experimental modelling, finite-element method, dimensional analysis*

1. Introduction

Structural analysis for a given structure is normally conducted using an idealized structural system, by which the actual behaviour of that structure is approximated. This idealization of structural system reduces the accuracy of analysis of the stress and strain situation in structural elements, and leads to a situation in which the theoretical behaviour deviates from the actual behaviour of a structure. Depending on the structural system under study, we need to estimate when these deviations can be neglected and when they have a significant role in the analysis of the actual behaviour of structures and a significant influence on the level of safety of structures.

In the structural analysis of trusses, the basic assumption is that members are linked together in nodes formed as pinned connections [1, 2, 5]. However, structural pinned connections are very rarely realized in practice where connections of members in nodes are usually rigid, which is especially true for truss beams. Elements of an idealized truss with pinned connections in nodes are influenced only by longitudinal forces and normal stresses caused by such forces. However, additional bending moments and additional stresses caused by such moments are experienced in real-life situations due to rigid connections of members in nodes of a real truss beam. Extreme normal stresses occur in the end sections of members and in node areas, and they exert a certain influence on the level of safety of the truss.

The influence of modelling on the real state of stress and strain in beams will be analyzed using a reinforced-concrete Warren truss shown in Fig. 1.

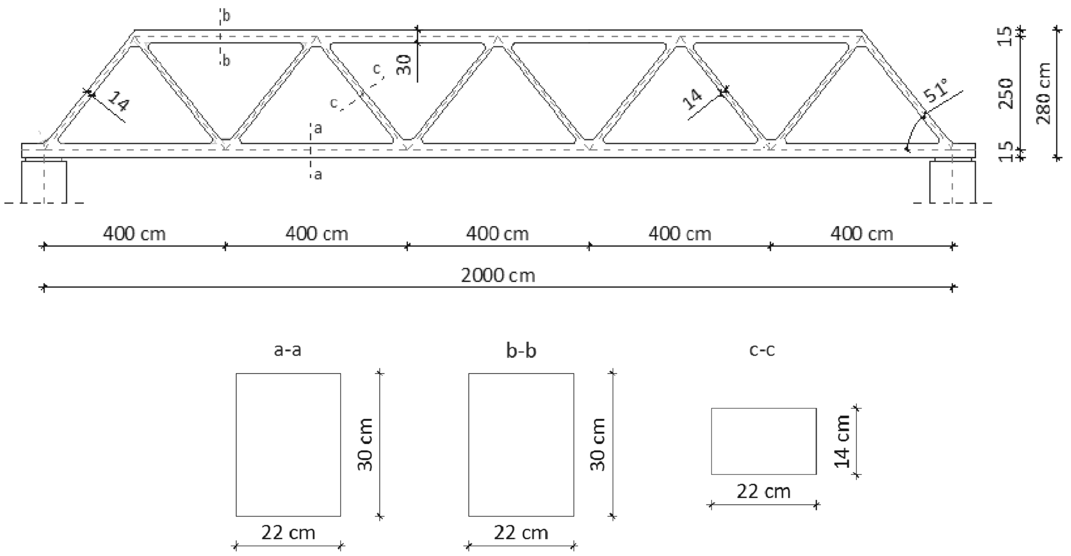


Fig. 1 Reinforced-concrete Warren truss

The analysis of stress in truss was conducted theoretically on three models: analytic calculation of an idealized truss system (in-plane truss with pinned connections) [2], analytic calculation of real truss system (in-plane frame, truss with rigid connections) [6], and finite-element method [14, 16]. The analysis was also conducted experimentally [3, 4, 6, 7, 10, 11] by laboratory testing of a medium density fibreboard truss model.

The analysis and experimental testing of truss beam were conducted for symmetrical load with forces F in the nodes of the upper chord of the beam.

The experimental testing of the beam [3, 4, 6, 7, 10, 11] was conducted on a medium density fibreboard scale model. Results obtained by experimental testing were compared with the results previously obtained by calculation based on dimensional analysis [13, 15], and the influence of modelling on the beam safety level is presented through a comparison of all results.

2. Analytical modelling of a truss

2.1 Idealizations during truss modelling

The simplest analytical examination of a truss is the analysis of an idealized truss according to the theory based on the following assumptions [1, 2, 3]:

- members are characterized by regular shapes and constant cross section,
- members are linked together in nodes via ideal hinges (without friction),
- loads are defined in nodes of the system,
- material is ideally elastic,
- hypothesis of small displacement and small strain values is applied,
- balance is in an ideal (initial, non-deformable) condition.

Truss elements assume compressive and tensile longitudinal forces only, while all transverse forces and internal bending moments are equal to zero. Links between elements are established as ideal hinges, and hence the mentioned internal force distribution is enabled. However, in real-life situations, it is very difficult to realize an ideal pinned connection where gravity axes of members and centres of gravity of connections are linked precisely at a single point of a node. Additional bending moments occur as a consequence of eccentricity and pinned connection stiffness. Together with additional bending moments occurring during realization, we should also take into account bending moments occurring due to load acting directly on elements between nodes, e.g. bending as a consequence of the weight of the member, or bending due to wind action and snow load.

In Section 2.2, the truss is calculated as an idealized plane truss beam based on the previously mentioned assumptions [1, 2], while in Section 2.3 it is calculated as a more realistic truss beam with rigid nodes [5, 18].

