APPLICATION OF THE EXTENDED MRP THEORY TO A BABY FOOD COMPANY

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Abstract

Actual markets require companies to think about new ways to improve their business or to get additional advantages from their existing competences. Such improvements should not be limited to optimisation of individual activity cells but should be the result of broader analyses. Companies should consider their whole supply chains and make deep observation of dependencies between individual activity cells. Material requirements planning (MRP) Theory has proved to be a successful tool for describing and evaluating multistage, multilevel production systems with the use of Net Present Value (NPV) calculation. Recently, this theory has been extended in a way that it also deals with other vital parts of global supply chains, such as distribution, consumption and the reverse logistics. We call this approach the Extended MRP Theory (EMRP Theory). This paper shows how EMRP Theory can be used in analysing business processes for a Spanish company dedicated to baby food production.

Key words: *MRP Theory, EMRP Theory, supply chain, NPV, practical application, baby food company*

1. INTRODUCTION

The Material Requirements Planning Theory (MRP Theory) is strongly established in academic circles. The origins of the Theory's pillars date back to 1967, when Grubbström introduced the connections between Laplace transforms and economic problems (Grubbström, 1967). Modern Theory as it is known today, was inspired by Orlicky's detailed description of Material Requirements Planning (Orlicky, 1975), and born in 90's with an intensive research of proper connections of the product structures, lead times and cash flows (Grubbström and Tang, 2000). MRP's product structure can be conveniently captured by input-output matrices, which are originating from Leontief's works (Leontief, 1966) and Koopman's activity analysis (Koopmans, 1951). Production lead times can be incorporated with the transformation of the relevant time functions into Laplace transforms in the frequency domain. Balance equations, which describe time developments of the total and available inventory, backlogs and allocations, are called fundamental equations of the MRP Theory. MRP Theory in its original meaning solves production-inventory problems.

