TWO-PART TARIFFS AND MULTISTAGE PROGRAMMING

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Abstract
This article demonstrates the relationship between the utility function's parameters which represent consumer's taste and the reservation price. We describe the changes in the reservation price analytically and supplement the geometrical illustrations by economic interpretations. We formulate the monopolist's problem of setting the price of a capital good and the price of a service that the capital good provides as the two stage programming problem and illustrate the influence of consumers' preferences and their income on the service market client structure and monopolist's decisions on couple of illustrative examples.

Key words: *two-part tariff, reservation price, price of a capital good, service price, homogeneous and heterogeneous consumers, two-stage programming*

1. Introduction

A two-part tariff problem includes pricing of capital goods and services it provides. The best known example includes monopolist's setting of park admission prices and individual rides prices in Disneyland [5]. In maximizing the profit monopolist takes into account consumers' reactions. Every consumer compares the admission fee and the highest price he or she is willing to pay for a capital good. After paying a lump sum fee for the right to buy a product, the consumer confronts the problem of efficient allocation of the remaining income. Diversity of tastes and wealth causes analytical difficulties in maximizing monopolist's profit. This article formulates the pricing policy of two-part tariffs as the two stage programming problem [1,4] and solves that problem. In characterizing consumers' tastes we start from subjective preferences and analyze how changes in consumers' tastes influence the monopolist's choices. The standard approach found in the literature usually starts from the demand functions or indirect utility functions accompanied by capital goods and services pricing description. However, it neglects the fact that the fundamental concept in representing the consumer's preferences is the binary preference relation which is represented by a real-valued utility function. This paper underlines a connection between the reservation price and the utility function's parameters which represent consumer's taste. Changes in these parameters influence the highest price each consumer is willing to pay for capital good. The price of capital good determines the consumer structure in the service market provided by the capital good. It is important to highlight the significance of consumers' wealth on consumers' decision to buy a service which implies that income effect should not be neglected. Since the monopolist's profit is affected by the consumers' reactions, it is natural to formulate the two-part tariff problem as the two stage programming problem. The fact that the consumer must pay a lump sum fee for the right to buy a product implies the problem of determining the reservation price. Analytical and geometrical analysis of a relationship between the service price and the reservation price is being accompanied by the analysis of the effect of a change in a consumer's taste on the reservation price. If the price of a capital good is higher than the reservation price, corresponding consumer won't enter the service market. We establish the unambiguous relationship between the price of the capital good and the service market client structure, by at the same time taking into account the influence of consumers' wealth on the decision to buy a product. We formulate the problem of maximizing the monopolist's profit as the two stage programming problem and solve it. We offer illustrative examples in which we demonstrate the influence of consumers' preferences and their structure on the price of capital good and the monopolist's service price.

2. The reservation price

The consumer’s problem of efficient allocation of limited income usually reduces to the choice of the most preferred bundle from the consumer’s budget set. Composite commodity theorem highlights the desired good or service and non-satiated consumer maximizes utility subject to the given budget constraint:

$$\begin{aligned} v(p,M) &= \max_{x,m\geq 0} u(x,m) \\ px+m &= M. \end{aligned} \tag{1}$$

The budget constraint is determined by the consumer's nominal income M and the prices p the consumer faces. Quantity of service x and the consumption of all other goods m are arguments of the direct utility function, $u(x,m)$, and as a result of optimization we get the indirect utility function, $v(p,M)$, and Marshallian demand functions, $x^M(p,M)$ and $m^M(p,M)$. Marshallian demand functions describe solutions to the above optimization problem for various prices and income. Existence and uniqueness of these solutions are the result of characteristics of preferences and properties of the budget set. Preferences are described by strongly increasing and strictly quasiconcave utility function. Sometimes the consumer cannot enjoy the consumption of service without the capital good which has its price, p^c. Best known examples of capital goods are admission fee for Disneyland and the Polaroid camera [6].

