FINANCIAL STRUCTURE OPTIMIZATION BY USING A GOAL PROGRAMMING APPROACH

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Abstract

This paper proposes a new methodology for solving the multiple objective fractional linear programming problems using Taylor's formula and goal programming techniques. The proposed methodology is tested on the example of company's financial structure optimization. The obtained results indicate the possibility of efficient application of the proposed methodology for company's financial structure optimization as well as for solving other multi-criteria fractional programming problems.

Key words: *multiple objective, fractional linear programming, goal programming, Taylor's formula, financial structure optimization*

1. INTRODUCTION

The first study dealing with the problem of fractional linear programming (Charnes and Cooper. 1962) gave rise to a large number of papers referring to FLP with one or more objectives.

In the problem the FLP goal function is fractionally linear and the constraints are linear making a convex set. As in the FLP problem the goal function is non-linear it is not possible to apply the simplex method for LP, various methods have been developed to solve it: Charnes and Cooper (1961), Bitran and Novaes (1973), Martos (1964).

The multiple objective linear fractional programming problem (MOLFP) is considered in various articles, for example: Gupta and Bhatia (2001), Guzel and Sivri (2005), Kornbluth and Steuer (1981), Nikowski and Zolkiski (1985), Saad (2007).

Solving MOLFP problems entails some calculating difficulties; therefore they are converted into single objective LFP problems and then solved using the method of Bitran and Novaes (1973) or Charnes and Cooper (1962).

In a goal programming problem, when the goals are linear fractional functions, the formulation of the goal programming problem to be solved is quite complex because of non-linear constraints (R. Caballero and M. Hernandez, 2006). In the literature there are very few references to goal programming with fractional goals, except for the papers of Hannan (1977, 1981), Soyster end Lev (1978), and an article by Kornbluth and Steuer (1981).

In this paper we seek to find an efficient algorithm to solve MOLFP problems by using goal programming techniques. To solve the problem of non-linear functions in the constraints set we propose linearization by application of Taylor's formula around the optimal point of the given function on the given set of constraints.

The proposed methodology is tested on the example of company's financial structure optimization.

The first section of this paper considers the theoretical approach to company's financial structure optimization. In the second section we first define the multi-criteria programming model and the main four approaches to solving MP models, among which goal programming is the most prominent. The third section presents the MOLFP model and the linearization of fractional linear function using Taylor series approach. In the fourth section the proposed methodology is tested on the practical problem of company's financial structure optimization. The concluding section presents the most important results obtained in the course of research.

2. COMPANY'S FINANCIAL STRUCTURE OPTIMIZATION

For a long time the effect of company's financial structure on estimation of its value has been in the focus of attention of numerous economists and business people. The reasons for this interest lie in the fact that different levels of company indebtedness have different implications. Thus the consequences of increased indebtedness level are increased financial risk, reduced credit rating, and increased cost of financing.

There are different theories for company financial structure. According to some of them the company value directly depends on its financial structure, while according to some other theories financial structure has at all no effect on company value.

According to the traditional theory represented in this paper, there is direct interdependence of financial structure and company value. This results from the exposure to financial risk, and thus companies with higher levels of indebtedness are required to offer higher return rates on the issued debt instruments to compensate the risk. In addition to that, there is also the financing cost with the own capital, which is higher than the financing cost rate of the borrowed capital. As the total financing cost is calculated as the sum of financing costs by both the own and borrowed capital, which are weighted by their shares in value, it is evident that there is an optimal financial structure entailing the minimal total cost. This results from the different intensity of the weighted average capital cost shares.

This means that the financial structure has important implications on the company value. Companies should choose such financial structure that will ensure minimal debt and minimal ratio of the current liabilities and capital, and maximal profitability and turnover (Lai Y. J. and Hwang C. L., 1994).

Generally, financial structure is determined by financial markets, financial institutions, supply of instruments in financial markets, and demand for these instruments in different sectors.