2.2 Analytical solution for an idealized truss (pinned connections in nodes) [1, 2]

The model analyzed using the node method based on the truss theory [1, 2], with longitudinal forces in beam elements, is presented in Fig. 2. Stress values in truss elements are defined by the expression $\sigma_0 = \frac{N}{A}$.

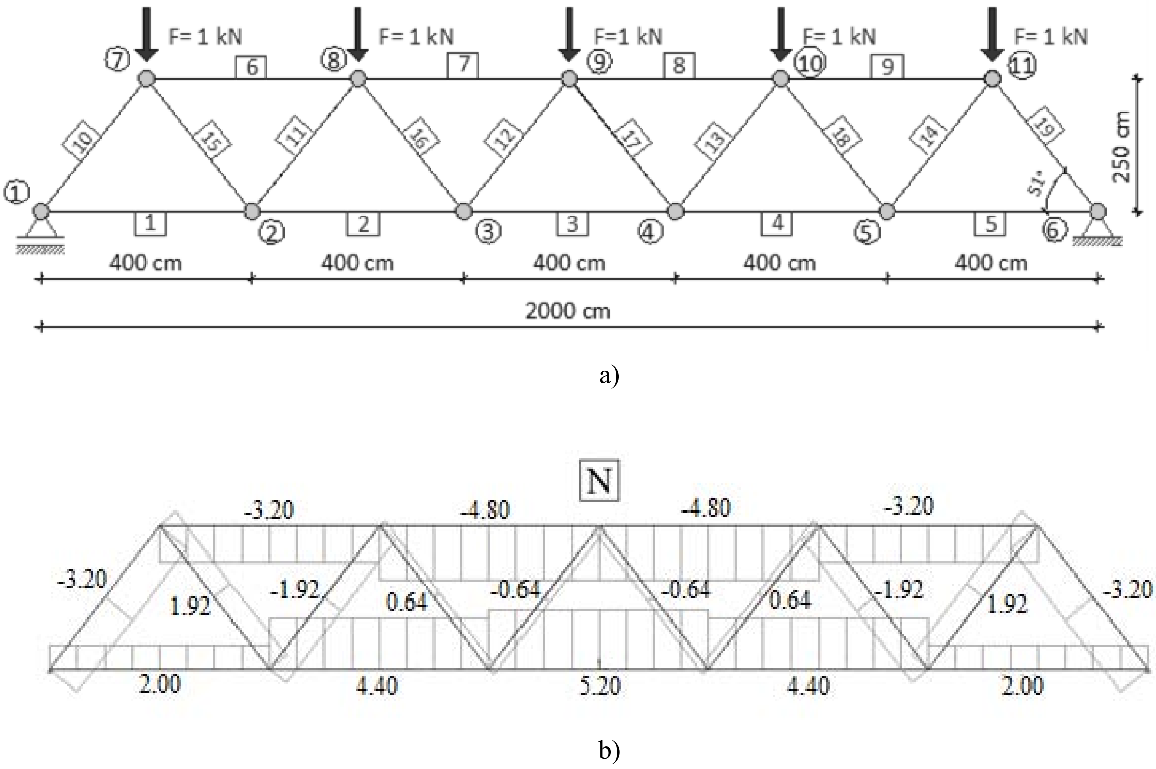


Fig. 2 a) idealized model of a truss subjected to symmetrical load.
b) diagram of longitudinal forces N[kN] ("-" compression, "+" tension)

2.3 Analytical solution for a real-life truss (rigid joints in nodes)

The model calculated by computer [5], with longitudinal forces and bending moments in beam elements, is presented in Fig. 3. Stress values in beam elements are defined by the expression $\sigma_1 = \frac{N}{A} \pm \frac{M}{W}$.

