USAGE OF INTERVAL CAUSE-EFFECT RELATIONSHIP COEFFICIENTS IN THE QUANTITATIVE MODEL OF STRATEGIC PERFORMANCE

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Abstract

This paper proposes the method to obtain values of the coefficients of cause-effect relationships between strategic objectives in the form of intervals and use them in solving the problem of the optimal allocation of organization's resources. We suggest taking advantage of the interval analytical hierarchy process for obtaining the intervals. The quantitative model of strategic performance developed by M. Hell, S. Vidučić and Ž. Garača is employed for finding the optimal resource allocation. The uncertainty originated in the optimization problem as a result of interval character of the cause-effect relationship coefficients is eliminated through the application of maximax and maximin criteria. It is shown that the problem of finding the optimal maximin, maximax, and compromise resource allocation can be represented as a mixed 0-1 linear programming problem. Finally, numerical example and directions for further research are given.

Key words: *Interval analytical hierarchy process, Quantitative model of strategic performance, Cause-effect relationship coefficients, Resource allocation problem, Extreme optimism (pessimism) criterion, Mixed 0 1 linear programming problem, Balanced Scorecard*

1. INTRODUCTION

Strategic performance management is a relatively young field of managerial science. It deals with problems of effective strategy implementation and validation of its contribution to organization’s success (De Wall, 2006). The most popular methodology in strategic performance management is the Balanced Scorecard (BSC) concept (Kaplan, Norton, 1996).

The approach to strategic performance optimization suggested in (Hell, Vidučić, Garača, 2009) attracts special attention. On the basis of the BSC methodology they built the quantitative model of strategic performance (QMSP) and proposed an approach to optimization of organization effectiveness through the optimal allocation of available resources. Matrix algebra and linear programming were used for formal problem definition. Description of the approach to the optimal resource allocation with QMSP is presented in the following Section.

2. OPTIMAL ALLOCATION OF AVAILABLE RESOURCES WITH THE QMSP

According to the QMSP strategy is presented as a graph of objectives (let all objectives be enumerated from 1 to n). Objectives must be linked with cause-effect relationships. An example of a cause-effect relationships graph is shown in Fig. 1.

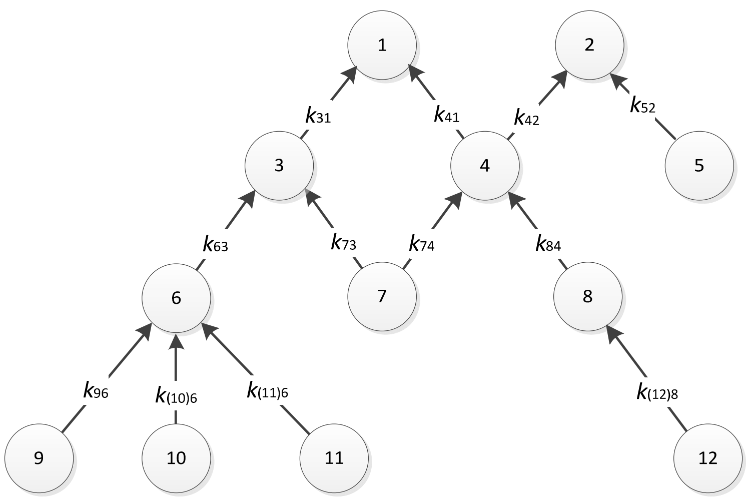

Fig. 1. An example of a cause-effect relationships graph

Each relationship has a weight coefficient $k_{ij} \geq 0$ which indicates the maximum allowable accomplishment level of the j-th superior objective when the accomplishment level of the i-th subordinate one is equal to 1 (100%) and accomplishment levels of the other subordinate objectives are equal to 0. It's assumed that the weight coefficients are normalized to the sum of 1:

$$\sum_{i \in N_j} k_{ij} = 1 , \ j = \overline{1, n}.$$

where N_j is a set of numbers of the objectives which are subordinate to the j-th objective.

Let an organization have s types of resources and vector $(R_1, R_2, \ldots, R_s)^T$ show an available amount of each resource. Let the technological coefficient r_{ij} be an amount of the i-th resource that is needed to achieve 100% level of the j-th objective's accomplishment.

Under these assumptions the linear programming problem (LPP) (1.1)–(1.4) for finding the vector x^* of strategic objectives' accomplishment levels corresponding to the optimal resource allocation can be formulated:

1. Criterion (maximization of the weight sum of strategic objectives' accomplishment levels):

$$I_1 = \sum_{j=1}^{n} w_j x_j \to \max_x, \tag{1.1}$$

   where w_j is a weight coefficient of the j-th objective (it's assumed that $w_j \geq 0, \ \sum_{j=1}^{n} w_j = 1$).