Company financial structure is determined by the relation of finance resources and company assets. Vertical financial structure reveals the relations of different assets, capital, and liabilities. For example, the share of liquid assets in total assets, the relation of short-term assets to long-term assets, the share of debtors in the total assets, the share of debt in the total capital, etc.

Horizontal financial structure refers to interrelation of different assets on one side and capital and liabilities on the other. For example: liquid assets to current liabilities, long-term liabilities to long-term assets, capital to long-term assets.

The key factors of financial structure are: business activity, competitiveness, development of financial system, management ability, etc.

The ability of the company to carry out its activities in the market depends on its capital structure efficiency. Debt and capital are the two main components of the total company capital. Debt is the amount obtained from financial resources such as individuals, banks or other financial institutions. Capital is the owners’ share in the company including share capital, share premium, preference share capital, reserves, and capital surplus.

The leverage of capital and debt in the capital structure differs among companies. Capital structure also varies according to industry or market situation in which the company operates.

The main goal of capital structure optimization is selecting the proportion of different forms of liabilities and capital which will maximize the company value and minimize the average cost of capital. Although this issue has been extensively researched, there is no general formula or theory that unquestionably ensures an optimal capital structure for any company.

3. MULTICRITERIA FRACTIONAL LINEAR PROGRAMMING

3.1. Model of multiple objective linear fractional programming (MOLFP)

Pal et al. (2003) define the general form of MOLFP, in the following way:

If $Z_k(x) = \dfrac{c_k x + \alpha_k}{d_k x + \beta_k}, \quad x \in R^n,\ c_k, d_k \in R^n,\ \alpha_k, \beta_k \in R,$ then

$$\text{Max } Z(x) = (z_1(x), z_2(x), \ldots, z_K(x)), \tag{1}$$

$$\text{s.t.} \qquad Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \tag{2}$$

$$x \geq 0,\ A \in R^{m \times n},\ b \in R^m. \tag{3}$$

3.2. Solving MOLFP model by goal programming

To solve the model (1) – (3) by the goal programming method we have to find marginal solutions for all the goal functions in the given constraints set with goal function values: $z_1^*, z_2^*, \ldots, z_k^*$. After that we form the goal programming model in one of the four possible ways:

(i) The Min – max form:

(MFG1)

$$\text{Min max } g_k(n_k, p_k) \tag{4}$$

$$\text{s.t.} \qquad \frac{c_k x + \alpha_k}{d_k x + \beta_k} + n_k - p_k = \overline{Z}_k,\ k = 1, 2, \ldots, K \tag{5}$$

$$Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \tag{6}$$

$$x \geq 0,\ n_k \geq 0,\ p_k \geq 0,\ n_k \cdot p_k = 0,\ k = 1, 2, \ldots, K. \tag{7}$$

Aspiration level $\overline{Z}$ is determined by the decision maker or is equal to Z^*.

(ii) The minimization of the sum of deviations form:

(MFG2)

$$\text{Min } \sum_{k=1}^{K} g_k(n_k, p_k) \tag{8}$$

$$\text{s.t.} \qquad \text{constraints (5) – (7)} \tag{9}$$

(iii) The minimization of the weighted sum of deviations form:

(MFG3) Min $\sum_{k=1}^{K} w_k g_k(n_k, p_k)$ (10)

s.t. constraints (5) – (7), (11)

where w_k (k = 1, 2, …, K) are weights determined by the decision maker.

(iv) The preemptive priority form:

In this form the K objectives are rearranged according to decision maker's priority levels, the highest priority goal is considered first, then the second and so on. The general lexicographical goal program is:

(MFG4) Min $a = \left\{ \sum_{k \in P_i} w_k g_k(n_k, p_k): \ i = 1, 2, ..., I \right\}$ (12)

s.t. constraints (5) – (7), (13)

where I is the number of priority levels and $k \in P_i$ means that the kth goal is in the ith priority level.

Models (MFG1), (MFG2), (MFG3) and (MFG4) are nonlinear programming models which cannot be solved by the simplex method. The non-linear functions in the constraints set are a particular problem, which significantly complicates the solving process.