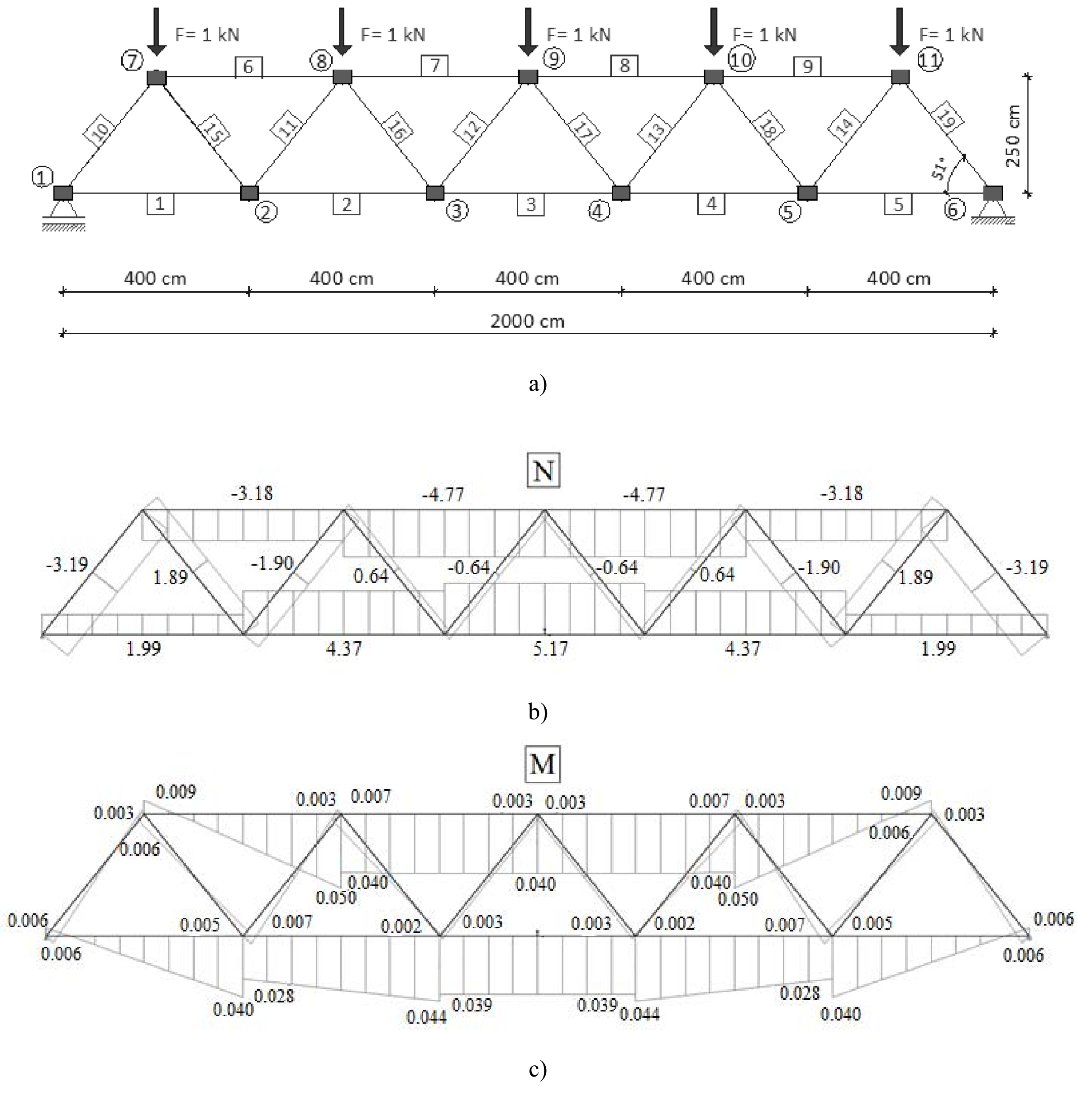


Fig. 3 a) realistic truss model, b) diagram of longitudinal forces [kN] ("-" compression, "+" tension).
c) diagram of bending moments [kNm]

2.4 Comparison of results

Calculated stresses and a comparison of additional stresses σ_1 based on basic stresses σ_0, in typical truss sections for the symmetrical beam load of F = 1.0 kN are presented in Table 1.

Table 1 Comparison of basic and additional stresses in beam, for the symmetrical beam load of F=1.0 kN ("-" compression, "+" tension)

Member	Cross-section	Stresses [MPa]		$\frac{\sigma_1 - \sigma_0}{\sigma_0} \cdot 100\%$
		σ_1	σ_0	
1 & 5	1-2, 6-5	0.0320909	0.0303030	5.90
	2-1, 5-6	0.0426363		40.70
2 & 4	2-3, 5-4	0.0748637	0.0666667	12.30
	3-2, 4-5	0.0794091		19.11
3	3-4, 4-3	0.0901970	0.0787879	14.48
6 & 9	7-8, 11-10	-0.0509697	-0.0484848	5.13
	8-7, 10-11	-0.0633636		30.69
7 & 8	8-9, 10-9	-0.0843636	-0.0727273	16.00
	9-8, 9-10	-0.0845151		16.21
10 & 19	1-7, 6-11	-0.1121660	- 0.1039610	7.89
	7-1, 11-6	-0.1077134		3.61
11 & 18	2-8, 5-10	-0.0715677	- 0.0623701	14.75
	8-2, 10-5	-0.0661410		6.05
12 & 17	3-9, 4-9	-0.0242578	- 0.0207792	16.74
	9-3, 9-4	-0.0242578		16.74
13 & 16	4-10, 3-8	0.0232189	0.0207792	11.74
	10-4, 8-3	0.0305937		47.23
14 & 15	5-11, 2-7	0.0684600	0.0623701	9.76
	11-5, 7-2	0.0701298		12.44

It can be seen from Table 1 that stress values obtained by an analytical procedure vary depending on the way the truss is modelled. Additional stresses σ_1, as related to basis stresses σ_0, are up to 40.70 percent greater at truss chords, and up to 47.23 percent greater at the truss fill zone.

3. Numerical modelling of truss

Numerical modelling was conducted by means of the finite-element method [12, 17, 18] using the SAP computer program [14, 16]. The elements are connected with rigid joints, as this is more realistic. The truss is divided into 2088 elements, out of which 704 rectangular elements form the lower chord, 572 rectangular and 12 triangular elements form the upper chord, and 800 rectangular elements form the fill (one fill is made of 80 elements), as shown in Fig. 4. Stress diagrams for typical cross sections of a symmetrically loaded truss are presented in Fig. 5 [16, 17, 18]. Displacements obtained by the finite-element method [16, 17, 18] are presented in Fig. 6.