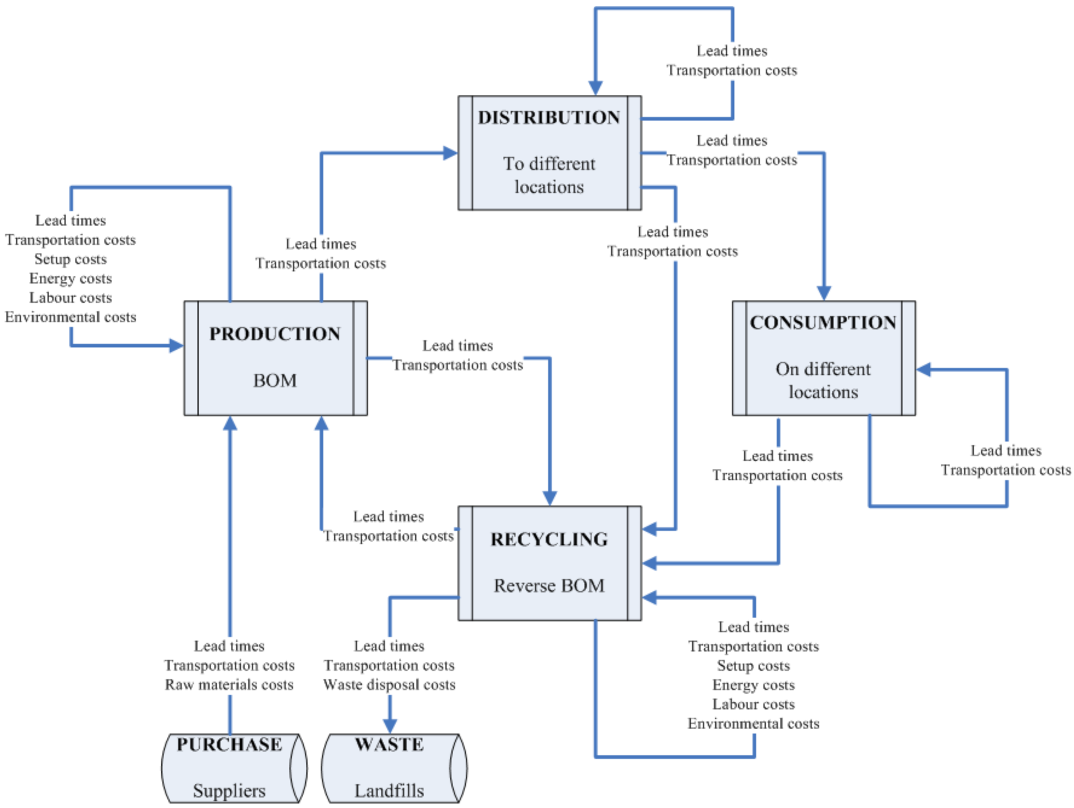

Figure 1: Cyclical model of 4 sub-processes of EMRP Theory.

Recently further developments of the Theory, which extends it from production-inventory problems to global supply chains, have appeared (Grubbström, M. Bogataj and L. Bogataj, 2007). Individual activity cells in real life are geographically dispersed (M. Bogataj, Grubbström and L. Bogataj, 2011; Kovačić and Bogataj, 2012a), giving attractiveness and stickiness to regions (Drobne and Bogataj, 2011). Consequently, transportation lead times and corresponding costs between any pair of activity cells have to be an integral part of the Net Present Value (NPV) calculation (M. Bogataj, Grubbström and L. Bogataj, 2011). Post sales activities such as consumption and reuse of waste items create a closed loop where items circulate through several cycles of production, distribution, consumption and recycling (Kovačić and Bogataj, 2011). Such extension embracing whole supply chain is called Extended MRP Theory (EMRP Theory) as it is schematically presented.

2. METHODOLOGY

The model denoted at Figure 1 can be described in a mathematical way with a pair of generalized input-output matrices:

$$\breve{\mathbf{H}}(s)=\begin{bmatrix}\breve{\mathbf{H}}_{11}(s) & \breve{\mathbf{H}}_{12}(s) & 0 & \breve{\mathbf{H}}_{14}(s)\\ 0 & 0 & \breve{\mathbf{H}}_{23}(s) & \breve{\mathbf{H}}_{24}(s)\\ 0 & 0 & 0 & \breve{\mathbf{H}}_{34}(s)\\ 0 & 0 & 0 & 0\end{bmatrix} \qquad \breve{\mathbf{G}}(s)=\begin{bmatrix}\breve{\mathbf{G}}_{11}(s) & 0 & 0 & \breve{\mathbf{G}}_{14}(s)\\ 0 & \breve{\mathbf{G}}_{22}(s) & 0 & 0\\ 0 & 0 & \breve{\mathbf{G}}_{33}(s) & 0\\ 0 & 0 & 0 & \breve{\mathbf{G}}_{44}(s)\end{bmatrix} \tag{1}$$

Generalized input and output matrices $\breve{\mathbf{H}}(s)$ and $\breve{\mathbf{G}}(s)$ consist of products' physical structures and material flow coefficients together with their corresponding lead times. Both matrices coincide in dimension and both have non-zero sub-matrices only on places where physical operations in the system are possible. Every sub-matrix $\breve{\mathbf{H}}_{ij}(s)$ describes the need of items required by sub-system *i* from running processes in sub-system *j*, where: 1 correspond to production; 2 to distribution; 3 to consumption and 4 to recycling sub-system. Similarly, every sub-matrix $\breve{\mathbf{G}}_{kl}(s)$ relates to outputs of sub-system *l* as requirements of processes in sub-system *k*. Such models are a mixture of assembly and arborescent systems. Assembly processes can be described using Bill of Materials (BOM) and arborescent processes can be described using reverse Bill of Materials (reverse BOM). Inside such systems, many different types of lead times and transaction costs can occur. Detailed description of input and output matrices' structures can be found in Kovačić and Bogataj (2011).