Purchasing the capital good reduces the budget of the consumer, who speculates whether to buy a capital good or spend entire available budget on other goods. In doing so, he compares maximal utility from possessing capital good and utility from spending the whole budget on other goods and buys the capital good only when the following inequality holds,

$$v(p,M-p^c) \geq u(0,M). \tag{2}$$

By inverting the indirect utility function we get the expenditure function, $e(p,u)$. The expenditure function and the compensated demand functions, $x^H(p,u)$ and $m^H(p,u)$, are derived from the model of minimizing the expenditure subject to the given utility level, u,

$$\begin{aligned} e(p,u) &= \min_{x,m\geq 0} px+m \\ u(x,m) &= u. \end{aligned} \tag{3}$$

Since the expenditure function is strongly increasing in the utility level, the inequality (2) is equivalent to the following:

$$e(p, v(p, M - p^c)) \geq e(p, u(0, M)), \tag{4}$$

$$M - p^c \geq e(p, u(0, M)), \tag{5}$$

$$M - e(p, u(0, M)) \geq p^c. \tag{6}$$

The highest price the consumer is willing to pay for capital good, so called reservation price, p^r, is thus equal to the difference between the nominal income and the minimal expenditures that ensures the level of utility that corresponds to the situation where the consumer does not buy the capital good [2],

$$p^r = M - e(p, u(0, M)). \tag{7}$$

The relevant indifference curve for the consumer is the one that goes through the bundle (0,*M*) and the consumer chooses that bundle in which the isoexpenditure curve with slope determined by price of service is tangent to the indifference curve. The reservation price is represented by the distance between the intercepts of the indifference curve and the tangent on the vertical axis.

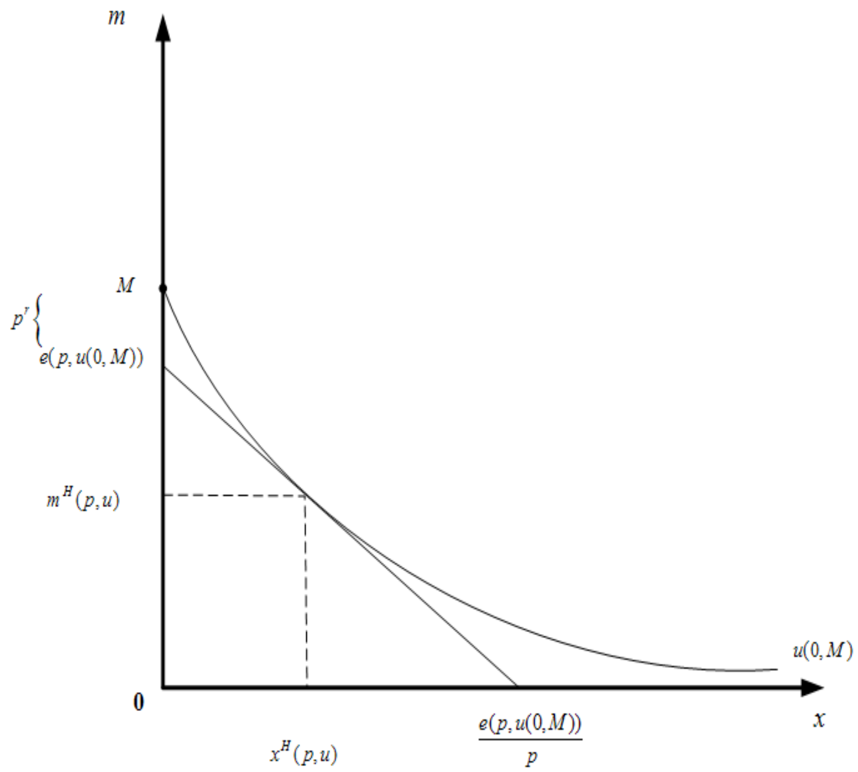

Figure 1. The reservation price

When the price of a service increases, it is intuitive to expect that the consumer is willing to pay less for the capital good. Because of the law of diminishing marginal rate of substitution between goods the point of tangency moves on the left and the distance between the corresponding intercepts or the reservation price declines. This result is formally confirmed by using Shepard's lemma [7,8],

$$\frac{dp^r}{dp} = -\frac{\partial e(p, u(0, M))}{\partial p} = -x^H(p, u(0, m)) < 0. \tag{8}$$