2. Resource constraints:

$$\sum_{j=1}^{n} r_{ij} x_j \leq R_i, \ i = \overline{1, s}. \tag{1.2}$$

3. Structure constraints:

$$x_j \leq \sum_{i \in N_j} k_{ij} x_i , \ j = \overline{1, n}. \tag{1.3}$$

4. Assumption that accomplishment levels belong to the interval [0,1]:

$$0 \leq x_j \leq 1, \ j = \overline{1, n}. \tag{1.4}$$

It's obvious that the optimal amount of the i-th resource which must be spent to achieve the j-th objective is equal to $x_j^* r_{ij}, j = \overline{1, n}$.

One of the main issues concerning usage the QMSP in practice is a problem of estimation of the cause-effect relationship coefficients $\{k_{ij}\}$. There are several papers dealing with this question. In (Rodrigues, Alfaro, Ortiz, 2009) it's proposed to use data-mining algorithm (principal components analysis with the least squares method) for identification and measuring relationships between key performance indicators which correspond to strategic objectives; in (Jassbi, Mohamadnejad, Nasrollahzadeh, 2011) fuzzy Decision Making Trial and Evaluation Laboratory (DEMATEL) is employed as a framework for estimation of cause-effect relationships between strategic objectives; in (Suwignjo, Bititci, Carrie, 2000) analytic hierarchy process (AHP) is used for similar purpose. Approaches which were suggested in the last two papers can be used without historical data so they are suitable for preliminary evaluation of the relationship coefficients. The former have such an advantage that it allows to deal with vague assessments and the latter conforms to the QMSP well. In this paper we present the method which combines both mentioned advantages. It is described in the next Section. Numerical example is demonstrated in Section 4.

3. THE METHOD PROPOSED

The method proposed consists of two stages. In the first stage the interval analytic hierarchy process (IAHP) is employed in order to obtain variation intervals of the cause-effect relationship coefficients. In the second stage special criteria (maximin and maximax) are used for finding the optimal resource allocation.

STAGE 1: obtaining estimates of cause-effect relationship coefficients' values in the form of intervals.

Step 1.1. Let $p \coloneqq 1$.

Step 1.2. Renumber objectives belonging to N_p with the numbers $1,2,\ldots,n_p$ ($n_p = |N_p|$). Let $n'_p \coloneqq n_p + 2$ and $e_p(\cdot)\colon \{1,2,\ldots,n'_p\} \to \{N_p, n+1, n+2\}$ is such a function that $e_p(i) = j$, where *j* is an old (global) number of an objective, i is a new (local) number of an objective; $e_p\big(n_p +$ *1=n+1, epnp+2=n+2*. Note, that at the first stage of the method objectives are designated with the new numbers.

Step 1.3. Form an expert group consisting of persons who can estimate an influence of an achievement of the objectives belonging to N_p, as well as other favorable and unfavorable factors, on an achievement of the *p*-th objective. Let this group include m_p experts.

Step 1.4. Estimate the competence of the experts: assign a coefficient of competence c_q to the q-th expert ($q = \overline{1, m_p}$). We consider that $c_q \geq 0$ and $\sum_{q=1}^{m_p} c_q = 1$. This step is not necessary because it can be assumed that competence coefficients are uncertain.

Step 1.5. For each pair of objectives $i, j = \overline{1, n_p},\ i < j$ which are subordinate to the *p*-th one, ask each expert to answer the question: "How many times an achievement of the *i*-th objective is more important for an achievement of the *p*-th one than an achievement of the *j*-th objective?" The answer of the q-th expert ($q = \overline{1, m_p}$) must be expressed as a number $a_{ijp}^q \in \{1/9, 1/8, \ldots, 1/2, 1, \ldots, 8, 9\}$ in accordance with the fundamental scale proposed in (Saaty, 2008) (see Table 1). If the *i*-th objective has no less effect on the *p*-th one than the *j*-th objective, then a_{ijp}^q value is taken from the table, otherwise an expert estimates prevalence of the j-th objective over the i-th one in accordance with the table and reciprocal value is taken for a_{ijp}^q.