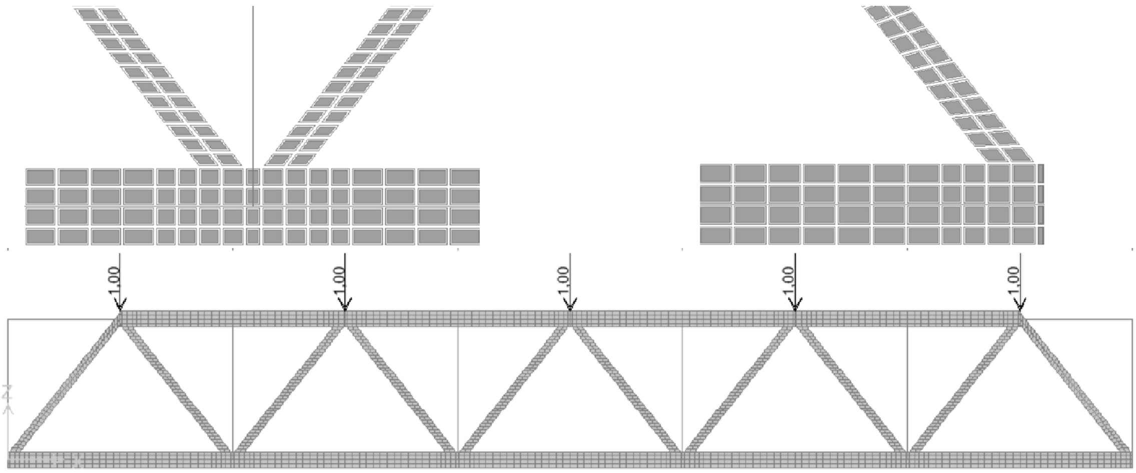
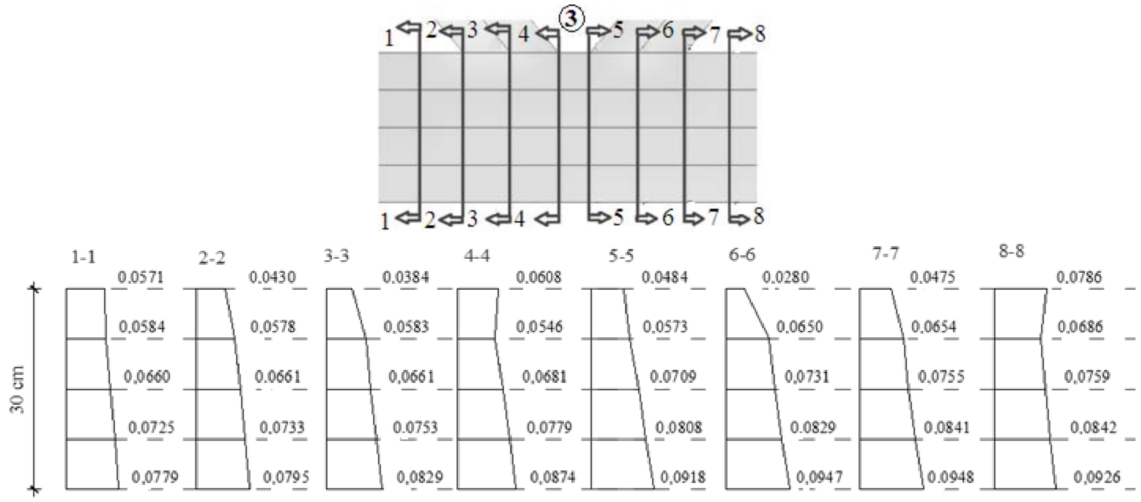
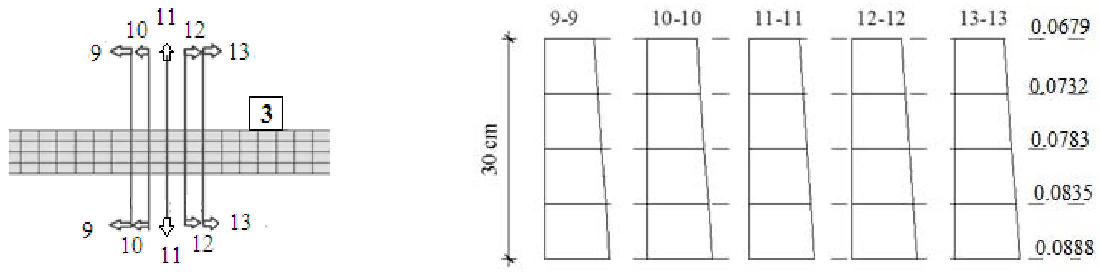


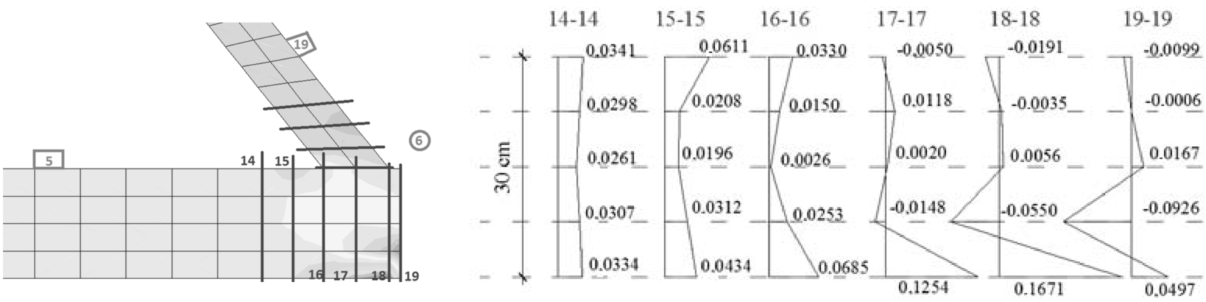
Fig. 4 Truss model calculated using the finite-element method [16, 17, 18], subjected to symmetrical forces F[kN]



a) Stresses σ[MPa] near the third node [16, 17, 18]



b) Stresses σ[MPa] at the third chord element [16, 17, 18]



c) Stresses σ[MPa] at the sixth node [16, 17, 18]