The ordinal approach to the consumer theory starts with the subjective preferences that are represented by the preference function or the ordinal utility function. Market changes induce active consumer to substitute the more expensive good with the cheaper one. In so doing savings are determined by the substitution parameter, ρ, that enters the numerical characterization of preferences with constant elasticity of substitution,

$$u(x,m) = (x^{\rho} + m^{\rho})^{\frac{1}{\rho}}, \quad 0 < \rho < 1. \tag{9}$$

The consumer minimizes the expenditure in order to achieve a given level of utility when he finds that bundle on the relevant indifference curve that equals the marginal rate of substitution between goods and the marginal rate of market transformation,

$$\frac{u_x}{u_m} = p. \tag{10}$$

The second equation of the system that the consumer solves includes the constraint and the expenditure function for the above utility function is the following

$$e(p,u) = (p^{1-\sigma} + 1)^{\frac{1}{1-\sigma}} u. \tag{11}$$

Notice that the elasticity of substitution enters the expression (11) for the expenditure function, [3]

$$\sigma = \frac{1}{1-\rho}. \tag{12}$$

The expenditure function and the equality that describes the reservation price detect the unambiguous relationship between the reservation price and the elasticity of substitution,

$$p^r = (1 - (p^{1-\sigma} + 1)^{\frac{1}{1-\sigma}})M. \tag{13}$$

It can be easily verified that an increase in the elasticity of substitution adversely affects the consumer's willingness to buy the capital good. In other words, the reservation price is a decreasing function of the elasticity of substitution.

In order to prove it, the following function from the equality that describes the reservation price can be introduced, $f(\sigma) = (p^{1-\sigma} + 1)^{\frac{1}{1-\sigma}}$.

Taking logs, we have

$$\ln f(\sigma) = \frac{1}{1-\sigma} \ln(p^{1-\sigma} + 1). \tag{14}$$

Differentiating the expression with respect to σ gives

$$\frac{f^{'}(\sigma)}{f(\sigma)} = \frac{1}{1-\sigma}\left[\frac{\ln(p^{1-\sigma}+1)}{1-\sigma} - \frac{p^{1-\sigma}\ln p}{p^{1-\sigma}+1}\right]. \tag{15}$$

Inequalities $(p^{1-\sigma}+1)\ln(p^{1-\sigma}+1) > p^{1-\sigma}\ln(p^{1-\sigma}+1)$ and $p^{1-\sigma}\ln(p^{1-\sigma}+1) > p^{1-\sigma}\ln p^{1-\sigma}$ imply $(p^{1-\sigma}+1)\ln(p^{1-\sigma}+1) > p^{1-\sigma}\ln p^{1-\sigma} = (1-\sigma)p^{1-\sigma}\ln p$. It follows $f^{'}(\sigma) > 0$ and