3.3. Linearization of fractional linear function using the Taylor series approach

In the models (MFG1), (MFG2), (MFG3) and (MFG4) fractional functions z_k(x) = $\frac{c_k x + \alpha_k}{d_k x + \beta_k}$ are transformed into linear functions by using Taylor series (Toksary, 2008). The fractional functions in the constraints of the specified models are replaced with the linearized functions.

Linearization procedure is carried out in two steps:

Step 1. Determine $x_k^* = (x_{k1}^*, x_{k2}^*, ..., x_{kn}^*)$ which is the value that maximizes the FL objective function $z_k(x)$,

k = 1, 2, …, K.

Step 2. Transform $z_k(x)$ by using first-order Taylor polynomial series (Toksari, 2008). Consequently:

$$z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k} \cong \hat{z}_k(x) = Z_k(x_k^*) + \left[(x_1 - x_{k1}^*) \frac{\partial Z_k(x_k^*)}{\partial x_1} + (x_2 - x_{k2}^*) \frac{\partial Z_k(x_k^*)}{\partial x_2} + ... + (x_n - x_{kn}^*) \frac{\partial Z_k(x_k^*)}{\partial x_n} \right] \quad (14)$$

In this way obtained models (MFG1), (MFG2) and (MFG3) are linear programming models that can be solved by the simplex method, and the model (MFG4) can be solved by the multiphase simplex method (Lee, 1972) or the sequential simplex method (Ignizio, 1982).

4. PRACTICAL APPLICATION: FINANCIAL PLANNING

4.1. The problem

Consider a firm which is expected to reach US$ 60.0 million of capital in the next year. In order to increase the firm value, the firm's financial manager wants to improve the financial condition of the company by optimally constructing the financial structure. Based on the expected sales for the next year, it is desired to maximize the manager's satisfaction with some financial ratios. The Table 1 shows the variables which are considered. The manager's preferences of key financial ratios are summarized in the Table 2. The four conflicting fractional goals are as follows: (1) minimization of the current ratio, (2) minimization of the debt ratio, (3) maximization of the turnover ratio and (4) maximization of the profitability ratio.

Table 1: Definition of variables in the balance sheet (B/S)

Assets	Variable	Expected values	Liabilities and equality	Variable	Expected values
Current assets	x_{11}	$150 \leq x_{11} \leq 250$	Current liabilities	x_{21}	$75 \leq x_{21} \leq 300$
Fixed assets	x_{12}	$x_{12} \leq 300$	Long-term liabilities	x_{22}	$x_{21} + x_{22} \geq 250$ $100 \leq x_{22} \leq 300$
Total assets	$x_{11}+x_{12}$	$x_{11} + x_{12} \geq 350$	Shareholders equity	x_{23}	$75 \leq x_{23} \leq 125$
			Retained earning added	x_{24}	$100 \leq x_{24} \leq 140$
			Total liabilities and equity	$x_{21}+x_{22}+x_{23}+x_{24}$	

4.2. MOFLP model

The above data are the basis for the following MOFLP model:

Min $z_1(x) = \dfrac{x_{11}}{x_{21}}$, {Current ratio} (15)

Min $z_2(x) = \dfrac{x_{21}+x_{22}}{x_{23}+x_{24}}$, {Debt ratio} (16)

Max $z_3(x) = \dfrac{60}{x_{11}+x_{12}}$ {Turnover ratio} (17)

Max $z_4(x) = \dfrac{x_{24}}{60}$ {Profitability ratio} (18)

s.t. $x_{11} + x_{12} = x_{21} + x_{22} + x_{23} + x_{24}$, (19)

$150 \le x_{11} \le 250$, (20)

$x_{12} \le 300$, (21)

$x_{11} + x_{12} \ge 350$, (22)

$75 \le x_{21} \le 300$, (23)

$100 \le x_{22} \le 300$, (24)

$x_{21} + x_{22} \ge 250$, (25)

$75 \le x_{23} \le 125$, (26)

$100 \le x_{24} \le 140$, (27)

$x_{11}, x_{12}, x_{21}, x_{22}, x_{23}, x_{24} \ge 0$. (28)

4.3. The model solving

Marginal solutions, obtained by maximizing each of the four objective functions individually on a given set of constraints, are presented in table 2.