Fig. 5 Stress diagrams obtained by the finite-element method

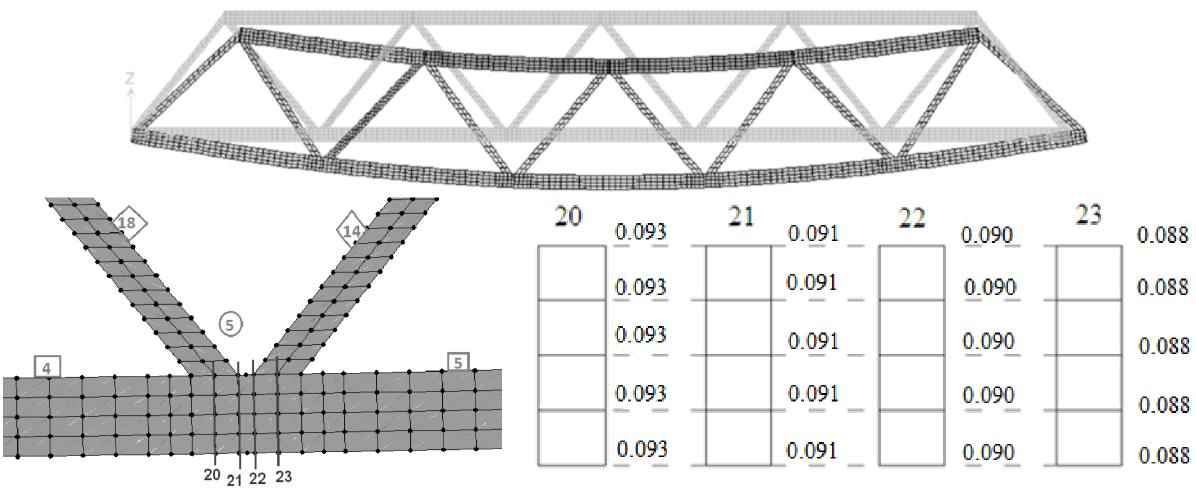


Fig. 6 Displacements [mm] obtained by the finite-element method [16, 17, 18]

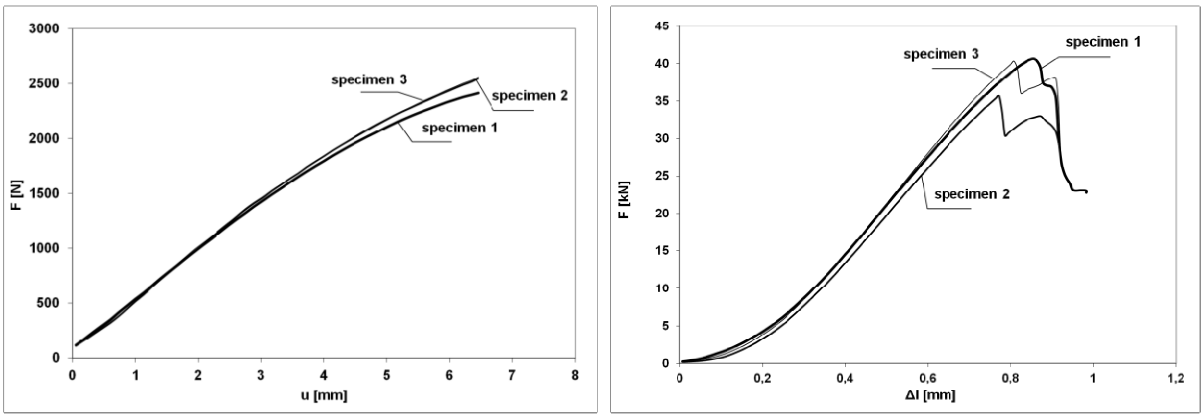
4. Experimental results

A truss subjected to symmetrical load was tested in laboratory. The truss model (scale: 1:10), made of a medium density fibreboard [6] 22 mm in thickness, was prepared for laboratory testing, cf. Fig. 8 b and c. The tensile strength of the medium density fibreboard was defined on specimens measuring l/b/h=38/3.8/3.8 cm which were subjected to bending test. The tensile strength of $\sigma_M = 25.9\ \mathrm{MPa}$ was obtained, cf. Fig. 7a. The compressive strength was tested on specimens measuring d/b/h=7.6/3.8/7.6 cm, and it amounts to $\sigma_M = 132.2\ \mathrm{MPa}$, cf. Fig. 7b. Elastic constants of the material, i.e. elastic modulus E and Poisson ratio v, were determined on prismatic specimens measuring d/b/h = 7.6/3.8/22.8 cm, cf. Fig. 7.c. [7, 9, 11]. The elastic modulus of the medium density fibreboard amounts to E = 2800 MPa, and the Poisson ratio is v = 0.27.

a) Measuring tensile strength on specimens

b) Measuring compressive strength on specimens

c) Measuring elastic modulus of the MDF



d) Force-displacement diagram for specimens 1, 2, 3, with load applied parallel to fibres

e) *F-Δl* diagram for specimens 1, 2, 3 at compressive load parallel to fibres

Fig. 7 Testing properties of a medium density fibreboard [7, 9, 11]

Force-displacement diagrams for specimens 1, 2, 3, with load applied parallel to fibres, are presented in Fig. 7d. The *F-Δl* diagram for specimens 1, 2, 3, at compressive load parallel to fibres, is shown in Fig. 7e.