One of the most significant advantages of the MRP Theory is its ability to evaluate cash flows. When Laplace complex frequency *s* is replaced with continuous interest rate *ρ*, this can be best done with the use of the NPV calculation:

$$\begin{aligned}\text{NPV} = &\mathbf{p}\left(\breve{\mathbf{G}}(\rho) - \breve{\mathbf{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho) - \hat{\mathbf{K}}\tilde{\mathbf{v}}(\rho) - \mathbf{U}^{\mathrm{T}}\left(\tilde{\mathbf{\Pi}}_G(\rho) + \tilde{\mathbf{\Pi}}_H(\rho)\right)\tilde{\mathbf{P}}(\rho) - \\ &- \mathbf{c}_{\mathrm{L}}\hat{\mathbf{L}}\tilde{\mathbf{v}}(\rho) - \mathbf{U}^{\mathrm{T}}\left(\breve{\mathbf{E}}_H(\rho) - \breve{\mathbf{E}}_G(\rho)\right)\tilde{\mathbf{P}}(\rho)\end{aligned} \tag{2}$$

Overall NPV of the system consists of several partial NPVs:

- $\mathbf{p}\left(\breve{\mathbf{G}}(\rho) - \breve{\mathbf{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho)$ is NPV of the revenues where $\mathbf{p}$ is a vector of prices for items appearing at each level and $\tilde{\mathbf{P}}(\rho)$ is a vector which holds contingents' sizes at each activity level.
- $\hat{\mathbf{K}}\tilde{\mathbf{v}}(\rho)$ is NPV of setup costs where $\tilde{\mathbf{v}}(\rho)$ are given timings.
- $\mathbf{U}^{\mathrm{T}}\left(\tilde{\mathbf{\Pi}}_G(\rho) + \tilde{\mathbf{\Pi}}_H(\rho)\right)\tilde{\mathbf{P}}(\rho)$ is NPV of transportation costs, where $\tilde{\mathbf{\Pi}}_H(\rho)$ and $\tilde{\mathbf{\Pi}}_G(\rho)$ are generalized input and output transportation matrices, and $\mathbf{U}^{\mathrm{T}}$ is a vector of unit values.
- $\mathbf{c}_{\mathrm{L}}\hat{\mathbf{L}}\tilde{\mathbf{v}}(\rho)$ is NPV of labour force where $\mathbf{c}_{\mathrm{L}}$ is price and $\hat{\mathbf{L}}$ quantity of labour needed at individual activity level.
- $\mathbf{U}^{\mathrm{T}}\left(\breve{\mathbf{E}}_H(\rho) - \breve{\mathbf{E}}_G(\rho)\right)\tilde{\mathbf{P}}(\rho)$ is NPV of used and produced/recovered energy. Energy can occur as input or output of the system, using corresponding input and output generalized energy matrices $\breve{\mathbf{E}}_H(\rho)$ and $\breve{\mathbf{E}}_G(\rho)$, respectively.

Strong theoretical background of the MRP Theory in both of its traditional and extended forms has not been widely applied to real world problems, although we can find some successful applications to paper mill production, which prove Theory's ability of solving such problems (Grubbström, 1990). EMRP Theory approach has only been tested using numerical examples and simulations (Kovačić and Bogataj, 2012b). This paper presents the first attempt to model and observe behaviour of real world problem using EMRP Theory which also consists of the environmental component. We will use a real case study of a Spanish food company, for this purpose.

3. PROBLEM DESCRIPTION

The observed food company is located in the southeast of Spain, and most of their clients are geographically dispersed throughout the country. Their main focus is on production of baby food, juices, jams and cereal bars. Exceptionally high standards in baby food production require exhaustive quality control and quarantine for raw materials and the final products. Production residues, final products, which do not satisfy expected standards, and raw materials with freshness date over the shelf life, are disposed of to landfills. The results of disposal are lost revenues and additional disposal costs and environmental taxes. Such a complex production process is causing noticeable lead times. Relatively long lead times together with high prices of raw materials (ingredients) and a wide distribution area in Spain make this baby food company a strong base for practical application of EMRP Theory.