Table 1: The fundamental scale of absolute numbers (adapted from (Saaty, 2008))

Intensity of Importance	*Definition*	*Explanation**
1	Equal importance	Accomplishments of the i-th and the j-th objectives contribute equally to the p-th one
2	Weak or slight	
3	Moderate importance	Experience and judgement slightly favour an accomplishment of the i-th objective over the j-th one
4	Moderate plus	
5	Strong importance	Experience and judgement strongly favour an accomplishment of the i-th objective over the j-th one
6	Strong plus	
7	Very strong or demonstrated importance	An accomplishment of the i-th objective is favoured very strongly over the j-th one; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favouring accomplishment of the i-th objective over the j-th one is of the highest possible order of affirmation

****Explanation is given for the case when the i-th objective has no less effect on the p-th one than the j-th objective.***

An expert is able to give his/her answer in the form of interval if accurate estimation is difficult. For example, if the q-th expert believe that an achievement of the i-th objective is more important for an achievement of the p-th objective than an achievement of the j-th objective with the prevalence of the i-th objective over the j-th one varying from «Moderate» to «Strong» then his assessment can be fixed as interval $[\underline{a}_{ijp}^q, \overline{a}_{ijp}^q] := [3, 5]$. If the q-th expert gave an accurate assessment a_{ijp}^q then $\underline{a}_{ijp}^q = \overline{a}_{ijp}^q := a_{ijp}^q$. An output of this step is a set of intervals $[\underline{a}_{ijp}^q, \overline{a}_{ijp}^q]$ $(i, j = \overline{1, n_p},\ i > j, q = \overline{1, m_p})$.

Step 1.6. Given that some favorable developments may occur (adjective "favorable" means that developments have a positive impact on an achievement of the p-th objective in addition to an achievement of its subordinate objectives), ask each expert to estimate an impact of such events on an achievement of the p-th objective in comparison with the influence of the i-th objective's achievement $(i = \overline{1, n_p})$. A result of this step is a set of intervals $[\underline{a}_{(n_p+1)jp}^q, \overline{a}_{(n_p+1)jp}^q]$ $(j = \overline{1, n_p},\ q = \overline{1, m_p})$.

Step 1.7. Similarly to the previous step obtain estimates of impact of unfavorable developments (adjective "unfavorable" means that they limit an achievement level of the p-th objective) compared with its subordinate objectives and favorable events (if $N_p = \emptyset$ then it is only estimate that should be obtained). An output of this step is a set of intervals $[\underline{a}_{(n_p+2)jp}^q, \overline{a}_{(n_p+2)jp}^q]$ $(j = \overline{1, n_p + 1}, q = \overline{1, m_p})$.

Step 1.8. With data collected in Steps 1.5–1.7, given that $\underline{a}_{iip}^{q} = \overline{a}_{iip}^{q} = 1, \underline{a}_{ijp}^{q} = 1/\overline{a}_{jip}^{q}, \overline{a}_{ijp}^{q} = 1/\underline{a}_{jip}^{q}$ $(i, j = \overline{1, n_p'})$, for the p-th objective and the q-th expert $(q = \overline{1, m_p})$ form an interval judgment matrix:

$$A_p^q = ([\underline{a}_{ijp}^{q}, \overline{a}_{ijp}^{q}])_{n_p' \times n_p'} = \begin{pmatrix} [1,1] & \cdots & [\underline{a}_{1n_p'p}^{q}, \overline{a}_{1n_p'p}^{q}] \\ \vdots & [1,1] & \vdots \\ [\underline{a}_{n_p'1p}^{q}, \overline{a}_{n_p'1p}^{q}] & \cdots & [1,1] \end{pmatrix}.$$

Remark 1. Possibility to give interval estimates is particularly relevant in Steps 1.6 and 1.7 because an expert has to deal with potential events precise evaluation of which may give unreliable results.

Remark 2. If an expert is able to give variation intervals for estimated values without pairwise comparisons then Steps 1.5–1.8 should be omitted.

Step 1.9.Calculate individual estimates for the weight coefficients' variation intervals $[\underline{k}_{jp}^{q}, \overline{k}_{jp}^{q}] (j = \overline{1, n_p'}, \; q = \overline{1, m_p})$. For the q-th expert $(q = \overline{1, m_p})$: IF expert didn't use the procedure of pairwise comparisons THEN individual intervals have already been obtained ELSE according to the method proposed in (Entani, 2009) obtain individual estimates of variation intervals as a solution of the following LPP:

$$I_2 = \sum_{j=1}^{n_p'} \left(\overline{k}_{jp}^{q} - \underline{k}_{jp}^{q} \right) \to \min_k, \tag{2.1}$$

$$\text{s.t.} \;\; \sum_{j=\overline{1,n_p'}, j \neq i} \overline{k}_{jp}^{q} + \underline{k}_{ip}^{q} \geq 1, \;\; i = \overline{1, n_p'}, \tag{2.2}$$