$$\frac{dp^r}{d\sigma} < 0. \tag{16}$$

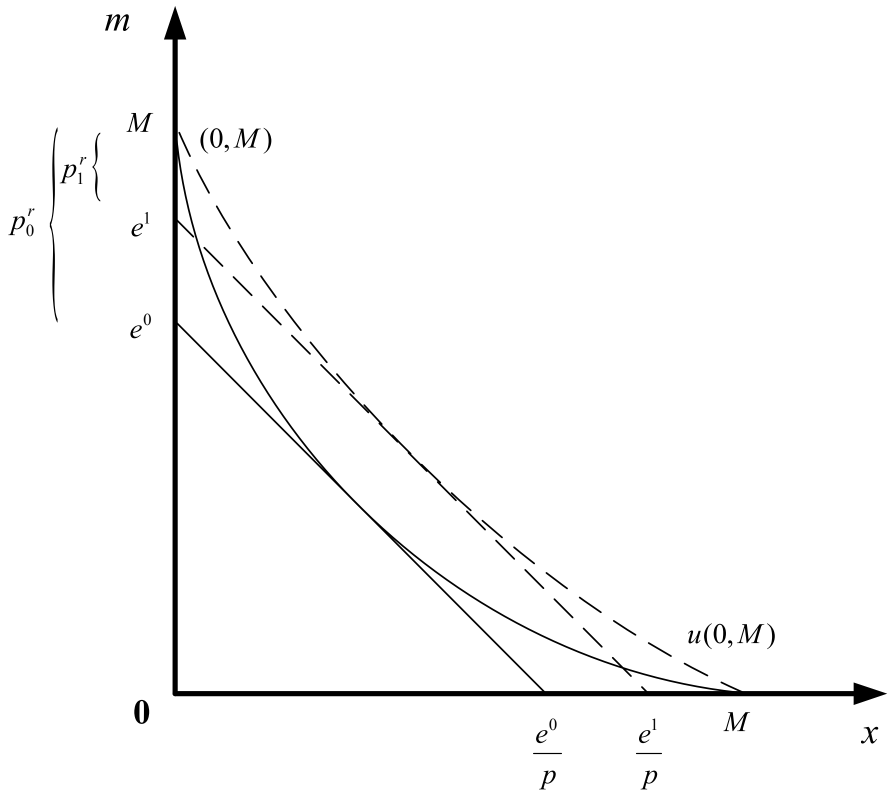

Figure 2. The reservation price and the elasticity of substitution

Knowing how the elasticity of substitution is related to the curvature of the indifference curves, the previous statement can be graphically illustrated.

The larger the elasticity of substitution, the flatter are the indifference curves and they are more like linear indifference curves that characterize perfect substitutability between the goods. In that case, the intercept of the tangent on the vertical axis approaches the intercept of the indifference curve on the vertical axis and the reservation price declines. The elasticity of substitution for perfect substitutes is infinite and the rational consumer completely substitutes the service with the consumption of all other goods when the price of a service is higher than the constant marginal rate of substitution between goods. In that case the consumer is not willing to pay anything for the capital good. The more difficult is the substitution between goods, the higher price is the consumer willing to pay for the capital good.

3. The profit maximization

The owner of a firm that sells the capital good and the service that the capital good provides determines the prices that maximize his or her profit. Let us assume that there are no other firms in the market and let us neglect fixed costs and the costs of capital goods for the moment. Let us also assume that the marginal costs of a service that the capital good provides are constant, *c*. The profit that the monopolist earns from a single consumer consists of two parts, the amount that the consumer pays for the capital good and the profit from services,

$$\pi = p^c + (p - c)x^M(p, M - p^c). \tag{17}$$

The quantity of a service that the consumer demands is determined from his or her Marshallian demand function and the residual budget. If the price of a capital good is higher than the reservation price of a single consumer, that consumer won't enter the market and the profit that the owner of a firm earns from this consumer is equal to zero. If we analyze a homogeneous group of consumers with respect to subjective preferences and wealth, the analysis becomes the simplest one. The problem of two-part tariff is then reduced to the following optimization problem:

$$\max_{p^c \leq p^r, p>0} \pi = p^c + (p-c)x^M(p, M-p^c). \tag{18}$$

If the elasticity of substitution for this group of consumers is $\sigma = 2$, the reservation price for CES utility function (9) is

$$p^r = \frac{M}{1+p}. \tag{19}$$

In that case $\frac{\partial \pi}{\partial p^c} = \frac{p^2+c}{p(1+p)} > 0$ and price of a service is equal to reservation price

$$p^c = p^r = \frac{M}{1+p}. \tag{20}$$

In the optimization problem (18) we only need to determine optimal price of a service. Optimizing the function $\pi = \frac{1+2p-c}{(1+p)^2}M$ we get

$$p = c, p^c = \frac{M}{1+c}. \tag{21}$$

Equality between optimal price of a service and marginal costs is valid in general case of normal goods [5]. Note that the price of a service just covers marginal costs and the firm earns profit from selling the capital good,

$$\pi = \frac{M}{1+c}. \tag{22}$$

The equality (22) expresses the maximal average profit and the total profit depends on a number of homogeneous consumers. In reality consumers differ with respect to taste and wealth and the profit maximizing firm determines the price of products on a basis of consumers' reactions. If the firm decides to charge a unique price for the capital good, it loses consumers who are not willing to pay that much for the capital good. Because of the various consumers' reactions, it is natural to formulate the two-part tariff problem as the two stage programming problem.