We will limit this research to only one company's final product: a jar of baby meat food with a net weight of 250 grams. Production takes place 3 times per month in batches of 152,000 jars and production setup costs of one batch equal to 20,000 €. Production lead time for one batch is one day. Before the final products can be delivered to the market, they have to be quarantined for 11 days. Approximately 1.1% of the total production are residues or products which do not meet required quality standards. They have to be disposed of at the costs of 0.145 €/jar.

Retail price of the final product is 0.65 €. Average transportation cost of one jar to retailers equals to 0.031 €. The company has to pay additional environmental tax of 0.0035 €/jar. If we suppose that payments for final products appear at approximately the same time as the company has to pay for transportation costs and environmental taxes, we can simplify the model considering the retail price as: 0.65 € - 0.031 € - 0.0035 € = 0.6155 €.

Detailed structure of the final product is denoted at Table 1, together with its ingredients' purchase prices and lead times. We can split lead times into two parts:

- Delivery lead times: are times between setting orders and receptions of ingredients and raw materials in the warehouses.
- Quarantine lead times: are times needed to verify the quality of ingredients according to internal standards.

Table 1: Structure of the final product and lead times of its components.

Ingredient	Quantity/jar	Price/unit	Delivery lead times	Quarantine lead times
A	0.1050 kg	0.0060 €	1 day	0 days
B	0.0300 kg	0.8434 €	6 days	1 day
C	0.0375 kg	0.5000 €	5 days	1 day
D	0.0275 kg	6.9277 €	10 days	28 days
E	0.0275 kg	0.6988 €	5 days	1 day
F	0.0100 kg	0.6024 €	9 days	31 days
G	0.0025 kg	1.8072 €	5 days	14 days
H	0.0100 kg	0.7229 €	10 days	14 days
Package	1 piece	0.0843 €	16 days	0 days

4. ANALYSIS

Described model can now be conveniently presented using extended BOM (Figure 2).