The functions z_1, z_2 and z_3 are not linear. The linearization of the function $z_1(x)$ by application of the first order Taylor's formula looks like this:

$$z_1(x) = \frac{x_{11}}{x_{21}} \cong z_1(x_1^*) + (x_{11} - x_{11}^*)\frac{\partial z_1(x_1^*)}{\partial x_{11}} + (x_{12} - x_{12}^*)\frac{\partial z_1(x_1^*)}{\partial x_{12}} + (x_{21} - x_{21}^*)\frac{\partial z_1(x_1^*)}{\partial x_{21}} +$$

$$+(x_{22} - x_{22}^*)\frac{\partial z_1(x_1^*)}{\partial x_{22}} + (x_{23} - x_{23}^*)\frac{\partial z_1(x_1^*)}{\partial x_{23}} + (x_{24} - x_{24}^*)\frac{\partial z_1(x_1^*)}{\partial x_{24}} = 0.8571 + \tag{29}$$

$$+(x_{11} - 150)\cdot 0.0057 + (x_{12} - 300)\cdot 0 + (x_{21} - 175)\cdot(-0.0049) + (x_{22} - 100)\cdot 0 +$$

$$+(x_{23} - 75)\cdot 0 + (x_{24} - 100)\cdot 0 = = 0.005714285x_{11} - 0.004897959x_{21} + 0.857142857$$

Table 2: Marginal solutions

Marginal solution	Variable values		z_1	z_2	z_3	z_4
x_1^*	$x_{11} = 150$	$x_{12} = 300$	0.8571	1.5714	0.1333	1.6667
	$x_{21} = 175$	$x_{22} = 100$				
	$x_{23} = 75$	$x_{24} = 100$				
x_2^*	$x_{11} = 215$	$x_{12} = 300$	1.4333	0.9434	0.1165	2.3333
	$x_{21} = 150$	$x_{22} = 100$				
	$x_{23} = 125$	$x_{24} = 140$				
x_3^*	$x_{11} = 220$	$x_{12} = 205$	2.9333	1.4286	0.1412	1.6667
	$x_{21} = 75$	$x_{22} = 175$				
	$x_{23} = 75$	$x_{24} = 100$				
x_4^*	$x_{11} = 250$	$x_{12} = 300$	3.3333	1.5581	0.1091	2.3333
	$x_{21} = 75$	$x_{22} = 260$				
	$x_{23} = 75$	$x_{24} = 140$				

The functions z_2 and z_3 are linearized analogously. Thus:

$$z_2(x) = \frac{x_{21}+x_{22}}{x_{23}+x_{24}} \cong 0.003773584x_{21} + 0.003773584x_{22} - 0.00359985x_{23} - \\ -0.003559985x_{24} + 0.9434 \quad (30)$$

$$z_3(x) = \frac{60}{x_{11}+x_{12}} \cong -0.000332179x_{11} - 0.000332179x_{12} + 0.282352575 \quad (31)$$

Based on this calculation we can form the linear goal programming model:

(i) The Min-Max form:

Min max $g_k(n_k, p_k)$, $k = 1, 2, 3, 4.$ (32)

s.t. $$0.0057x_{11} - 0.0049x_{21} + 0.8571 + n_1 - p_1 = \bar{z}_1 \quad (33)$$

$$0.0038x_{21} + 0.0038x_{22} - 0.0036x_{23} - 0.0036x_{24} + 0.9434 + n_2 - p_2 = \bar{z}_2 \quad (34)$$

$$-0.00033x_{11} - 0.00033x_{12} + 0.2824 + n_3 - p_3 = \bar{z}_3 \quad (35)$$

$$\frac{x_{24}}{60} + n_4 - p_4 = \bar{z}_4 \quad (36)$$

constraints (19) – (28), (37)

$n_k \geq 0$, $p_k \geq 0$, $n_k \cdot p_k = 0$, $k = 1, 2,3,4$, (38)

where $g_1(n_1, p_1) = p_1$, $g_2(n_2, p_2) = p_2$, $g_3(n_3, p_3) = n_3$, $g_4(n_4, p_4) = n_4$, and $\bar{z}_k = z_k^*$, $k = 1, 2, 3, 4$.