$$\sum_{j=\overline{1,n_p'}, j \neq i} \underline{k}_{jp}^{q} + \overline{k}_{ip}^{q} \leq 1, \;\; i = \overline{1, n_p'}, \tag{2.3}$$

$$\underline{k}_{ip}^{q} \leq \underline{a}_{ijp}^{q} \overline{k}_{jp}^{q}, \;\; i, j = \overline{1, n_p'}, \tag{2.4}$$

$$\overline{k}_{ip}^{q} \geq \overline{a}_{ijp}^{q} \underline{k}_{jp}^{q}, \;\; i, j = \overline{1, n_p'}, \tag{2.5}$$

$$\underline{k}_{ip}^{q} \geq \varepsilon, \;\; i = \overline{1, n_p'}, \tag{2.6}$$

where ε is a small constant (for example, $\varepsilon \coloneqq 0.0001$).

Step 1.10. Calculate aggregated estimates for the weight coefficients' variation intervals $[\underline{k}_{jp}, \overline{k}_{jp}] (j = \overline{1, n_p'})$: IF coefficients of competence are assumed to be uncertain THEN go to Step 1.10.1 ELSE go to Step 1.10.2.

Step 1.10.1. Calculate aggregated estimates for the weight coefficients' variation intervals according to the formulae: $\underline{k}_{jp} = \min_{q=\overline{1,m_p}} \underline{k}_{jp}^{q}, \; \overline{k}_{jp} = \max_{q=\overline{1,m_p}} \overline{k}_{jp}^{q}$ $(j = \overline{1, n_p'})$. Go to the Step 1.11.

Step 1.10.2. Calculate aggregated estimates for the weight coefficients' variation intervals according to the formulae: $\underline{k}_{jp} = \sum_{q=1}^{m_p} c_q \underline{k}_{jp}^{q}, \; \overline{k}_{jp} = \sum_{q=1}^{m_p} c_q \overline{k}_{jp}^{q}$ $(j = \overline{1, n_p'})$.

So, the result of Steps 1.2–1.10 is a set of aggregated variation intervals for values of the coefficients of cause-effect relationships between the p-th objective and its subordinates as well as aggregated interval values of an impact of other favorable and unfavorable factors which influence an achievement of the p-th objective: $[\underline{k}_{jp}, \overline{k}_{jp}]$ $(j = \overline{1, n'_p})$.

Remark 3. For the obtained intervals the following inequalities are satisfied:

$\sum_{j=1}^{n'_p} \overline{k}_{jp} \geq 1, \sum_{j=1}^{n'_p} \underline{k}_{jp} \leq 1.$

Step 1.11. IF $p < n$ THEN $p := p + 1$, go to Step 1.2 ELSE go to Stage 2.

Remark 4. It can be assumed that the strategy map is "closed" (no external factors affect an achievement of the objectives). If so, we don't have to obtain values of an impact of favorable and unfavorable factors to achievement of organizational objectives (they are equal to 0).

STAGE 2: calculation of the optimal resource allocation on condition that interval values of the cause-effect relationship coefficients as well as interval values of an impact of other favorable and unfavorable factors are provided.

Interval character of the model parameters introduces uncertainty to the resource allocation problem: it is given that $k_{e_j(i)j} \in [\underline{k}_{ij}, \overline{k}_{ij}]$ $(j = \overline{1,n}, i = \overline{1, n'_j})$ and $\sum_{i \in N_j} k_{ij} + k_{(n+1)j} + k_{(n+2)j} = 1$ $(j = \overline{1,n})$, but it isn't known what value from the intervals the coefficients will possess. To eliminate uncertainty it is necessary to use special criteria. The first criterionwhich can be used is based upon extreme optimism principle (maximax criterion). According to this criterion resource allocation is optimal if it provides the best result (word "result" means the weight sum of strategic objectives' accomplishment levels, see formula (1.1)), under the assumption that actual values of uncertain coefficients will be the most favorable. So, its usage gives an opportunity to evaluate the maximum possible result of the strategy developed (considering that resource amounts are limited) and allocate resources so that this result can be realized under favorable conditions. If decision maker isn't satisfied with it, then the strategy has to be revised (or resource amounts have to be increased).