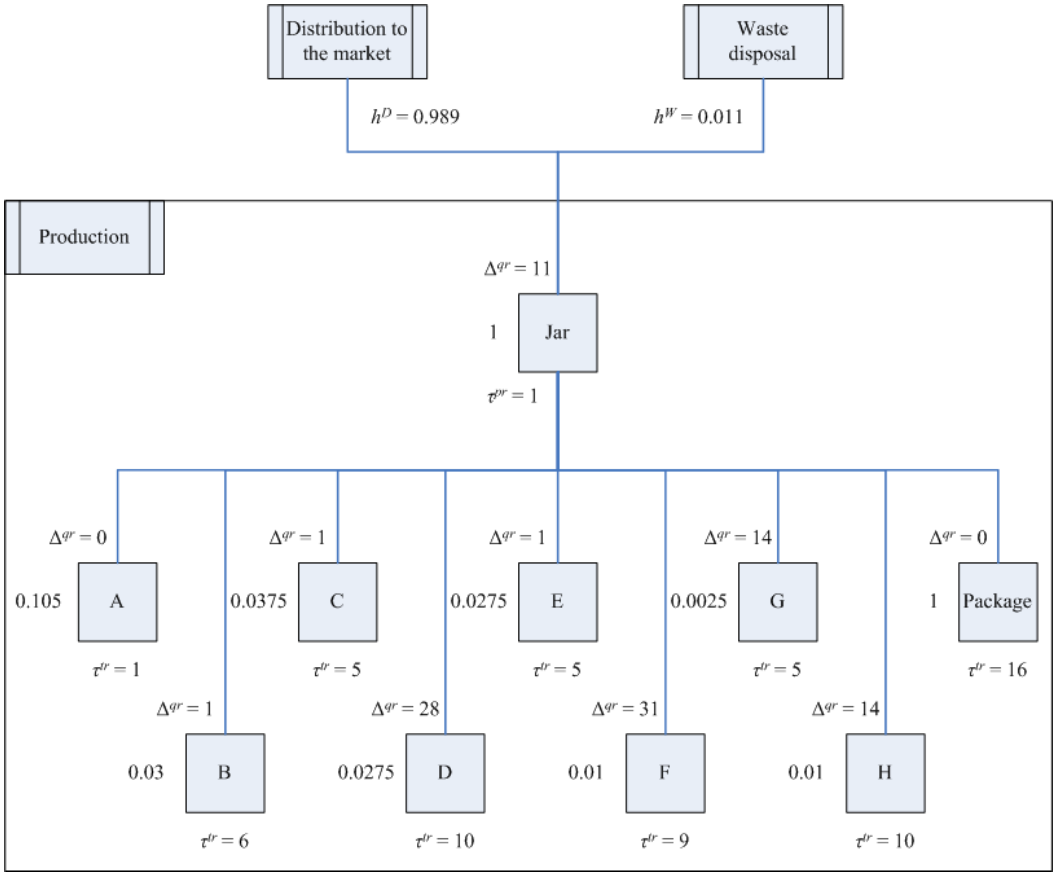

Figure 2: Extended BOM for production and distribution of 1 jar of baby food (structures and lead times).

According to the above description of the model we can now conveniently write its structures using input-output matrices **H** and **G** and activity vector **P** in the following way:

$$\mathbf{H} = \left[\begin{array}{c|c} & \\ 0.105 & \\ 0.03 & \\ 0.0375 & \\ 0.0275 & \\ 0.0275 & \\ 0.01 & \\ 0.0025 & \\ 0.01 & \\ 1 & \\ \hline & \end{array}\right] \qquad \mathbf{G} = \left[\begin{array}{c|c} 0.989 & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ \hline & 0.011 \end{array}\right] \qquad \mathbf{P} = \begin{bmatrix} 152,000 \\ 152,000 \end{bmatrix} \tag{3}$$

We should point out that we limit the observation to only 2 sub-systems: production and recycling (waste disposal). Net production **z** of the system is thus:

$$\mathbf{z} = \mathbf{y} - \mathbf{x} = \mathbf{GP} - \mathbf{HP} = (\mathbf{G} - \mathbf{H})\mathbf{P} = \begin{bmatrix} 150,328 \\ -15,960 \\ -4,560 \\ -5,700 \\ -4,180 \\ -4,180 \\ -1,520 \\ -380 \\ -1,520 \\ -152,000 \\ \hline 1,672 \end{bmatrix} \tag{4}$$

During each cycle, we produce 150,328 jars of baby food, which are suitable for launch to the market, and 1,672 jars are waste which should be disposed of. For each production cycle, we need 15,960 kg, 4,560 kg, 5,700 kg, 4,180 kg, 4,180 kg, 1,520 kg and 380 kg of ingredients A, B, C, D, E, F, G and H, respectively. Additionally, we need 152,000 pieces of a package to complete each production cycle.

We can capture price properties of items at each stage inside the vector **p** using the following form:

$$\mathbf{p} = \left[0.6155 \quad 0.006 \quad 0.8434 \quad 0.5 \quad 6.9277 \quad 0.6988 \quad 0.6024 \quad 1.8072 \quad 0.7229 \quad 0.0843 \;\vdots\; -0.145\right] \tag{5}$$

Further, setup costs only appear with the production process, so vector **K** can be written as:

$$\mathbf{K} = \left[-20,000 \;\vdots\; 0\right] \tag{6}$$

Since initiation times t and lengths of the cycles T are known, we can calculate given timings $\tilde{\mathbf{v}}(\rho)$:

$$\tilde{\mathbf{v}}(\rho) = \tilde{\mathbf{t}}(\rho)\tilde{\mathbf{T}}(\rho) = \left[\begin{array}{c|c} e^{-\rho t_1} & \\ \hline & e^{-\rho t_2} \end{array}\right]\left[\begin{array}{c|c} (1-e^{-\rho T_1})^{-1} & \\ \hline & (1-e^{-\rho T_2})^{-1} \end{array}\right] = \left[\begin{array}{c|c} e^{-41\rho} & \\ \hline & e^{-52\rho} \end{array}\right]\left[\begin{array}{c|c} (1-e^{-10\rho})^{-1} & \\ \hline & (1-e^{-10\rho})^{-1} \end{array}\right] =$$
$$= \left[1,213.07 \;\vdots\; 1,211.98\right] \tag{7}$$