The model (32) – (38) is converted to a linear program as follows:

$$\text{Min } \lambda \tag{39}$$

$$\text{s.t.} \quad \text{constraints (33) – (38)} \tag{40}$$

$$\lambda \geq g_k(n_k, p_k),\ k = 1,2,3,4. \tag{41}$$

Model (39) – (41) is solved by the simplex method.

(ii) The minimization of the sum of deviations form:

$$\text{Min } \sum_{k=1}^{4} g_k(n_k, p_k) \tag{42}$$

$$\text{s.t.} \quad \text{constraints (33) – (38),} \tag{43}$$

where $g_1(n_1, p_1) = p_1$, $g_2(n_2, p_2) = p_2$, $g_3(n_3, p_3) = n_3$, and $g_4(n_4, p_4) = n_4$. This model can be solved using the simplex method.

(iii) The minimization of the weighted sum of deviations form:

$$\text{Min } \sum_{k=1}^{4} w_k g_k(n_k, p_k) \tag{44}$$

$$\text{s.t. constraints (33) – (38),} \tag{45}$$

where $g_1(n_1, p_1) = p_1$, $g_2(n_2, p_2) = p_2$, $g_3(n_3, p_3) = n_3$, $g_4(n_4, p_4) = n_4$, and w_k ($k = 1, 2, 3, 4$) are determined by the decision maker. In our model we put $w_1 = 0.4$, $w_2 = 0.3$, $w_3 = 0.2$ and $w_4 = 0.1$. The model (44) – (45) is solved by using the simplex method.

(iv) The preemptive priority form: The four objectives are ranked according to their priority: the goal 1 has priority 1, the goal 2 has priority 2, the goal 3 has priority 3, and the goal 4 has priority 4. The general lexicographical goal program is:

$$\text{Min a} = \left\{ \sum_{k \in P_i} w_k g_k(n_k, p_k) : i = 1,2,3,4 \right\} \tag{46}$$

$$\text{s.t.} \quad \text{constraints (33) – (38),} \tag{47}$$

where $k \in P_i$ means that kth goal is in the ith priority level. In our model the weights are the same as in the previous case. The model (46) – (47) is solved by using the sequential simplex method.

4.4. The solutions

The solutions of the models (39) – (41), (42) – (43), (44) – (45) and (46) – (47) are showed in the Table 3.

Table 3: Goal programming solutions

Goal programming solution	Variable values		z_1	z_2	z_3	z_4
(*i*)	$x_{11} = 161.75$	$x_{12} = 300.00$	1.0783	1.1806	0.1299	2.1438
	$x_{21} = 150.00$	$x_{22} = 100.00$				
	$x_{23} = 83.12$	$x_{24} = 128.63$				
(*ii*)	$x_{11} = 165.00$	$x_{12} = 300.00$	1.1000	1.1628	0.1290	2.3333
	$x_{21} = 150.00$	$x_{22} = 100.00$				
	$x_{23}= 75.00$	$x_{24} = 140$				
(*iii*)	$x_{11} = 165.00$	$x_{12} = 300.00$	1.1000	1.1628	0.1290	2.3333
	$x_{21} = 150.00$	$x_{22} = 100.00$				
	$x_{23} = 75.00$	$x_{24} = 140.00$				
(*iv*)	$x_{11} = 183.59$	$x_{12} = 300.00$	1.0590	1.3000	0.1241	2.2539
	$x_{21} = 173.35$	$x_{22} = 100.00$				
	$x_{23} = 75.00$	$x_{24} = 135.23$				