It is obvious that for finding the optimal maximax resource allocation, structural constraints (1.3) in the problem (1.1)–(1.4) must be replaced by the following constraints:

$$x_j \leq \max_{k_j \in Q_j} \left(\sum_{i=1}^{n_j} k_{ij}\, x_{e_j(i)} + k_{(n_j+1)j}\right),\ j = \overline{1,n}, \tag{3.1}$$

$$Q_j = \{k_j \mid \sum_{i=1}^{n'_j} k_{ij} = 1, \underline{k}_{ij} \leq k_{ij} \leq \overline{k}_{ij}, i = \overline{1, n'_j}\}. \tag{3.2}$$

The right side of inequality (3.1) can be interpreted as an LPP with respect to variables $k_{1j}, k_{2j}, \dots, k_{n'_j j}$ provided that the point $k_j = (k_{1j}, k_{2j}, \dots, k_{n'_j j})$ belongs to the polygon Q_j. Using the property of the LPP that the optimal k_j is one of the extreme points of Q_j, constraints (3.1),(3.2) can be replaced by the following ones:

$$x_j \le \delta_j^p \left(\sum_{i=1}^{n_j} k_{ij}^{p*} x_{e_j(i)} + k_{(n_j+1)j}^{p*} - 1\right) + 1,\ j = \overline{1,n}, p = \overline{1,q_j}, \tag{4.1}$$

$$\delta_j^p \in \{0,1\},\ j = \overline{1,n}, p = \overline{1,q_j}, \tag{4.2}$$

$$\sum_{p=1}^{q_j} \delta_j^p \ge 1,\ j = \overline{1,n}, \tag{4.3}$$

where $\{k_j^{p*}\}_{p=1}^{q_j} = Q_j^*$ is the set of extreme points of the polygon Q_j ($q_j = |Q_j|$).

The solution of (1.1),(1.2),(4.1)–(4.3),(1.4) with respect to variables x and δ and the corresponding resource allocation satisfy the principle of extreme optimism.

Another criterion which allows eliminating uncertainty is based upon extreme pessimism principle (maximin or Wald criterion). According to it resource allocation is optimal if it provides the best result in the assumption that the actual values of uncertain parameters will be the least favorable. Usage of this criterion thereby gives an opportunity to understand what is the maximum guaranteed result of the strategy developed (considering that resource amounts are limited) and allocate resources so that this result will be achieved under *any* conditions.

To find the optimal maximin resource allocation, structural constraints (1.3) in (1.1)–(1.4) must be replaced by constraints like (3.1),(3.2), with maximization being replaced by minimization. They, in turn, can be transformed to the following constraints:

$$x_j \le \sum_{i=1}^{n_j} k_{ij}^{p*} x_{e_j(i)} + k_{(n_j+1)j}^{p*},\ j = \overline{1,n}, p = \overline{1,q_j}. \tag{5}$$

The solution of (1.1),(1.2),(5),(1.4) with respect to variable x and the corresponding resource allocation satisfy the principle of extreme pessimism.

In view of these considerations the following actions must be performed within the scope of Stage 2.

Step 2.1. Construct Q_j^* which is the set of the extreme points of the polygon Q_j ($j = \overline{1,n}$). Thereto:

1. Construct the set of points Q_j':

$$Q_j' = \{k_j' | k_{ij}' \in \{\underline{k}_{ij}, \overline{k}_{ij}\}\}.$$

2. For each $i = \overline{1,n_j'}$ construct the set of points Q_{ij}'':

$$Q_{ij}'' = \{k_j'' | k_{(p=i)j}'' = 1 - \sum_{q=\overline{1,n_j'},q\ne i} k_{qj}', k_{(p\ne i)j}' = k_{pj}', k_j' \in Q_j', p = \overline{1,n_j'}\}.$$

3. Construct the required set of the extreme points of the polygon Q_j:

$$Q_j^* = Q_j \cap (\cup_{i=1}^{n_j'} Q_{ij}'').$$

Step 2.2. Formulate the problem (6.1)–(6.8):

$$I_3 = \alpha \underbrace{\sum_{j=1}^{n} w_j (x_j^G + x_j^O)}_{I_3^O} + (1-\alpha) \underbrace{\sum_{j=1}^{n} x_j^G w_j}_{I_3^G} + \varepsilon \sum_{j=1}^{n} \sum_{p=1}^{q_j} \delta_j^p \to \max_{x^O, x^G, \delta}, \quad (6.1)$$

s.t.