When we add delivery, quarantine and production lead times to physical structures of matrices **H** and **G**, we can write them in generalized form:

$$\breve{\mathbf{H}} = \left[\begin{array}{c:c} 0.105e^{\rho(1+0+1)} & \\ 0.03e^{\rho(6+1+1)} & \\ 0.0375e^{\rho(5+1+1)} & \\ 0.0275e^{\rho(10+28+1)} & \\ 0.0275e^{\rho(5+1+1)} & \\ 0.01e^{\rho(9+31+1)} & \\ 0.0025e^{\rho(5+14+1)} & \\ 0.01e^{\rho(10+14+1)} & \\ 1e^{\rho(16+0+1)} & \\ \hdashline & \end{array}\right] \qquad \breve{\mathbf{G}} = \left[\begin{array}{c:c} 0.989e^{-\rho*11} & \\ & \\ \hdashline & 0.011e^{-\rho*11} \end{array}\right] \tag{8}$$

Company can borrow money at the interest rate of 3.50% (ρ = 0.035 per year). Above information allows us to calculate overall NPV of the cyclical system with an infinite number of repeating cycles:

$$\text{NPV}_{infinite} = \mathbf{p}\left(\breve{\mathbf{G}}(\rho) - \breve{\mathbf{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho) - \hat{\mathbf{K}}\tilde{\mathbf{v}}(\rho) = 18{,}559{,}553.20 \text{ €} \tag{9}$$

The NPV of only the first cycle is:

$$\text{NPV}_{first\_cycle} = \mathbf{p}\left(\breve{\mathbf{G}}(\rho) - \breve{\mathbf{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho) - \hat{\mathbf{K}}\tilde{\mathbf{v}}(\rho) = 17{,}788.30 \text{ €} \tag{10}$$

Long-term production of the final product is profitable if all the parameters stay constant for an infinite number of repeating cycles. Further, we can analyse the behaviour of the observed system and its sensibility to the change of individual parameters. Figure 3 denotes how change of continuous interest rate reflects in overall NPV of the system. Such analysis can help company recognize marginal interest rate which ensures long-term sustainability of its supply chain. Very high interest rates could under some circumstances turn profitability of individual final product to negative. We denote such a situation at Figure 4 where we can clearly see that extremely high interest rate in combination with long payment lead times (delays) would make production of baby meat food unprofitable.

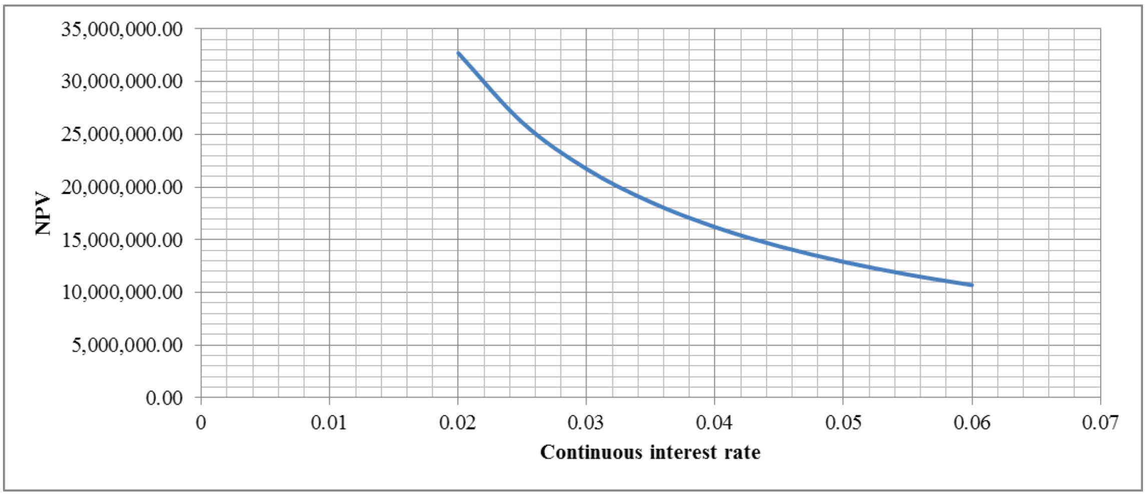

Figure 3: The impact of continuous interest rate on overall NPV of the system.