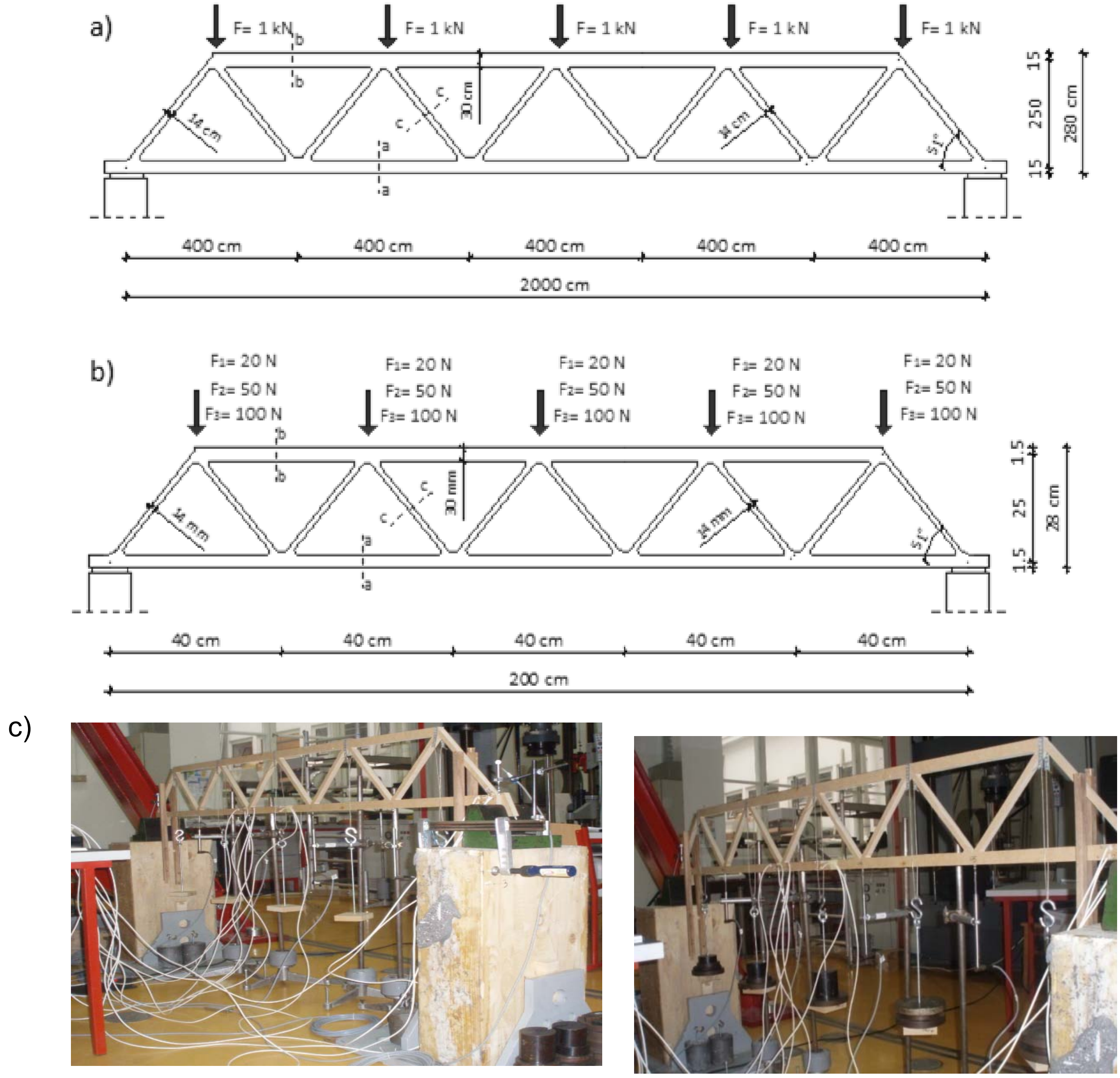


Fig. 8 Prototype of the structure (a) and the model tested in laboratory (b), (c) [4, 7, 9, 11]

Eleven resistance strain gauges for strain measurements, and six inductive strain gauges for displacement measurements, were placed on the model; cf. Fig. 9 [4, 7, 9, 11].

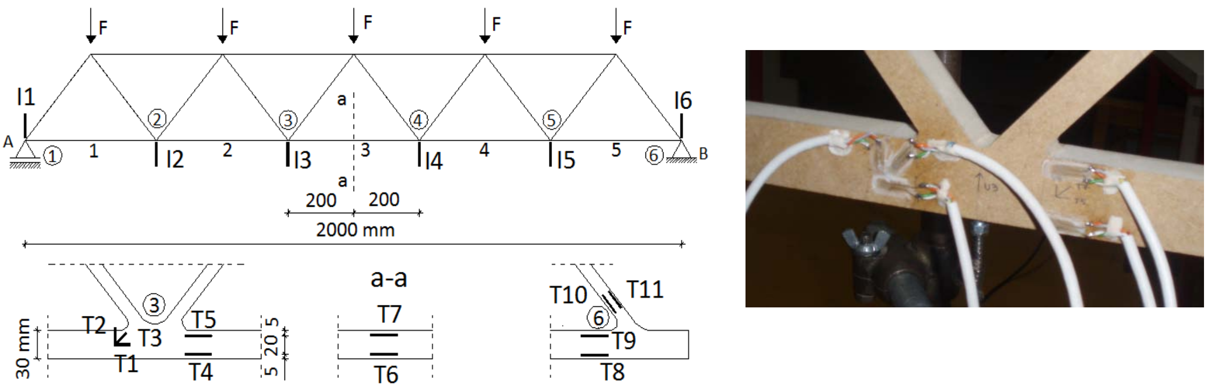


Fig. 9 Truss model with resistance strain gauges and inductive strain gauges [4, 7, 9, 11]