$$\sum_{j=1}^{n} r_{ij} (x_j^G + x_j^O) \le R_i\,,\ i = \overline{1,s}, \quad (6.2)$$

$$x_j^G \le \sum_{i=1}^{n_j} k_{ij}^{p*} x_i^G + k_{(n_j+1)j}^{p*},\ j = \overline{1,n}, p = \overline{1,q_J}, \quad (6.3)$$

$$x_j^G + x_j^O \le \delta_j^p \left(\sum_{i=1}^{n_j} \left(k_{ij}^{p*} (x_i^G + x_i^O) \right) + k_{(n_j+1)j}^{p*} - 1 \right) + 1,\ j = \overline{1,n}, p = \overline{1,q_J}, \quad (6.4)$$

$$\delta_j^p \in \{0,1\},\ j = \overline{1,n}, p = \overline{1,q_J}, \quad (6.5)$$

$$\sum_{p=1}^{q_j} \delta_j^p \ge 1,\ j = \overline{1,n}, \quad (6.6)$$

$$0 \le x_j^G, x_j^O,\ j = \overline{1,n}, \quad (6.7)$$

$$x_j^G + x_j^O \le 1,\ j = \overline{1,n}, \quad (6.8)$$

where ε is a small constant (for example, $\varepsilon := 0.0001$).

Step 2.3. Find the maximax result solving the problem (6.1)–(6.8) in which the parameter α is taken near to 1, but $1-\alpha$ is much greater than ε (for example, $\alpha := 0.99$ if $\varepsilon = 0.0001$) [1]. Fix the obtained optimal values of sub-criteria I_3^O and I_3^G as a pair $(I_{3(\alpha=1-)}^{O*}, I_{3(\alpha=1-)}^{G*})$, where $I_{3(\alpha=1-)}^{O*}$ is the maximax result, $I_{3(\alpha=1-)}^{G*}$ is a result which will be obtained at the worst actual values of the coefficients $\{k_{ij}\}$, if resources are allocated so as to provide the maximax result.

Technique for solving the problem (6.1)–(6.8) is quite simple. Constraints (6.4) are converted to linear ones with the method proposed in (Glover, 1975): each product of binary and continuous variable δx (δ is binary, *x* is continuous) is replaced by continuous variable *z* constrained with linear inequalities:

$$L\delta \le z \le U\delta,$$

$$x - U(1-\delta) \le z \le x - L(1-\delta),$$

where *L* and *U* are the lower and upper bounds for *x*, respectively (in our case $L = 0, U = 1$).

Derived mixed 0-1 LPP can be solved, for example, with the branch and bound method or some specialized algorithm (see, for example (Eckstein, Nediak, 2007), (Wilbaut, Hanafi, 2009)).

Step 2.4. Find maximin result, solving the problem (6.1)–(6.8) in which the parameter α is taken near to zero, but it is much greater than ε (for example, $\alpha := 0.01$ if $\varepsilon = 0.0001$). Fix the obtained optimal values of sub-criteria I_3^O and I_3^G as a pair $(I_{3(\alpha=0+)}^{O*}, I_{3(\alpha=0+)}^{G*})$, where $I_{3(\alpha=0+)}^{G*}$ is maximin (maximum guaranteed) result, $I_{3(\alpha=0+)}^{O*}$ is the result which will be obtained at the best actual values of the coefficients $\{k_{ij}\}$, if resources are allocated so as to provide the maximin result.

[1] The above combination of parameters ensures that the resources will be distributed such that: in the first place, provide the maximax result, in the second place (if there are variety resource allocations which ensure the maximax result), provide the maximum guaranteed result, in the third place, ensure that the maximax result will be achived at the maximum number of combinations of points from the sets $\{Q_j^*\}_{j=1}^{n}$.

Step 2.5. Find the set of compromise results in which there is a balance between maximum possible and guaranteed results. To do this, for each $q = \overline{1, d-1}$, where d is the number of segments in partition of the interval $[I^{O*}_{3(\alpha=0+)}, I^{O*}_{3(\alpha=1-)}]$, formulate and solve the problem (6.1)–(6.8),(7.q) with $\alpha := 0$. Constraint (7.q) has the following form:

$$\sum_{j=1}^{n}(x_j^G + x_j^O)w_j \geq I^{O*}_{3(\alpha=0+)} + q\,\frac{I^{O*}_{3(\alpha=1-)} - I^{O*}_{3(\alpha=0+)}}{d}, \qquad (7.q)$$

Similarly to Steps 2.3 and 2.4 fix the set of pairs $(I^{O(q)*}_{3(\alpha=0)}, I^{G(q)*}_{3(\alpha=0)})_{q=1}^{d-1}$.