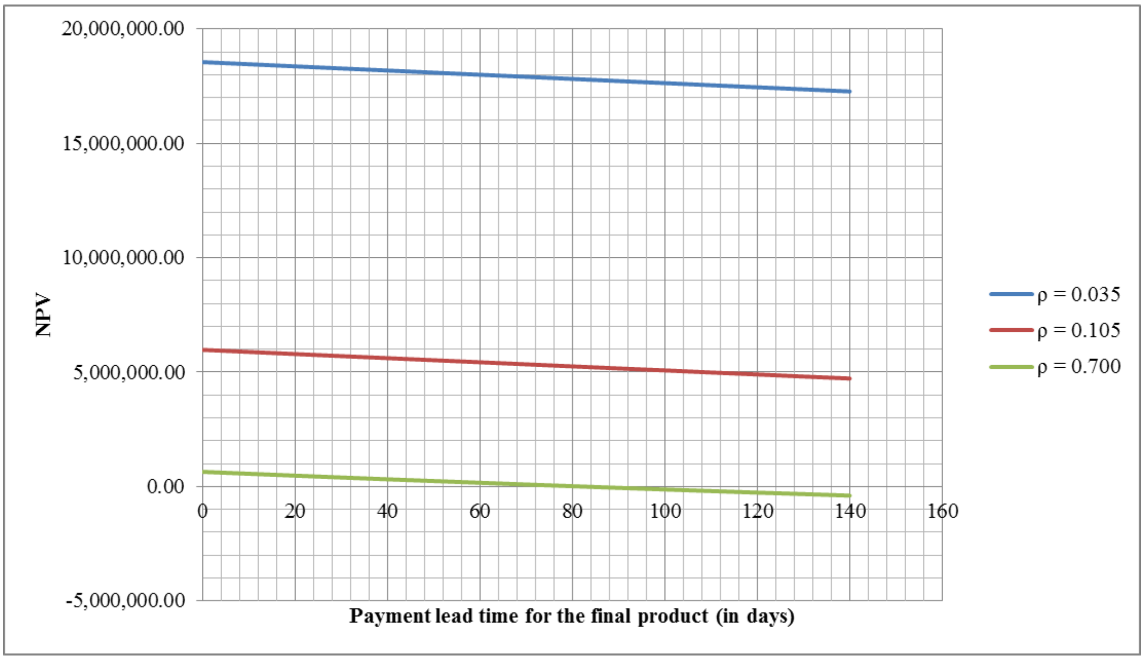

Figure 4: The impact of payment lead times on the NPV at different continuous interest rate levels.

Company produces relatively small amounts of waste (1.1%). Nevertheless, the performance of the system could be still slightly improved with the proper investment into modification of production line (Figure 5). Deeper insight into the production process reveals that most of the waste is generated inside of the transportation pipes. With proper technological improvements, it would be possible to decrease the quantity of ingredients which stay inside the pipes and have to be disposed of at the end of the production process. We assume that such technological changes would change the balance between setup costs and waste. Simulation at Figure 5 shows the area with possible solutions, but not all of them are feasible. Feasibility is subject to technological constraints. The presented model is a usable tool for choosing technological improvement, which will result in an increase of the NPV. If such solution does not exist the process is optimal, or it can be improved by changing one or more of the other parameters.