4. Two-part tariffs problem as the two stage programming problem

The problem in question, as presented beforehand, can be formulated as the two stage programming problem. In the first stage, the monopolist selling the capital good and the service makes the decision about the price of the capital good and the price of the service with the goal of maximizing his profit. These prices trigger different reactions of consumers who are divided into homogenous groups where each group is characterized by its income and elasticity of substitution. Once the prices of the capital good and the service are set, in the second stage each consumer group independently determines the level of its demand so as to maximize the correspondent utility function. Therefore, when making the decision about the price of the capital good and the price of the service, the monopolist has to take into account the reactions of the consumers since his profit is directly affected by consumers' reactions.

Let *n* denote the number of consumer groups, s_i size of the consumer group *i*, M_i income of the consumer group *i*, σ_i elasticity of substitution for the consumer group *i*, c marginal cost of a service that the capital good provides and let α be a sufficiently large number. The monopolist has to make a decision about price of the capital good, p^c, and price of the service, *p*. The decision variables of each consumer from consumer group *i* consider quantity of service consumed, x_i, expenditure on all other goods, m_i, and reservation price, p_i^r. Also, they make the decision about whether or not to consume the good, δ_i, where $\delta_i = 1$ if consumers from group *i* are consuming the good in question and *0* otherwise.

Now we can formulate the problem as the following two stage programming problem:

$$\max_{p\geq 0, p^c \geq 0} \sum_{i=1}^{n} s_i \cdot \left[p^c + \left(p - c \right) \cdot x_i \right] \cdot \delta_i \tag{23}$$

s.t.

$$\max_{x_i, m_i, p_i^r} \left(x_i^{1-\frac{1}{\sigma_i}} + m_i^{1-\frac{1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}} \tag{24}$$

s.t.

$$px_i + m_i = M_i - p^c \cdot \delta_i \tag{25}$$

$$p_i^r = \left[1 - \left(p^{1-\sigma_i} + 1 \right)^{\frac{1}{1-\sigma_i}} \right] \cdot M_i \tag{26}$$

$$\delta_i = \begin{cases} 1; & \text{if } p^c \leq p_i^r \\ 0; & \text{otherwise} \end{cases} \tag{27}$$

$$x_i \leq \alpha \cdot \delta_i \tag{28}$$

$$x_i, m_i \geq 0, \delta_i \in \{0,1\} \tag{29}$$

The monopolist has to determine the price of the capital good, p^c, and the price of the service, *p*, in order to maximize his profit (23). However, when making the decision the monopolist has to take into account that each of the *n* consumer groups will make a decision about consumption independently according to their individual objectives and constraints, which will directly influence monopolist's profit. A certain consumer group will consume the good only if that the price of the capital good is less or equal than its reservation price. Therefore, the monopolist's profit consists of the sum of profits per each consumer group, where profit per consumer groups *i* is *0* if that group is not consuming the good ($\delta_i = 0$), and equal to $s_i \cdot \left[p^c + \left(p - c \right) \cdot x_i \right]$ if the group is consuming the good. Hereby $\left[p^c + \left(p - c \right) \cdot x_i \right]$ is the profit per each member of the group and s_i is the number of members in that group.