The truss model was tested in three phases using load values of F_1=20 N, F_2=50 N, and F_3=100 N. During the test, the load was released after each loading phase. The model testing is presented in Fig. 8b, c [4, 7, 9, 11]. The final load F_3 is ten times smaller than the load of the structure prototype, F = 1 kN. The selected load results in the stress value that is sufficiently lower than the medium density fibreboard strength defined during the testing, and so, there is no danger that the structural model will fail.

Node stresses and displacements obtained by model testing at the load value of F=100 N [4,7,9,11] are presented in Table 2.

Table 2 Node stresses and displacements at symmetric model load of F=100 N [4, 7, 9, 11]

Measurement point	Stresses σ[MPa]	Measurement point	Displacements [mm]
T1	0.606	I1	0.025
T4	0.744	I2	0.718
T5	0.648	I3	1.082
T6	0.645	I4	1.064
T7	0.522	I5	0.711
T8	-0.081	I6	-0.016
T9	0.498		
T10	-0.414		
T11	-0.372		

It was assumed that the modulus of elasticity for concrete $E_b = 2.3 \cdot 10^4$ MPa.

The dimensional analysis [3, 4] shows that stresses exerted on the medium density fibreboard must be ten times greater than those obtained by the analysis of the reinforced-concrete truss. Model displacements must be eight times greater than those obtained by the analysis of the reinforced-concrete truss.

5. Comparison of results

The relationship between loads, stresses and displacements of the laboratory model and of structure prototype is defined by dimensional analysis [13, 15]. The laboratory model of the structure, scaled 1:10, was prepared using the medium density fibreboard, and the scaling factor is:

$$K_L = \frac{L_m}{L_k} = \frac{1}{10}$$

The load relationship between the laboratory model and the structure prototype is:

$$K_F = \frac{F_m}{F_k} = \frac{1}{10}$$

The scale for stress is:

$$K_\sigma = K_F \cdot K_L^{-2} = \frac{1}{10} \cdot 100 = 10$$

Consequently, stresses acting on the laboratory model must be ten times greater than those obtained by the analysis of the structure prototype.

The relationship between elastic modulus of the laboratory model and the structure prototype is:

$$K_E = \frac{E_m}{E_k} = \frac{2800}{2.3 \cdot 10^4} \cong \frac{1}{8}$$

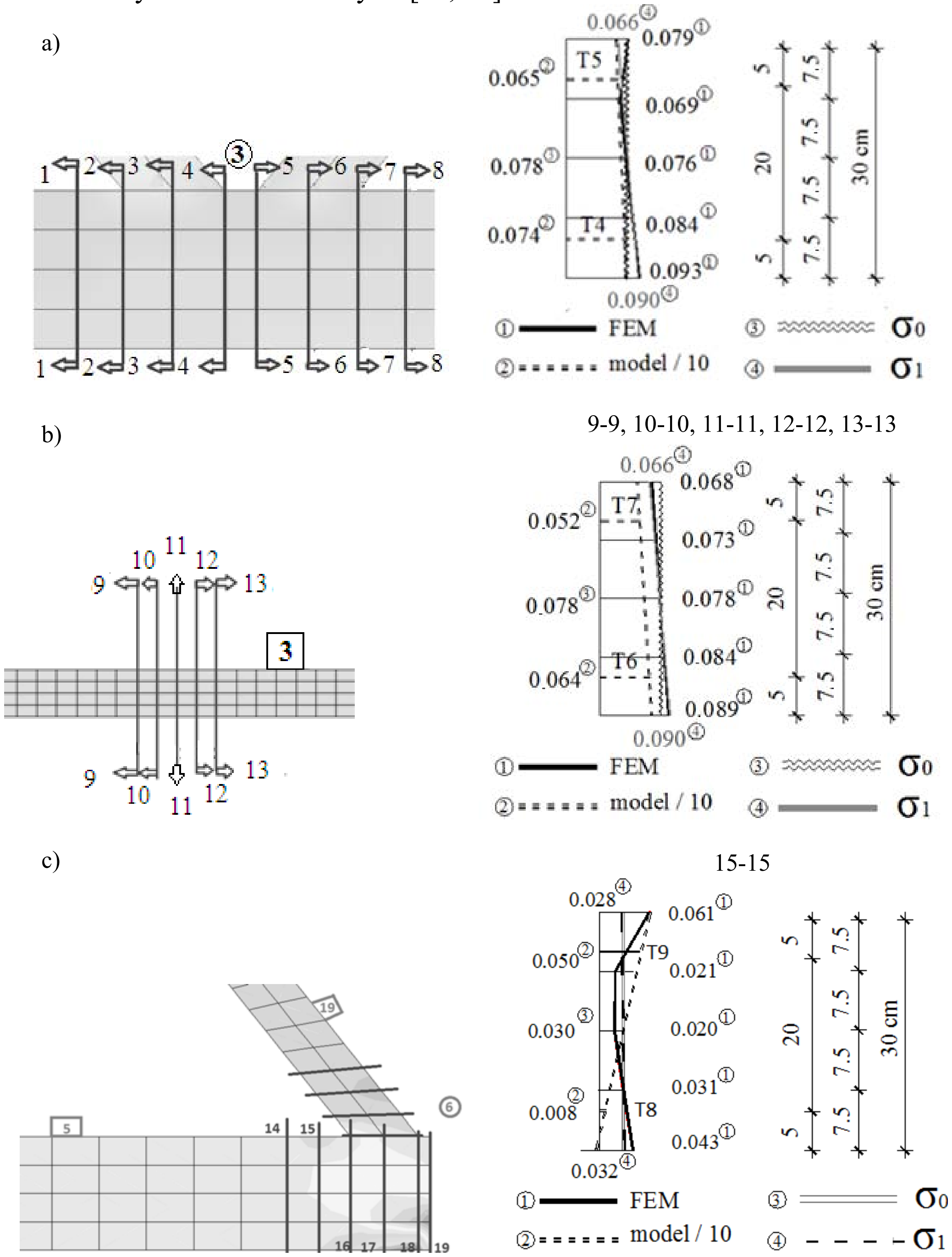
$$K_\gamma = K_\sigma \cdot K_L^{-1} = 10 \cdot 10 = 100$$