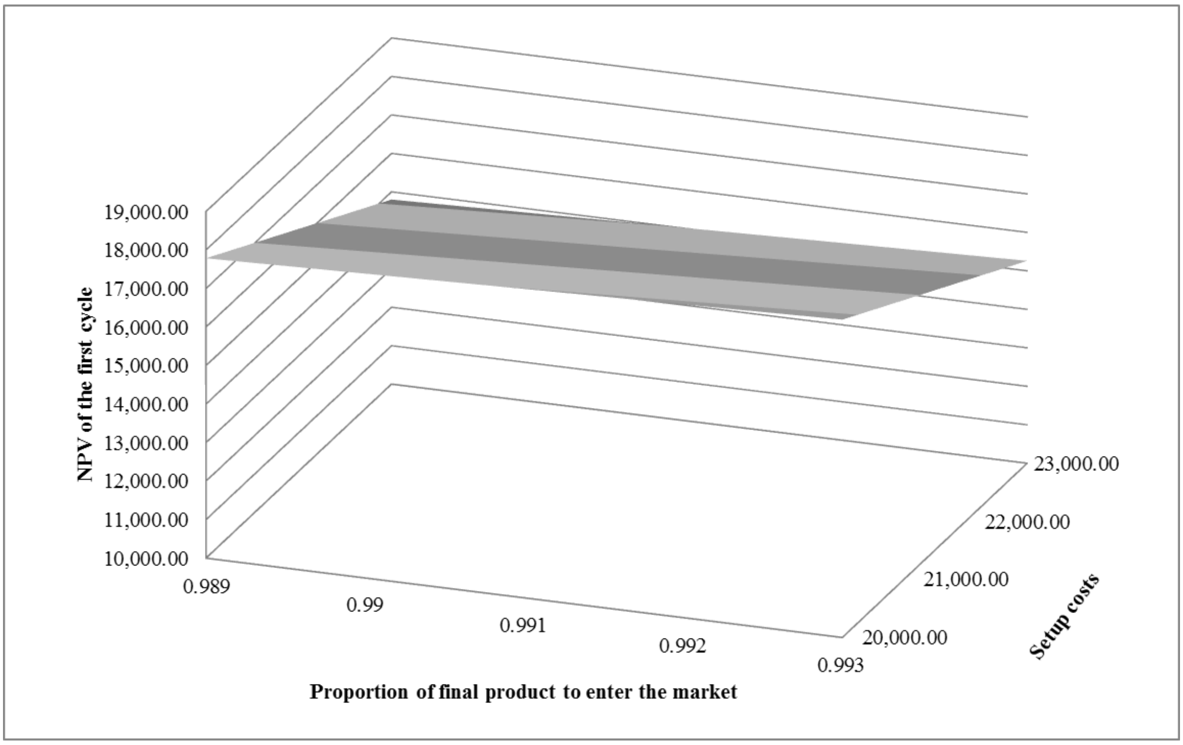

Figure 5: Area of all theoretically possible NPVs for the first production cycle in dependence of correlation between setup costs and generated waste.

5. CONCLUSION

In this work, we analyse the behaviour of supply chain on a real case study of a Spanish food producer. The main research focus is on production and waste disposal part of extended supply chain. We use EMRP Theory for this purpose, which makes this paper one of the few attempts to use this otherwise intensively and well developed theory on a practical application.

Although the presented paper in some ways simplifies the real situation, it clearly shows that EMRP Theory can be strong analytical and also decision tool when dealing with real-life supply chain problems. Systems performance is evaluated through the NPV calculation, which is one of the main pillars of the EMRP Theory.

Quantity of residues in the production process of the company is relatively small. In extended observation, distances between regions of activity cells or production and the final user increase. Large distances result in significant transportation lead times and costs. If complete logistics is not organised optimally the quantity of waste can be significant. This fact is especially apparent with perishable goods inside large international supply chains. Such sub optimality could result in a decrease of the NPV of complete supply chain, or even its negativity. EMRP Theory gives us a sufficient tool for evaluation of profitability of complete supply chain and for establishing long-term economic-environmental sustainability.

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