Table 4: Solution after the first phase

Goal programming solution	Variable values		z_1	z_2	z_3	z_4
(*iv*)-1	$x_{11} = 150.00$	$x_{12} = 300.00$	0.8571	1.5714	0.1333	1.6667
	$x_{21} = 175.00$	$x_{22} = 100.00$				
	$x_{23} = 75.00$	$x_{24} = 100.00$				

Let us explain the model (46) – (47) solving procedure by the sequential simplex method. First we solve the following model:

$$\text{Min } 0.4\, p_1 \tag{48}$$

$$\text{s.t. constraints (33) – (38).} \tag{49}$$

The obtained solution is in table 4.

In order to allow the improvement of the function z_2, in the second phase we solve the following problem:

$$\min 0.3\, p_2 \tag{50}$$

$$\text{s.t. constraints (33) - (38)} \tag{51}$$

$$0.005714285\, x_{11} - 0.\ 004897959\, x_{21} = 0 + 0.2 \tag{52}$$

The obtained solution is as follows:

Table 5: Solution after the second phase

Goal programming solution	Variable values		z_1	z_2	z_3	z_4
(iv)-2	$x_{11} = 163.57$	$x_{12} = 300.00$	1.0905	1.1706	0.1294	2.3095
	$x_{21} = 150.00$	$x_{22} = 100.00$				
	$x_{23} = 75.00$	$x_{24} = 138.57$				

In the third phase the following LP model is solved:

$$\min n_3 \tag{53}$$

$$\text{s.t. constraints (33) – (38)} \tag{54}$$

$$0.005714285\, x_{11} - 0.\ 004897959\, x_{21} = 0 + 0.2 \tag{55}$$

$$0.003773584\, x_{21} + 0.003773584\, x_{22} - 0.003559985\, x_{23}$$
$$- 0.003559985\, x_{24} = 0.183084999 + 0.05. \tag{56}$$

The obtained solution is in table 6.

In the fourth phase the following LP model is solved:

$$\min n_4 \tag{57}$$

$$\text{s.t. constraints (55) – (60)} \tag{58}$$

$$0.005714285\, x_{11} - 0.\ 004897959\, x_{21} = 0 + 0.2 \tag{59}$$

$$0.003773584\, x_{21} + 0.003773584\, x_{22} - 0.003559985\, x_{23} -$$

$$– 0.003559985\, x_{24} = 0.183084999 + 0.05 \quad (60)$$

$$– 0.000332179\, x_{11} – 0.000332179\, x_{12} = – 0.149321104 \quad (61)$$

Table 6: Solution after the third phase

Goal programming solution	Variable values		z_1	z_2	z_3	z_4
(*iv*)-3	$x_{11} = 163.57$	$x_{12} = 285.95$	1.0905	1.2529	0.1335	2.0755
	$x_{21} = 150$	$x_{22} = 100.00$				
	$x_{23} = 75.00$	$x_{24} = 124.53$				

The obtained solution is identical to the solution presented in the Table 6. The obtained solution cannot be further improved even with the reduced level of the goal 3 without reducing the value of the goals 1 and 2.

5. CONCLUSION

This paper proposes the methodology of fractional linear goal functions linearization by use of Taylor's formula to solve MOLFP problem by goal programming methods.

The proposed methodology is tested on the problem of company's financial structure optimization.

The obtained results reveal the possibility of an efficient application of the proposed methodology in solving the given problem.

The paper presents four approaches to the goal programming problem for solving company's optimal financial structure. The proposed methodology allows the use of lexicographic simplex method in the process of obtaining the preferred solution with active participation of the decision maker.

Further research will involve finding valid analytical proofs that linearization of fractional linear goal functions and subsequent solving of multi-criteria fractional linear programming model by using goal programming techniques leads to satisfactory solutions. We have shown numerically in the examples solved in this paper that the solutions obtained in this way are identical to the solutions obtained by applying methods without linearization.

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