The scale for displacements is:

$$K_u = \frac{u_m}{u_k} = K_\sigma \cdot K_L^{-2} \cdot K_E^{-1} = 100 \cdot \frac{1}{100} \cdot 8 = 8$$

Therefore, displacements on the laboratory model must be eight times greater than displacements on the structure prototype as obtained by the finite-element method.

Stress diagrams for all four modelling methods are presented in Fig. 10 for selected typical cross sections (hinge 3, member 3, and hinge 6). Stresses obtained experimentally were reduced by ten times in accordance with results obtained by dimensional analysis [13, 15]. Displacements obtained by the finite-element method [8, 12, 17, 18], and those obtained experimentally by model testing under the load of F = 100 N [7, 9, 11], are presented in Fig. 11. Displacements obtained experimentally were reduced by eight times in accordance with results obtained by dimensional analysis [13, 15].



Legend: 1) FEM (rigid connections), 2) experimental (rigid connections),
3) analytic (pinned connections), 4) analytic (rigid connections)

Fig. 10 Stress diagrams σ[MPa] obtained by analysis and experimentally

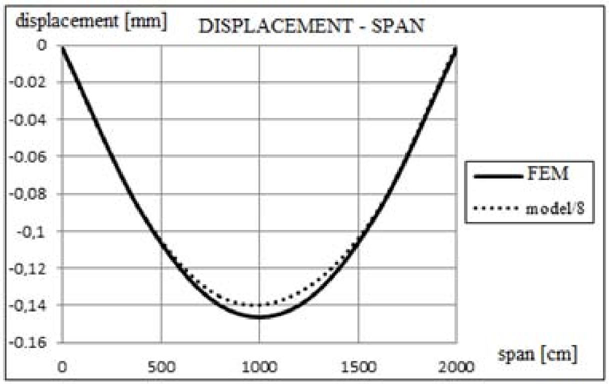


Fig. 11 Displacement diagrams obtained by FEM and experimentally

The analytic procedure for the truss model with pinned connections in nodes, and for the truss model with rigid connections in nodes, shows that longitudinal forces in truss elements are similar for both modelling alternatives. The only difference is the occurrence of bending moments in truss elements with rigid connections in nodes, which causes higher stress in truss elements. In the lower chord of the beam, the increase in stress due to rigid connections in nodes amounts to max. 40.70 percent, while this increase amounts to max. 30.69 percent in the upper chord. In some fill elements of the truss, this increase in stress is relatively greater than in truss chords. As basic stresses in fill elements are much lower than stresses in chords, it is more relevant to pay attention to additional stresses in chord elements of the truss.

It can be seen from a comparison of stress diagrams, Fig. 10, that stresses obtained experimentally are lower than stresses obtained by analysis. Stresses obtained by analysis correspond well to stresses obtained by the finite-element method, for a truss with rigid connections in nodes. A good correspondence of displacements obtained experimentally and by the finite-element method can be noted in Fig. 11.

6. Conclusion

It can be seen from the analysis of results obtained in this study that the fixity of members in rigid nodes increases stress in some truss members by as much as 40 percent or more, which reduces the level of safety of the truss. The impact of reinforcement in the area of the truss nodes and the impact of denser mesh in the numerical model around nodes in the truss were not taken into consideration in the analysis of the research results. It would be advisable to statically view the reinforced-concrete truss as a frame structure with rigid nodes, i.e. to view it as a frame structure with rigid nodes, rather than as a truss with pinned connections in nodes, as the former case is on the side of safety. In the area of the truss nodes it is necessary to provide adequate reinforcement to the actual state of stress in the nodes of the truss. The results of the study will be useful for the modelling and calculation of reinforced concrete trusses. The results of the study showed that, in some cases, the assumptions made in defining a computational model of the structure can significantly affect the results of stress in the structure, and thus the degree of structural safety.

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