Step 2.6. Plot the points $(I^{O*}_{3(\alpha=0+)}, I^{G*}_{3(\alpha=0+)})$, $(I^{O(q)*}_{3(\alpha=0)}, I^{G(q)*}_{3(\alpha=0)})_{q=1}^{d-1}$, $(I^{O*}_{3(\alpha=1-)}, I^{G*}_{3(\alpha=1-)})$ on the plane, drawing a two-dimensional graph in axes "Best possible result"–"Guaranteed result". Select the points for which closed north-east corner is empty (Pareto set). Decision maker based on his/her own preferences (willingness to sacrifice guaranteed result for better results at the favorable values of the coefficients) has to select some Pareto-optimal point. The optimal resource allocation is calculated with the formula $r_{ij}(x_j^{G*} + x_j^{O*})$ $(j = \overline{1,n})$, where x_j^{G*} and x_j^{O*} are solutions corresponding to the point selected.

4. NUMERICAL EXAMPLE

To demonstrate the application of the method proposed we will use the cause-effect relationships graph suggested in (Hell, 2008) (see Fig. 2). M. Hell optimized an allocation of resources of three types considering that $k_{ij} = 1/|N_j|$ $(j = \overline{1,n})$.

Suppose that the system is closed and there is no information about values of the cause-effect relationship coefficients (it's known only that if $|N_j| > 1$ then $k_{ij} \in [0, 1]$, $j = \overline{1,n}$,)[2]. What is the optimal resource allocation in this case? With the method proposed we calculated ten Pareto-optimal points (see Fig. 3).[3]

[2]Of course, during the real examination, it is unlikely that the aggregated expert opinions would be the intervals [0, 1]. This would increase the maximin result, but reduce the maximax one.

[3]All calculations were performed with the C#-program developed in Microsoft Visual Studio 2010. To solve mixed 0-1 LPP open-source library *lpsolve* was employed.