Once the monopolist determines the price of the capital good and the price of the service, in the second stage each of the *n* consumer groups independently makes a decision about the consumption of that good while trying to maximize its utility (24). Therefore, the second stage consists of *n* independent optimization problems, one per each consumer group. Since groups are homogeneous, it suffices to consider the behavior of one individual consumer per group. A member of group *i* has to make the decision about whether it will consume the good and the service offered by the monopolist, described by δ_i, as well as about the quantity of the service consumed, x_i, in order to maximize its utility (24). The decision about the quantity of the service implies the decision regarding the expenditure on other goods, m_i. The utility is being maximized under the set of constraints (25)-(29). Should members of the group *i* decide to buy the capital good, each of them has a budget of $M_i - p^c$ left for spending on buying the service (px_i) and other goods (m_i) as shown by budget constraint (25). If the members of group *i* are not buying the capital good, than $\delta_i = 0$ and the whole budget is being spent on other goods. Reservation price for group *i* is determined by (26). Members of group *i* will buy the good and the service only if the price of the capital good is less that the corresponding reservation price, as shown by (27) and (28). All decision variables should be non-negative, as required by (29).

We will illustrate the problem in question as well as it economic interpretation by using the following two examples.

Example 1. Let us consider a monopolist who has to determine the price of the capital good and the price of the service in presence of two homogeneous consumer groups, where the marginal costs of a service that the capital good provides is equal to *1*. Group *1* consists of 4 members, has an income of *10* and its elasticity of substitution is equal to 2. Group *2* consists of *2* members where each member has an income of *40* and elasticity of substitution 2. Note that both groups have the same elasticity of substitution, but differ in their income.

The monopolist has to determine the price of the capital good, p^c, and the service, *p*, in order to maximize its profit given by

$$\max_{p\geq 0,\, p^c\geq 0} 4\cdot\left[p^c+(p-1)\cdot x_1\right]\cdot\delta_1+2\cdot\left[p^c+(p-1)\cdot x_2\right]\cdot\delta_2 \tag{30}$$

Given the prices p^c and *p*, each consumer from group *1* will make the decision about whether to consume good offered by monopolist or not, as well as about its quantity, by solving the optimization problem (31)-(36):

$$\max_{x_1, m_1, p_1^r} \left(x_1^{\frac{1}{2}}+m_1^{\frac{1}{2}}\right)^2 \tag{31}$$

s.t.

$$px_1+m_1=10-p^c\cdot\delta_i \tag{32}$$

$$p_1^r=\left[1-\left(p^{-1}+1\right)^{-1}\right]\cdot 10 \tag{33}$$

$$\delta_1=\begin{cases}1; & \text{if } p^c\leq p_1^r\\ 0; & \text{otherwise}\end{cases} \tag{34}$$

$$x_1\leq\alpha\cdot\delta_1 \tag{35}$$

$$x_1, m_1\geq 0, \delta_1\in\{0,1\} \tag{36}$$

The similar problem for consumers from group *2* is as follows:

$$\max_{x_2, m_2, p_2^r} \left(x_2^{\frac{1}{2}} + m_2^{\frac{1}{2}} \right)^2 \tag{37}$$

s.t.

$$px_2 + m_2 = 40 - p^c \cdot \delta_i \tag{38}$$

$$p_2^r = \left[1 - \left(p^{-1} + 1 \right)^{-1} \right] \cdot 40 \tag{39}$$

$$\delta_2 = \begin{cases} 1; & \text{if } p^c \leq p_2^r \\ 0; & \text{otherwise} \end{cases} \tag{40}$$

$$x_2 \leq \alpha \cdot \delta_2 \tag{41}$$

$$x_2, m_2 \geq 0, \delta_2 \in \{0,1\} \tag{42}$$

Now $p_1^r(p) = \dfrac{10}{p+1}$ and $p_2^r(p) = \dfrac{40}{p+1}$. Furthermore, $x_1 = \dfrac{10 - p^c}{p(p+1)}$ and $x_2 = \dfrac{40 - p^c}{p(p+1)}$. Since $p_1^r \leq p_2^r$ for all p, in order to determine the optimal reservation prices and the optimal service price, we have to consider the following 3 cases:

Case 1: $0 \leq p^c \leq p_1^r$

Since $p^c \leq p_1^r$, we know that $\delta_1 = \delta_2 = 1$, so the monopolist determines the price of the capital good so as to maximize the profit function

$$\Pi = 4 \cdot \left[p^c + (p-1) \cdot x_1 \right] + 2 \cdot \left[p^c + (p-1) \cdot x_2 \right] = \frac{6\left[p^c \left(p^2 + 1 \right) + 20p - 20 \right]}{p(1+p)}$$

The maximum is achieved for $p \approx 1,7$ and $p^c = p_1^r \approx 3,7$ with $\Pi_1 = 37,13$.

Case 2: $p_1^r < p^c \leq p_2^r$

Since $p_2^r \leq p^c$, we know that $\delta_1 = 0$ and $\delta_2 = 1$, so the monopolist determines the price of the capital good so as to maximize the profit function

$$\Pi = 2 \cdot \left[p^c + (p-1) \cdot x_2 \right] = 2 \cdot \left[p^c + (p-1) \cdot \frac{40 - p^c}{p(1+p)} \right]$$

The maximum is achieved for p=1 and $p^c = p_2^r = 20$ with $\Pi_2 = 40$.

Case 3: $p_2^r < p^c$

Since $p_1^r < p_2^r < p^c$, we know that $\delta_1 = \delta_2 = 0$, and the monopolist's profit is equal to $\Pi_3 = 0$.

By comparing these three possibilities, we see that the maximum profit is achieved in *Case 2.* Therefore, the optimal price of the capital good is $p^c = 20$ and the optimal price for the service is *p=1*. Although both consumer groups have the same elasticity, here only consumers from group *2* consume the good. The quantity of service consumed is $x_2 = 10$ with expenditure on other goods $m_2 = 10$. However, if the income of group 2 rises to $M_1 = 12$, the maximum profit is obtained for $p \approx 1,62$ and $p^c \approx 4,57$ with $\Pi_2 = 42,16$, so if the income of group 1 is above 12, both groups will consume the good.

Example 2. Let us consider monopolist and two homogenous consumer groups where the marginal costs of a service that the capital good provides is equal to *2*. Hereby both groups consist of 4 members and both have elasticity of substitution 2. However, income of the first group is 20, while the income of the second one is 40. By conducting the same analysis as in Example 1, we conclude that the optimal prices of the service and the capital good are $p \approx 2,7$ and $p^c \approx 5,41$, with the maximum profit $\Pi_{\max} \approx 57,03$. Here, both groups are buying the good ($\delta_1 = 1$, $\delta_2 = 1$, $x_1 \approx 10.65$, $x_2 \approx 25.24$, $p_1^r \approx 5,41$, $p_2^r \approx 10,81$). However, if the elasticity of the second group increases to $\sigma_2 = 4$, the group structure changes and now only the first group is buying the good. Hereby the optimal values of the decision variables are $p = 2$, $p^c = \frac{20}{3}$, $\delta_1 = 1$, $\delta_2 = 0$, $x_1 = \frac{80}{9}$, $x_2 = 0$, $p_1^r \approx 10,75$ and $p_2^r \approx 10,806$ with $\Pi_{\max} = \frac{80}{9} \approx 26,67$.

5. Conclusion

In this paper subjective preferences that are represented by constant elasticity of substitution have been used as a starting point for analysis of the two-part tariff problem. Contributions are manifested in directly relating the reservation price and parameters that represent consumer's taste. Changes of reservation price due to changes in the service price or the consumer's taste are described analytically and illustrations of these changes are economically interpreted. At the same time, in the analysis of consumers' reactions, their wealth is taken into account. By comparing the reservation price and the service price, client structure on the service market is determined. Because of the heterogeneity of the consumers, the problem of setting the price of a capital good and the price of a service that the capital good provides is formulated as the two stage programming problem. Solving that problem is illustrated by economically interesting examples that show how the changes in consumer' taste and income influence the market client structure, the service price and the price of the capital good.

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