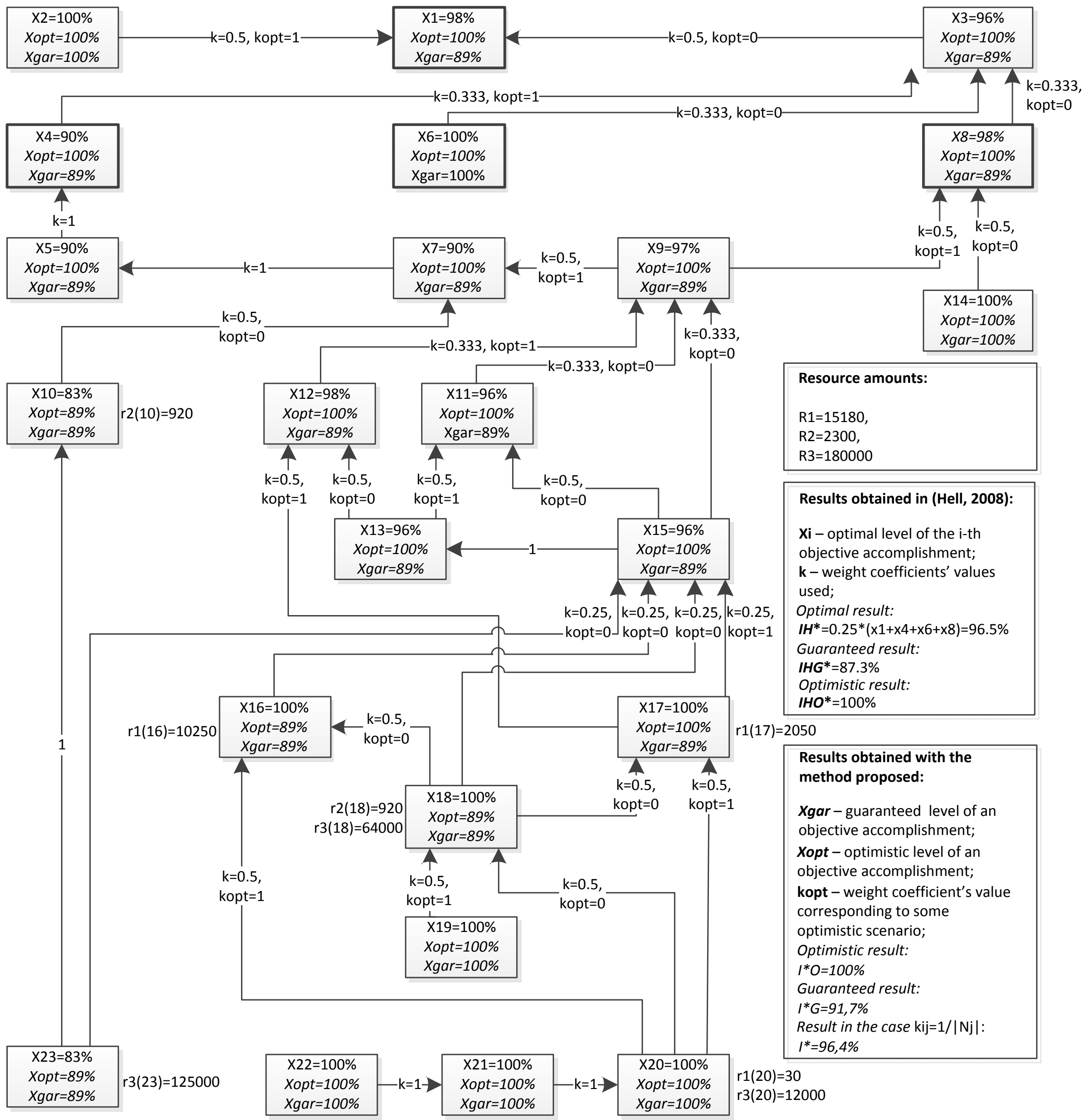

Figure 2: The cause-effect relationships graph (adapted from (Hell, 2008))

The graph shows that at the most favorable values of the coefficients $\{k_{ij}\}$ result can reach 100% and if resources are allocated so as to provide this result, guaranteed result will be equal to 91,7% (point "A").

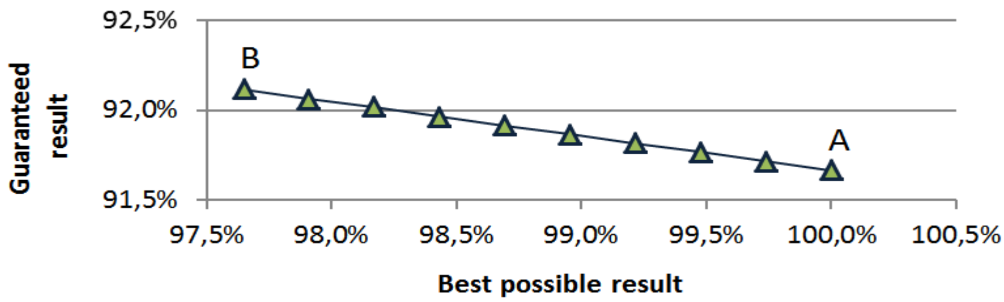

Figure 3: Pareto-optimal points

The graph shows that at the most favorable values of the coefficients $\{k_{ij}\}$ result can reach 100% and if resources are allocated so as to provide this result, guaranteed result will be equal to 91,7% (point "A"). On the other hand, resources can be allocated so as to ensure guaranteed result which is equal to 92,2%, however, at the most favorable values of the relationship coefficients, the result will be only 97,4% (point "B").

Thus the strategy proposed in (Hell, 2008) is balanced so good that

1) if resources are allocated according to some of the found points then the result will be more than 91,7% at *any* permissible values of the cause-effect relationship coefficients;
2) if resources are allocated in such a way to give maximum guaranteed result then the result will differ from the potential one if resources are allocated under full awareness (all coefficients are defined precisely) not more then by 7,8% = 100% – 92,2% (in fact, it's very crude estimation).

Losing 0.5% of guaranteed result in point "A" versus "B" we can win 2,6% under the most favorable circumstances, so we take point "A" as the optimal point (respective objectives' achievement levels, resource allocation and the values of the cause-effect relationship coefficients under some optimistic scenario are shown in Fig. 2). It's interesting that the resource allocation calculated in (Hell, 2008) will give the guaranteed result is equal to 87,3% (less than our guaranteed result by 4,4%) and best possible is equal to 100% (equal to our best possible result), whereas our allocation will give the result which is equal to 96,4% under the assumption that $k_{ij} = 1/|N_j|$ (less than the result of M. Hell only by 0,1%).

5. CONCLUSIONS AND FURTHER RESEARCH

This paper proposes the method to obtain values of the coefficients of cause-effect relationships between strategic objectives in the form of intervals and uses them in solving the problem of the optimal allocation of organization's resources. It should be noted that variation intervals can be obtained for the other parameters of QMSP (amounts of available resources $\{R_j\}$ and technological coefficients $\{r_{ij}\}$) and used in planning of strategy's implementation. Further research, in our opinion, should be associated with employing 1) Savage criterion which is based upon loss profit minimization principle; 2) stochastic programming methods that allow finding such resource allocation that maximizes the probability of the event A={result will be more than the fixed threshold $I \in \left[I^{G*}_{3(\alpha=0+)}, I^{O*}_{3(\alpha=1-)}\right]$}.

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